PROBABILITY DISTRIBUTIONS IN IMAGE SEGMENTATION

Prof. K. SRINIVASA RAO
Department of Statistics
Andhra University
Visakhapatnam
ksraoau@yahoo.co.in

WE DISCUSS

- **Need for Image Segmentation**
- **Image segmentation based on Mixture distributions**
- **Estimation of model parameters**
- **Initialization of model parameters**
- **Segmentation Algorithm**
- **Performance evaluation**
- **Comparative study**
- **Scope for further study**

IMAGE

The optical appearance of something produced in a mirror or through a lens is known as image.

DIGITAL IMAGE

- The concept of digital image was found in literature as early as in 1920.

In low level image analysis the entire image is considered as a union of several image regions. In each image region the image data is quantized by pixel intensities.

- The pixel intensity $z = f(x , y)$ for a given point (pixel), z is a random variable, because of the fact that the brightness measured at a point in the image is influenced by various random factors like vision, lighting, moisture, environmental conditions etc,.

- Digital image is a matrix, where each number represents the brightness at regularly spaced points in the image.
- These points are called pixels and the brightness value of a pixel is called its grey level.

Why Image Analysis

- The aim of image processing applications is to extract important features from image data from which a description, interpretation or understanding of the scene can be provided by the Machine .

- Image analysis helps to find the relationship between the objects inside the image. The image processing operations help in better recognition of object of interest. After identifying the objects of interest, the next step is to check whether each pixel belongs to the object of interest or not.

What is Image Analysis

- The first step of image analysis is to divide the image into regions so that various features such as size, shape, color, texture can be measured , and these features in turn can be used as inputs for classification.

- Image analysis involves

  i) feature extraction ii) Segmentation

IMAGE SEGMENTATION

- Image segmentation refers to decomposition of a scene into different components.
- Segmentation is a process of partitioning the image into non-intersecting regions such that each region is homogenous and the union of no two regions is homogenous.
- Several Segmentation techniques have been developed and utilized for image analysis, but there is no unique segmentation procedure, which serve all the situations. It is a key step to image analysis.

USES OF IMAGE SEGMENTATION

- **Image Understanding (Content Identification)**
- **Image Retrieval**

Applications of Image Segmentation

- **Medical Diagnostics**
- **Remote sensing**
- **Robotics**
- **Filming and Video**
- **Industrial Automation**
- **Animation**

SEGMENTATION METHODS

- Image Segmentation based on Histogram, Threshold and edge based techniques .
- Model based image segmentation methods.
- Image Segmentation based on other methods (graph, neural Networks, Fuzzy logic, genetic algorithms, saddle points etc.,).
- Broadly image segmentation can be classified into two categories namely, parametric and non-parametric image segmentation.
- Model based image segmentation is more efficient compared to the non-parametric methods of segmentation.

MODEL BASED IMAGE SEGMENTATION

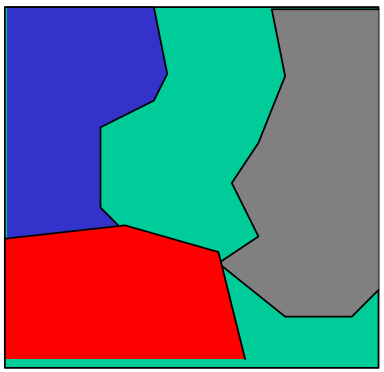
Image segmentation based on Gaussian or Gaussian Mixture model

- Yamazaki et al(1998)
- Jan puzicha et al(1998)
- Figureido et al(1999)
- Rahman Farnoosh et al(2000)
- Jacob et al(2002)
- Permuter.H et al (2003)
- Yudi Augusta (2003)
- Abhir et al(2003)
- Alfonos et al(2004)
- Belkas.K et al(2005)
- Rahman Farnoosh et al(2006)

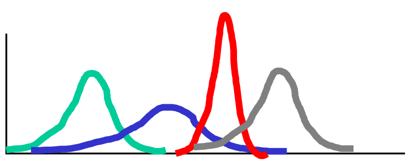
MODEL BASED IMAGE SEGMENTATION

- Much emphasis is given for image analysis through finite Gaussian mixture model. In finite Gaussian mixture model each image region is characterized by a Gaussian distribution and the entire image is considered to be a mixture of these Gaussian components. They assumed that the whole image is characterized by Gaussian mixture model in which the pixel intensities of each image region follows a Gaussian distribution.

Image Segmentation



Segment 1, θ_1
Segment 2, θ_2
Segment 3, θ_3
Segment 4, θ_4



π_l: Probability of choosing segment l at random (*a priori*)

$p(\mathbf{x}|\theta_l)$: Conditional density of feature vector **x**, given that it comes from segment *l, l*=1,…*g*

Model: $p(\mathbf{x}|\theta_l)$ is Gaussian, $\theta_l=(\mu_l,\Sigma_l)$

The total density for the feature vector of any pixel drawn at random…

$$p(\mathbf{x}) = \sum_l p(\mathbf{x} \mid \theta_l)\pi_l$$

This is known as a ***Mixture Model***

Parameter vector:

$$\Theta = (\alpha_1, \alpha_2, \ldots, \alpha_g, \theta_1, \theta_2, \ldots, \theta_g)$$

mixing weights ***Prameters***

The mixture model becomes:

$$p(\mathbf{x} \mid \Theta) = \sum_{l=1}^{g} \alpha_l p_l(\mathbf{x} \mid \theta_l)$$

With each component is Gaussian:

$$P_l(Z) = \frac{1}{\sqrt{2\pi\sigma}} \; e^{-\frac{1}{2}\left(\frac{z_l - \mu_l}{\sigma_l}\right)^2} \quad ; \; -\infty < z + \infty$$

$$-\infty < \mu_l + \infty \; ; 0 < \sigma_l$$

IMAGE SEGMENTATION

- A more comprehensive discussion on model based image segmentation is given by Pal S.K. & Pal N.R (1993) and Jahne (1995).

- There does not exist a single algorithm that works for all applications.

IMAGE SEGMENTATION

- In finite Gaussian mixture model the pixel intensities of the image region are considered to be meso-kurtic and symmetric. But in some images the pixel intensities of the image region may not be distributed as meso – kurtic even though they are symmetric.
- To have a more close approximation to the pixel intensities of each image region it is needed to consider that the pixel intensities of each region follows a non-Gaussian symmetric distribution .
- In Non-Gaussian symmetric distributions the kurtosis plays a vital role.
- In natural images the pixel intensities follows symmetric and platy kurtic distributions.
- Hence it is needed to develop image segmentation methods based on platy kurtic distributions.

Srinivasa Rao and Sheshashayee (2011a, 2011b) Developed and analysed Image segmentation methods based on Mixture of new symmetric mixture distribution.

NEW SYMMETRIC MIXTURE DISTRIBUTION

- It is assumed that the whole image consisting of several(K) image regions and the pixel intensities in each image region follows a new symmetric distribution.
- The probability density function of the pixel intensity in the image region is

$$f(z,\mu,\sigma^2) = \frac{\left(2+\left(\frac{z-\mu}{\sigma}\right)^2\right)e^{\frac{-1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}}{3\sigma\sqrt{2\pi}}, \qquad (1)$$

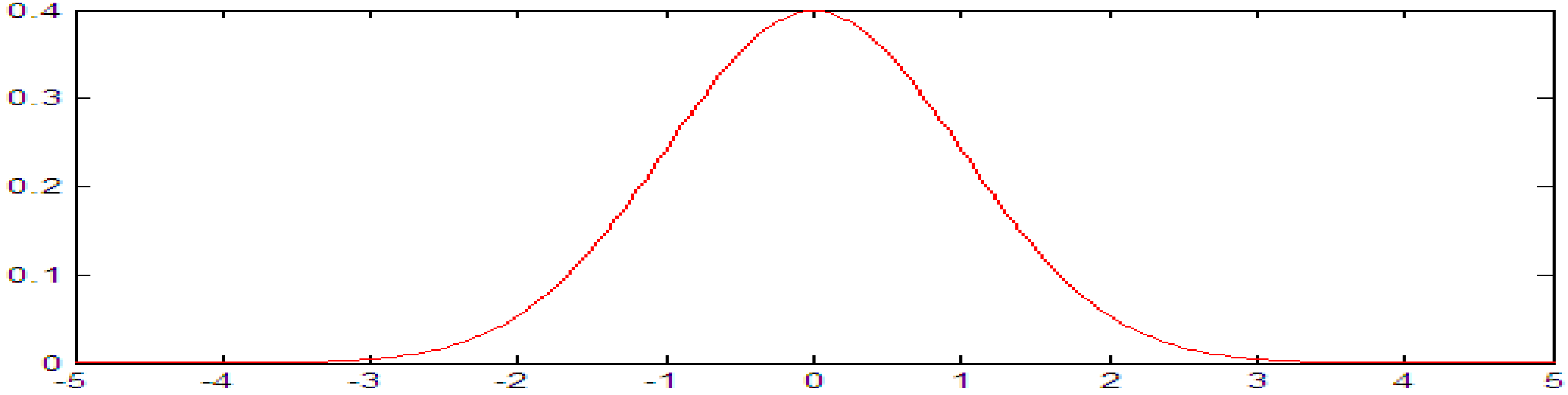
$$-\infty < z < \infty, -\infty < \mu < \infty, \sigma > 0$$

- The distribution function of the pixel intensity in the image region is

$$F(z;\mu,\sigma) = \frac{2}{3}\Phi\left(\frac{z-\mu}{\sigma}\right) - \frac{1}{3}e^{\frac{-1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}\left[1+\frac{(z-\mu)}{\sigma\sqrt{2\pi}}\right]$$

- where Φ is distribution function of the standard normal variate

Frequency curve of new symmetric distribution is



- Mean μ variance $= \left[\frac{5}{3}\right]\sigma^2$
- Kurtosis $\beta_2 = 2.52$
- It is a platy kurtic symmetric distribution.

NEW SYMMETRIC MIXTURE DISTRIBUTION

Since the whole image is collection of K image regions .
The pixel intensities of the image follow a new symmetric Mixture distribution.
The probability density function of new symmetric mixture distribution is

$$p(z) = \sum_{i=1}^{K} \alpha_i f_i(z / \mu_i, \sigma_i^2)$$

where, K is number of regions , $0 \leq \alpha_i \leq 1$ are weights such that $\sum \alpha_i = 1$ and $f_i(z, \mu, \sigma^2)$ is as given in equation (1). α_i is the weight associated with i^{th} region in the whole image

ESTIMATION OF THE MODEL PARAMETERS BY EM ALGORITHM

- we derive the updated equations of the model parameters using Expectation Maximization (EM) algorithm.
- The likelihood function of the observations $z_1, z_2, \ldots, z_N$ drawn from an image is

$$L(\theta)=\prod_{S=1}^{N} p(z_s, \theta^{(l)}), \qquad \log L(\theta) = \sum_{s=1}^{N} \log p(z_s, \theta^{(l)}) = \sum_{s=1}^{N} \log\left(\sum_{i=1}^{K} \alpha_i f_i(z_s, \theta_i) \right),$$

$\theta = (\mu_i, \sigma_i^2, \alpha_i; i = 1, 2, \ldots, K)$ is the parameter set

$$\log L(\theta) = \sum_{s=1}^{N} \log \left[\sum_{i=1}^{K} \frac{\alpha_i \left(2 + \left(\frac{z_s - \mu_i}{\sigma_i} \right)^2 \right) e^{\frac{-1}{2}\left(\frac{z_s - \mu_i}{\sigma_i} \right)^2}}{3\sigma_i \sqrt{2\pi}} \right]$$

The first step of the EM algorithm requires the estimation of the likelihood function of the sample observations.

E-STEP:

- In the expectation (E) step, the expectation value of log L(θ) with respect to the initial parameter vector $\theta^{(0)}$ is

$$Q\left(\theta;\theta^{(0)}\right)=E_{\theta^{(0)}}\left[\log L(\theta)\right]$$

- Given the initial parameters $\theta^{(0)}$, one can compute the density of pixel intensity z_i as

$$p(z_s,\theta^{(l)})=\sum_{i=1}^{K}\alpha_i^{(l)} f_i(z_s,\theta^{(l)}), \qquad \mathrm{L}(\theta)=\prod_{s=1}^{N} p(z_s,\theta^{(l)})$$

- This implies

$$\log L(\theta)=\sum_{s=1}^{N}\log\left(\sum_{i=1}^{K}\alpha_i f_i(z_s,\theta_i)\right)$$

- The conditional probability of any observation z_i, belongs to region K is

$$t_k(z_s,\theta^{(l)})=\left[\frac{\alpha_k^{(l)} f_k(z_s,\theta^{(l)})}{p(z_s,\theta^{(l)})}\right]=\left[\frac{\alpha_k^{(l)} f_k(z_s,\theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}\right]$$

$$Q\left(\theta;\theta^{(l)}\right)=\sum_{i=1}^{K}\sum_{s=1}^{N}\left(t_i(z_s,\theta^{(l)})\left(\log f_i(z_s,\theta^{(l)})+\log\alpha_i^{(l)}\right)\right)$$

M-STEP:

- For obtaining the estimates of the model parameters one has to maximize $Q\left(\theta;\theta^{(l)}\right)$ such that $\sum\alpha_i = 1$ with respect to the model parameters $\alpha_i, \mu_i, \sigma_i^2$

UPDATED EQUATIONS

- The updated equation of $\alpha_i, \mu_i, \sigma_i^2$ for $(\ell+1)^{\text{th}}$ iteration is

$$\alpha_i^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N}\left[\frac{\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}\right]$$

$$\mu_i^{(l+1)} = \frac{\sum_{s=1}^{N} z_s t_i(z_s,\theta^{(l)}) - \sum_{s=1}^{N} t_i(z_s,\theta^{(l)})\left(\frac{2(\sigma_i^2)^{(l)}\left(z_s-\mu_i^{(l)}\right)}{2(\sigma_i^2)^{(l)}+\left(z_s-\mu_i^{(l)}\right)^2}\right)}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})}$$

$$\left(\sigma_i^2\right)^{(l+1)} = \frac{2\sum_{s=1}^{N}(z_s-\mu_i^{(l+1)})^2\left(\frac{1}{2}-\frac{\left(\sigma_i^2\right)^{(l)}}{\left(2\left(\sigma_i^2\right)^{(l)}+\left(z_s-\mu_i^{(l+1)}\right)^2\right)^2}\right)\left(t_i(z_s,\theta^{(l)})\right)}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})}$$

where, $t_i(z_s,\theta^{(l)}) = \dfrac{\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l+1)},\left(\sigma_i^2\right)^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l+1)},\left(\sigma_i^2\right)^{(l)})}$

INITIALIZATION OF THE MODEL PARAMETERS

- To run EM algorithm we require initial estimates of the model parameters in each image region.

- The K-means algorithm / Hierarchal clustering and moment method of estimation is used for obtaining the initial estimates of the model parameters.

K-MEANS CLUSTERING ALGORITHM

1) Randomly choose K data points from the whole dataset as initial clusters. These data points represent initial cluster centroids.

2) Calculate Euclidean distance of each data point from each cluster centre and assign the data points to its nearest cluster centre

3) Calculate new cluster centre so that squared error distance of each cluster should be minimum.

4) Repeat step 2 and 3 until clustering centers do not change.

5) Stop the process.

- The initial estimate α_i is taken as $\alpha_i = \frac{1}{K}$, where i = 1,2,…,K. The parameters and are estimated by the method of moments as

$$\mu_i = \overline{z} \qquad \sigma_i^2 = \frac{4n_i}{3(n_i-1)} S^2$$ where, S^2 is the sample variance

SEGMENTATION ALGORITHM

- After refining the parameters, the prime step in image segmentation is allocating the pixels to the segments of the image. This operation is performed by Segmentation Algorithm. The image segmentation algorithm consists of four steps.
- Step 1) Plot the histogram of the whole image.
- Step 2) Obtain the initial estimates of the model parameters using K-Means algorithm and moment estimates for each image region .
- Step 3) Obtain the refined estimates of the model parameters for i=1,2, …, K using the EM algorithm with the updated equations.

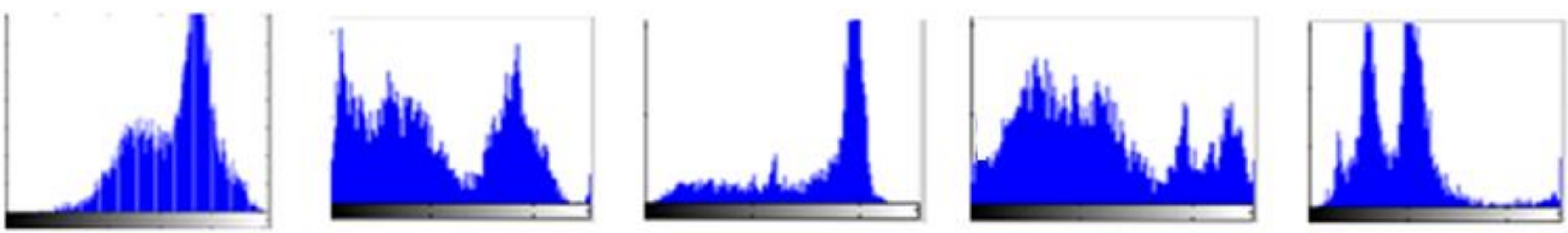
- Step 4) Assign each pixel into the corresponding j^{th} region (segment) according to the maximum likelihood of the j^{th} component L_j.

$$L_j = \max_{j \in k}\left[(3\sigma_j\sqrt{2\pi})^{-1}\left(2+\left(\frac{z_s-\mu_j}{\sigma_j}\right)^2\right)e^{\frac{-1}{2}\left(\frac{z_s-\mu_j}{\sigma_j}\right)^2}\right]$$

EXPERIMENTAL RESULTS

- To demonstrate the utility of the image segmentation algorithm developed in this chapter, an experiment is conducted with five images taken from Berkeley images dataset (http://www.eecs.berkeley.edu/Research/Projects/CS/Vision/bsds/BSDS300/html).

HISTOGRAMS OF THE IMAGES



HORSE MAN BIRD BOAT TOWER

ESTIMATION OF INITIAL PARAMETERS
ESTIMATION OF FINAL PARAMETERS BY EM ALGORITHM

Table-4.2.a

Estimated Values Of The Parameters For HORSE Image

Number of Image Regions (K =2)

Parameters	Estimation of Initial Parameters		Estimation of Final Parameters by EM Algorithm	
	Image Region		Image Region	
	1	2	1	2
α_i	0.500	0.500	0.410	0.590
μ_i	125.155	189.812	147.310	175.320
σ_i^2	703.538	357.761	1741.700	903.460

Table-4.2.b

Estimated Values Of The Parameters For MAN Image

Number of Image Regions (K =4)

Parameters	Estimation of Initial Parameters				Estimation of Final Parameters by EM Algorithm			
	Image Region				Image Region			
	1	2	3	4	1	2	3	4
α_i	0.250	0.250	0.250	0.250	0.417	0.289	0.002	0.292
μ_i	84.07	21.790	251.720	184.300	86.223	44.455	251.950	183.500
σ_i^2	904.78	199.370	27.973	443.520	2849.200	901.960	8.849	488.280

Table-4.2.c

Estimated Values Of The Parameters For BIRD Image

Number of Image Regions (K =3)

Parameters	Estimation of Initial Parameters			Estimation of Final Parameters by EM Algorithm		
	Image Region			Image Region		
	1	2	3	1	2	3
α_i	0.333	0.333	0.333	0.265	0.046	0.689
μ_i	43.840	190.620	103.850	66.366	187.530	91.638
σ_i^2	281.550	287.630	501.660	874.040	3722.300	864.320

Table-4.2.d

Estimated Values Of The Parameters For BOAT Image

Number of Image Regions (K =4)

Parameters	Estimation of Initial Parameters				Estimation of Final Parameters by EM Algorithm			
	Image Region				Image Region			
	1	2	3	4	1	2	3	4
α_i	0.250	0.250	0.250	0.250	0.070	0.320	0.395	0.215
μ_i	16.070	121.300	61.870	214.810	39.908	114.790	78.874	203.640
σ_i^2	102.800	489.780	351.810	734.930	723.850	3463.90	1535.300	1270.900

Table-4.2.e

Estimated Values Of The Parameters For TOWER Image

Number of Image Regions (K =3)

Parameters	Estimation of Initial Parameters			Estimation of Final Parameters by EM Algorithm		
	Image Region			Image Region		
	1	2	3	1	2	3
α_i	0.333	0.333	0.333	0.967	0.027	0.006
μ_i	84.940	238.480	183.010	84.690	234.440	153.650
σ_i^2	1186.800	281.890	159.120	948.690	282.420	5080

ESTIMATES OF THE PROBABILITY DENSITY FUNCTION OF THE IMAGES

- The estimated probability density function of the pixel intensities of the image HORSE is

$$f\left(z_s,\theta^{(l)}\right)=\frac{(0.410)\left(2+\left(\frac{z_s-147.310}{41.733}\right)^2\right)^1 e^{\frac{-1}{2}\left(\frac{z_s-147.310}{41.733}\right)^2}}{(125.199)\sqrt{2\pi}}+\frac{(0.590)\left(2+\left(\frac{z_s-175.320}{30.057}\right)^2\right) e^{\frac{-1}{2}\left(\frac{z_s-175.320}{30.057}\right)^2}}{(90.171)\sqrt{2\pi}}$$

- The estimated probability density function of the pixel intensities of the image MAN is

$$f\left(z_s,\theta^{(l)}\right)=\frac{(0.4167)\left(2+\left(\frac{z_s-86.223}{53.377}\right)^2\right) e^{\frac{-1}{2}\left(\frac{z_s-86.223}{53.377}\right)^2}}{(160.131)\sqrt{2\pi}}+\frac{(0.289)\left(2+\left(\frac{z_s-44.455}{30.032}\right)^2\right) e^{\frac{-1}{2}\left(\frac{z_s-44.455}{30.032}\right)^2}}{(90.097)\sqrt{2\pi}}+\frac{(0.002)\left(2+\left(\frac{z_s-251.950}{2.974}\right)^2\right) e^{\frac{-1}{2}\left(\frac{z_s-251.950}{2.974}\right)^2}}{(8.922)\sqrt{2\pi}}+\frac{(0.292)\left(2+\left(\frac{z_s-183.500}{22.097}\right)^2\right)^2 e^{\frac{-1}{2}\left(\frac{z_s-183.500}{22.097}\right)^2}}{(66.291)\sqrt{2\pi}}$$

- The estimated probability density function of the pixel intensities of the image BIRD is

$$f\left(z_s,\theta^{(l)}\right)=\frac{(0.265)\left(2+\left(\frac{z_s-66.366}{29.564}\right)^2\right)e^{\frac{-1}{2}\left(\frac{z_s-66.366}{29.564}\right)^2}}{(88.692)\sqrt{2\pi}}+\frac{(0.046)\left(2+\left(\frac{z_s-187.530}{61.010}\right)^2\right)e^{\frac{-1}{2}\left(\frac{z_s-187.530}{61.010}\right)^2}}{(183.030)\sqrt{2\pi}}+\frac{(0.689)\left(2+\left(\frac{z_s-91.638}{29.399}\right)^2\right)^1 e^{\frac{-1}{2}\left(\frac{z_s-91.638}{29.399}\right)^2}}{(88.197)\sqrt{2\pi}}$$

- The estimated probability density function of the pixel intensities of the image BOAT is

$$f\left(z_s,\theta^{(l)}\right)=\frac{(0.070)\left(2+\left(\frac{z_s-39.908}{26.904}\right)^2\right)e^{\frac{-1}{2}\left(\frac{z_s-39.908}{26.904}\right)^2}}{(80.713)\sqrt{2\pi}}+\frac{(0.320)\left(2+\left(\frac{z_s-114.790}{58.854}\right)^2\right)e^{\frac{-1}{2}\left(\frac{z_s-114.790}{58.854}\right)^2}}{(176.562)\sqrt{2\pi}}+\frac{(0.395)\left(2+\left(\frac{z_s-78.874}{39.182}\right)^2\right)e^{\frac{-1}{2}\left(\frac{z_s-78.874}{39.182}\right)^2}}{(117.546)\sqrt{2\pi}}+\frac{(0.215)\left(2+\left(\frac{z_s-203.640}{35.649}\right)^2\right)e^{\frac{-1}{2}\left(\frac{z_s-203.640}{35.649}\right)^2}}{(106.947)\sqrt{2\pi}}$$

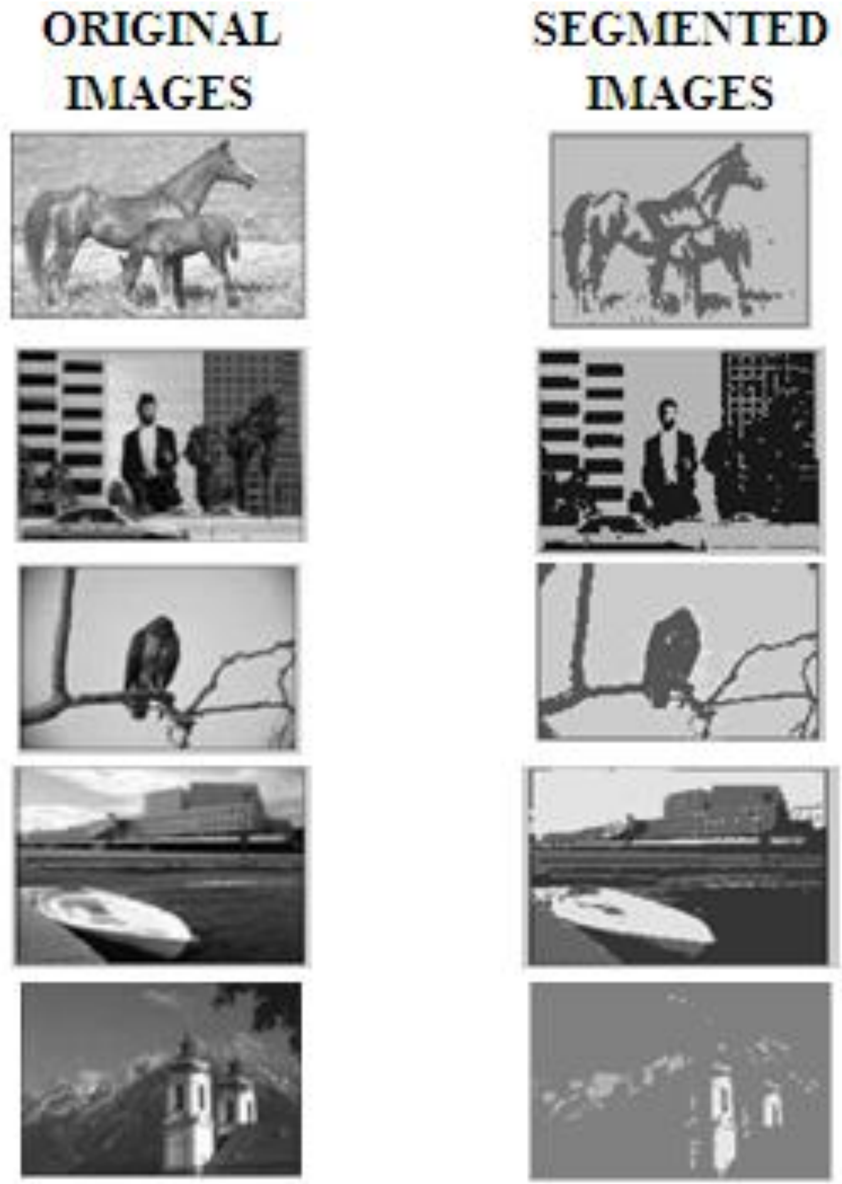
- The estimated probability density function of the pixel intensities of the image TOWER is

$$f\left(z_s, \theta^{(l)}\right) = \frac{(0.967)\left(2+\left(\frac{z_s - 84.690}{30.800}\right)^2\right) e^{\frac{-1}{2}\left(\frac{z_s - 84.690}{30.800}\right)^2}}{(92.4)\sqrt{2\pi}}$$

$$+ \frac{(0.027)\left(2+\left(\frac{z_s - 234.440}{16.805}\right)^2\right) e^{\frac{-1}{2}\left(\frac{z_s - 234.440}{16.805}\right)^2}}{(50.415)\sqrt{2\pi}}$$

$$+ \frac{(0.006)\left(2+\left(\frac{z_s - 153.650}{71.274}\right)^2\right) e^{\frac{-1}{2}\left(\frac{z_s - 153.650}{71.274}\right)^2}}{(213.822)\sqrt{2\pi}}$$

THE ORIGINAL AND SEGMENTED IMAGES



SEGMENTATION PERFORMANCE MEASURES

- It is observed that the PRI values of the proposed algorithm for the five images considered for experimentation are more than that of the values from the segmentation algorithm based on finite Gaussian mixture model with K-means. Similarly GCE and VOI values of the proposed algorithm are less than that of finite Gaussian mixture model for the images HORSE, MAN, BIRD, BOAT and TOWER.

IMAGES	METHOD	PERFORMACE MEASURES		
		PRI	GCE	VOI
HORSE	GMM	0.9142	0.1737	1.8643
	NSMM-K	0.9283	0.1634	1. 8403
	NSMM-H	0.9420	0.1054	1. 8249
MAN	GMM	0.9228	0.3107	1. 8389
	NSMM-K	0.9342	0.1734	1.7875
	NSMM-H	0.9521	0.0839	1.7366
BIRD	GMM	0.9106	0.1369	1. 7479
	NSMM-K	0.9140	0.1352	1.7259
	NSMM-H	0.9432	0.0702	1.6373
BOAT	GMM	0.9026	0.6485	1. 7882
	NSMM-K	0.9174	0.6483	1.7542
	NSMM-H	0.9356	0.1431	1.6980
TOWER	GMM	0.9102	0.1090	1. 8643
	NSMM-K	0.9246	0.0981	1.7988
	NSMM-H	0.9640	0.0137	1.7539

THE ORIGINAL AND RETRIEVED IMAGES

COMPARATIVE STUDY OF IMAGE QUALITY METRICS

IMAGE	Quality Metrics	GMM	NSMM-K	NSMM- H	Standard Limits
HORSE	Average Difference	0.5011	0.4413	0.3983	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	1.0000	1.0000	1.0000	Close to 1
	Mean Square Error	0.5011	0.4414	0.3940	Close to 0
	Signal to Noise Ratio	5.6542	5.9301	6.7480	As big as possible
	Image Quality Index	1.0000	1.0000	1.0000	Close to 1
MAN	Average Difference	0.8858	0.8002	0.7776	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	1.0000	1.0000	0.9999	Close to 1
	Mean Square Error	0.4995	0.5079	0.4092	Close to 0
	Signal to Noise Ratio	5.6828	5.6251	6.4092	As big as possible
	Image Quality Index	1.0000	1.0000	1.0000	Close to 1
BIRD	Average Difference	0.6939	0.6573	0.6022	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	1.0000	1.0000	1.0000	Close to 1
	Mean Square Error	0.5090	0.5050	0.4032	Close to 0
	Signal to Noise Ratio	4.6861	4.4842	5.6452	As big as possible
	Image Quality Index	1.000	1.0000	1.0000	Close to 1
BOAT	Average Difference	0.8039	0.7217	0.5267	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	1.0000	1.0000	0.9999	Close to 1
	Mean Square Error	0.5070	0.5070	0.5044	Close to 0
	Signal to Noise Ratio	4.6318	5.6573	6.2673	As big as possible
	Image Quality Index	1.000	1.0000	1.0000	Close to 1
TOWER	Average Difference	0.6936	0.6640	0.5187	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	0.9999	0.9999	0.9999	Close to 1
	Mean Square Error	0.5076	0.5076	0.4019	Close to 0
	Signal to Noise Ratio	4.6170	4.4647	5.6511	As big as possible
	Image Quality Index	1.0000	1.0000	1.0000	Close to 1

K. Srinivasa Rao and Sheshashayee (2011, 2012) developed image segmentation method based on a mixture of generalized new symmetric distribution

- An image segmentation algorithm is developed based on a generalized new symmetric mixture distribution with K-means algorithm. Here it is assumed that the whole image consisting of K-image regions and in each image region the pixel intensities follows a generalized new symmetric distribution. The generalized new symmetric distribution given by Srinivasa Rao K. et al (1997) includes Gaussian distribution as a particular case when the indexing parameter "r" becomes zero. It also includes new symmetric distribution , as a particular case when r=1. For different values of r it provides a generalized platy kurtic symmetric distribution.

- The probability density function of the pixel intensity is

$$f(z,\mu,\sigma^2,r)=\frac{\left(2r+\left(\frac{z-\mu}{\sigma}\right)^2\right)^r e^{\frac{-1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}}{\sigma(2r)^r(2\Pi)^{\frac{1}{2}}+\sum_{j=1}^{r}\binom{r}{j}(2r)^{r-j}2^{j+\frac{1}{2}}\Gamma(j+\frac{1}{2})\sigma},$$

$$-\infty<z<\infty,-\infty<\mu<\infty,\sigma>0,\ \mathrm{r}=0,1,2...$$

$$F(z)=\frac{\sum_{j=1}^{r}\binom{r}{j}(2r)^{(r-j)}2^{j+\frac{1}{2}}\Gamma(j+\frac{1}{2})F_j(z)+(2\pi)^{\frac{1}{2}}F_0(z)}{\sum_{j=1}^{r}\binom{r}{j}(2r)^{(r-j)}2^{j+\frac{1}{2}}\Gamma(j+\frac{1}{2})+(2\pi)^{\frac{1}{2}}(2r)^r}$$

- mean μ Variance $=\left[\frac{\frac{\sqrt{\pi}}{2}+\sum_{j=1}^{r}\binom{r}{j}r^{-j}\Gamma(j+\frac{3}{2})}{(\pi)^{\frac{1}{2}}+\sum_{j=1}^{r}\binom{r}{j}r^{-j}\Gamma(j+\frac{1}{2})}\right]2\sigma^2$

- Kurtosis $\beta_2=\frac{\left[\frac{3}{4}\pi^{\frac{1}{2}}+\sum_{j=1}^{r}\binom{r}{j}r^{-j}(j+\frac{1}{2})(j+\frac{3}{2})\Gamma(j+\frac{1}{2})\right]\left[\pi^{\frac{1}{2}}+\sum_{j=1}^{r}\binom{r}{j}r^{-j}\Gamma(j+\frac{1}{2})\right]}{\left[\frac{\pi^{\frac{1}{2}}}{2}+\sum_{j=1}^{r}\binom{r}{j}r^{-j}\Gamma(j+\frac{3}{2})\right]^2}$

FREQUENCY CURVES OF GENERALIZED NEW SYMMETRIC DISTRIBUTION

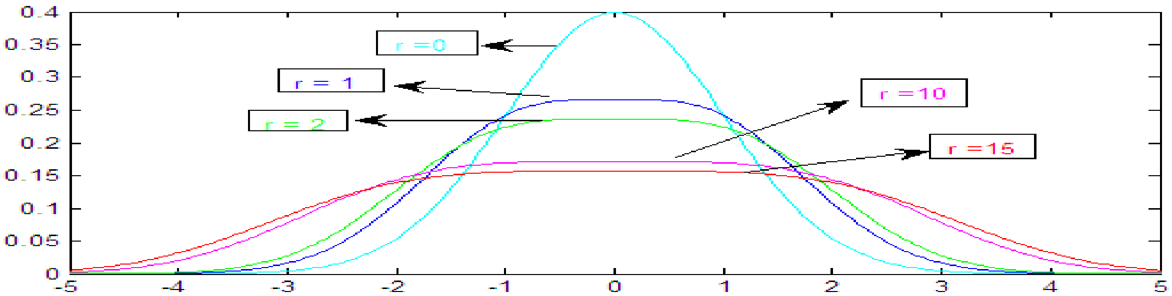


Table 5.1 : Values of β_2 for different values of r.

r	β_2	r	β_2	r	β_2	r	β_2
0	3.0000	5	1.1826	10	1.0952	15	1.0645
1	2.5200	6	1.1539	11	1.0869	16	1.0606
2	2.0260	7	1.1333	12	1.0800	17	1.0571
3	1.5105	8	1.1176	13	1.0740	18	1.0540
4	1.2398	9	1.1052	14	1.0689	19	1.0512

ESTIMATION OF THE MODEL PARAMETERS BY EM ALGORITHM

- The updated equations of the model parameters using Expectation Maximization (EM) algorithm. The likelihood function of the observations $z_1, z_2, \ldots, z_N$ drawn from an image is

$$L(\theta) = \prod_{s=1}^{N} p(z_s, \theta)$$

$$\log L(\theta) = \sum_{s=1}^{N} \log\left(\sum_{i=1}^{K} \alpha_i f_i(z_s, \theta_i)\right) \qquad \text{where, } \theta = (\mu_i, \sigma_i^2, r_i, \alpha_i; i = 1, 2, \ldots, K)$$

$$= \sum_{s=1}^{N} \log\left[\sum_{i=1}^{K} \frac{\alpha_i \left(2r_i + \left(\frac{z_s - \mu_i}{\sigma_i}\right)^2\right)^{r_i} e^{\frac{-1}{2}\left(\frac{z_s - \mu_i}{\sigma_i}\right)^2}}{\sigma_i (2r_i)^{r_i} (2\Pi)^{\frac{1}{2}} + \sum_{j=1}^{r_i} \binom{r_i}{j} (2r_i)^{r_i - j}\, 2^{j + \frac{1}{2}}\, \Gamma(j + \frac{1}{2}) \sigma_i}\right]$$

- In the expectation (E) step, the expectation value of log L(θ) with respect to the initial parameter vector $\theta^{(0)}$ is

$$Q\left(\theta;\theta^{(0)}\right)=E_{\theta^{(0)}}\left[\log L(\theta)\right]$$

- Given the initial parameters $\theta^{(0)}$, one can compute the density of pixel intensity z_i as

$$p(z_s,\theta^{(l)})=\sum_{i=1}^{K}\alpha_i^{(l)}f_i(z_s,\theta^{(l)})\ ,\qquad L(\theta)=\prod_{s=1}^{n}p(z_s,\theta)$$

This implies $$\log L(\theta)=\sum_{s=1}^{N}\log\left(\sum_{i=1}^{K}\alpha_i f_i(z_s,\theta_i)\right)$$

- The conditional probability of any observation z_i, belongs to any region K is

$$t_k(z_s,\theta^{(l)})=\left[\frac{\alpha_k^{(l)}f_k(z_s,\theta^{(l)})}{p(z_s,\theta^{(l)})}\right]=\left[\frac{\alpha_k^{(l)}f_k(z_s,\theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l)}f_i(z_s,\theta^{(l)})}\right]$$

- The expectation of the log likelihood function of the sample is

$$Q\left(\theta;\theta^{(l)}\right)=\sum_{i=1}^{K}\sum_{s=1}^{N}\left(t_i(z_s,\theta^{(l)})\left(\log f_i(z_s,\theta^{(l)})+\log\alpha_i^{(l)}\right)\right)$$

UPDATED EQUATIONS

- The updated equation of $\alpha_i, \mu_i, \sigma_i^2$ for $(\ell+1)^{\text{th}}$ estimate is

$$\alpha_i^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N}\left[\frac{\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}\right]$$

$$\mu_i^{(l+1)} = \frac{\sum_{s=1}^{N} z_s t_i(z_s,\theta^{(l)}) - \sum_{s=1}^{N} t_i(z_s,\theta^{(l)})\left(\frac{2r_i(\sigma_i^2)^{(l)}\left(z_s-\mu_i^{(l)}\right)}{\left(2r_i(\sigma_i^2)^{(l)}+\left(z_s-\mu_i^{(l)}\right)\right)^2}\right)}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})}$$

$$\left(\sigma_i^2\right)^{(l+1)} = \frac{2\sum_{s=1}^{N}(z_s-\mu_i^{(l+1)})^2\left(\frac{1}{2}-\frac{r_i(\sigma_i^2)^{(l)}}{\left(2r_i(\sigma_i^2)^{(l)}+\left(z_s-\mu_i^{(l+1)}\right)^2\right)^2}\right)\left(t_i(z_s,\theta^{(l)})\right)}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})}$$

where, $t_i(z_s,\theta^{(l)}) = \dfrac{\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l+1)},\left(\sigma_i^2\right)^{(l)},r_i)}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l+1)},\left(\sigma_i^2\right)^{(l)},r_i)}$

INITIALIZATION OF THE PARAMETERS

- The efficiency of the EM algorithm in estimating the parameters is heavily dependent on the number of regions in the image. The number of mixture components initially taken for K – Means algorithm / Hierarchical clustering algorithm by plotting the histogram of the pixel intensities of the whole image, the number of peaks in the histogram can be taken as the initial value of the number of regions K.

INITIALIZATION OF MODEL PARAMETERS

The shape parameter r_i can be estimated through sample kurtosis by using the following equation

$$\left[\frac{3\sqrt{\pi}}{4}+\sum_{j=1}^{r_i}\binom{r_i}{j}r_i^{-j}\left(j+\frac{1}{2}\right)\left(j+\frac{3}{2}\right)\Gamma\left(j+\frac{1}{2}\right)\right]\frac{(\pi)^{1/2}+\sum_{j=1}^{r_i}\binom{r_i}{j}r_i^{-j}\Gamma\left(j+\frac{1}{2}\right)}{\left[\frac{\sqrt{\pi}}{2}+\sum_{j=1}^{r_i}\binom{r_i}{j}_i^{-j}\left(j+\frac{1}{2}\right)\Gamma\left(j+\frac{1}{2}\right)\right]^2}=\frac{\left[\frac{1}{n_i}\sum_{i=1}^{n_i}(z_i-\bar{z})^4\right]}{\left[\frac{1}{n_i}\sum_{i=1}^{n_i}(z_i-\bar{z})^2\right]}$$

- The initial estimate α_i is taken as $\alpha_i = \frac{1}{K}$, where i = 1,2,...,K. The parameters are estimated by the method of moments as

$$\mu_i = \overline{z}$$

- $$\sigma_i^2 = \frac{n_i}{(n_i - 1)}\left[1 + \frac{\sum_{j=1}^{r_i}\binom{r_i}{j} r_i^{-j}\left(j+\frac{1}{2}\right)}{2\sum_{j=1}^{r_i}\binom{r_i}{j}_i^{-j} r_i^{-j}\left(j+\frac{1}{2}\right)\Gamma\left(j+\frac{1}{2}\right)}\right]S^2$$ where, s^2 is the sample variance

SEGMENTATION ALGORITHM

- After refining the parameters, the prime step in image segmentation is allocating the pixels to the segments of the image. This operation is performed by Segmentation Algorithm. The image segmentation algorithm consists of four steps.

- Step 1) Plot the histogram of the whole image.

- Step 2) Obtain the initial estimates of the model parameters using K-Means algorithm and moment estimates for each image region .

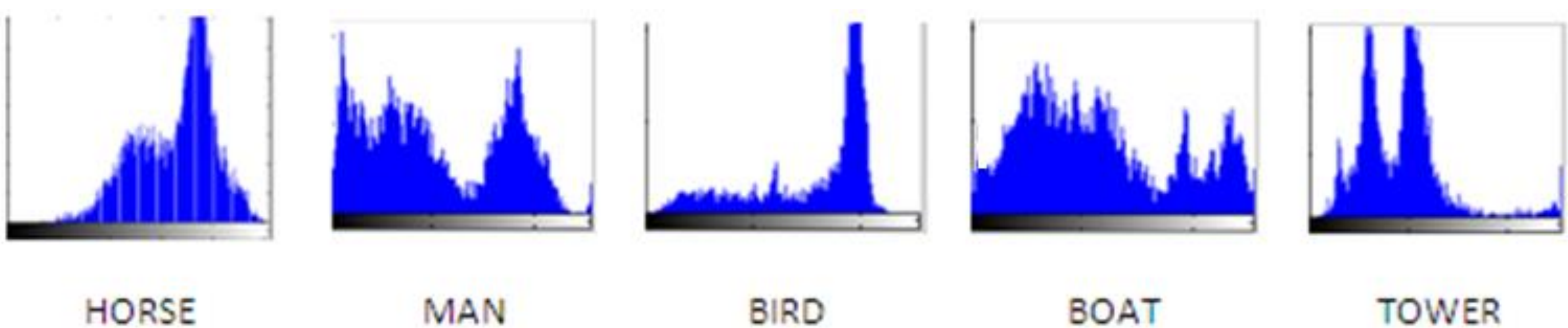
- Step 3)Obtain the refined estimates of the model parameters μ_i, σ_i^2, r_i and α_i for i=1, 2, ..., K using the EM algorithm with the updated equations.
- Step 4) Assign each pixel into the corresponding j[th] region (segment) according to the maximum likelihood of the j[th] component L_j.

$$L_j = \max_{j \in k} \left\{ \frac{\left(2r + \left(\frac{z_s - \mu_j}{\sigma_j}\right)^2\right)^r e^{\frac{-1}{2}\left(\frac{z_s - \mu_j}{\sigma_j}\right)^2}}{\sigma_j (2r)^r (2\Pi)^{\frac{1}{2}} + \sum_{j=1}^{r} \binom{r}{j} (2r)^{r-j} 2^{j+\frac{1}{2}} \Gamma(j + \frac{1}{2}) \sigma_j} \right\}$$

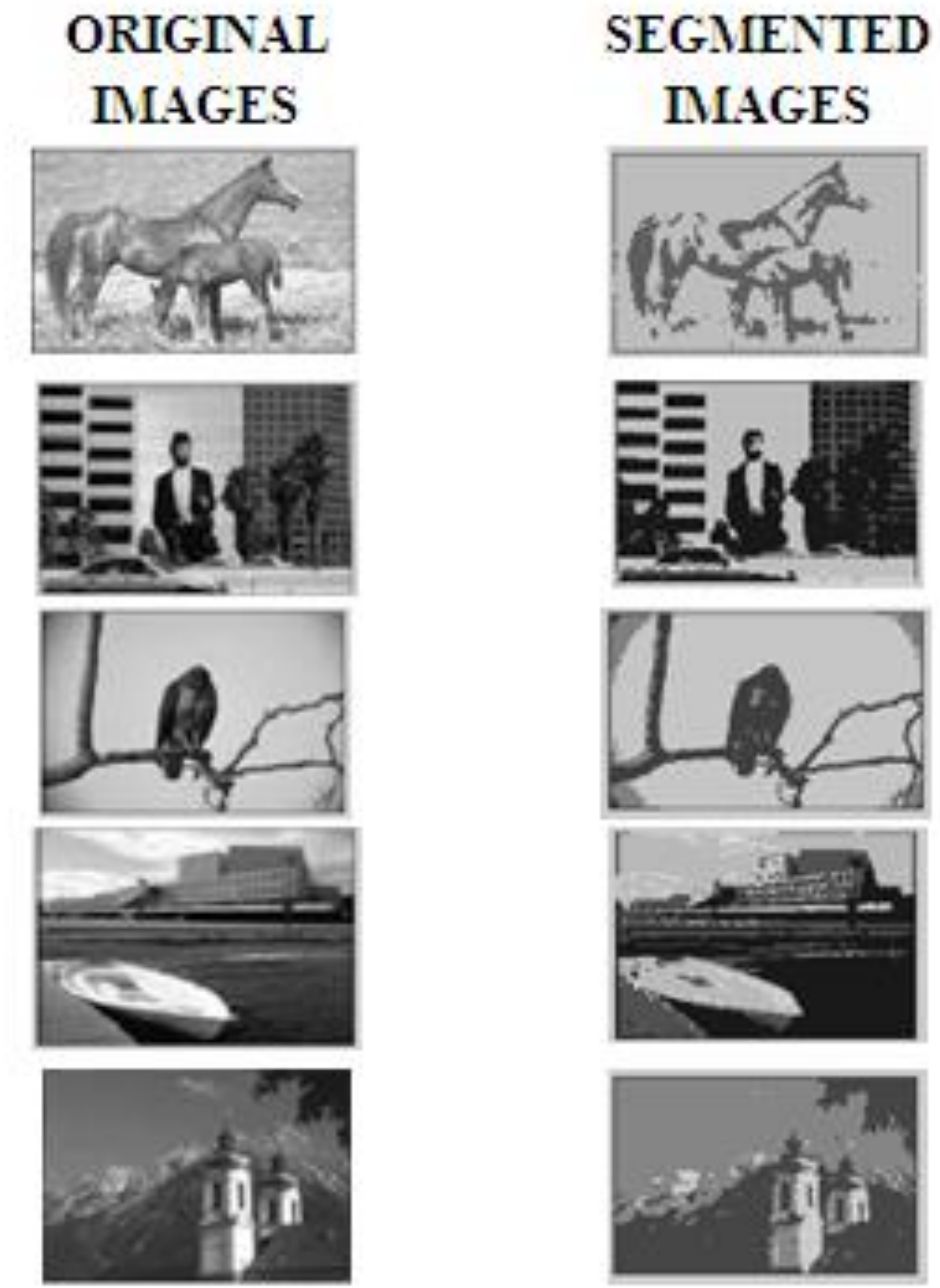
EXPERIMENTAL RESULTS

- To demonstrate the utility of the image segmentation algorithm developed in this chapter, an experiment is conducted with five images taken from Berkeley images dataset (http://www.eecs.berkeley.edu/Research/Projects/CS/Vision/bsds/BSDS300/html).

HISTOGRAMS OF THE IMAGES



THE ORIGINAL AND SEGMENTED IMAGES



SEGMENTATION PERFORMANCE MEASURES

- The performance evaluation of the segmentation technique is carried by obtaining the three performance measures namely, (i) probabilistic rand index (PRI), (ii) variation of information (VOI) and (iii) global consistence error (GCE).

  - PRI is a measure of similarity between the clusters. This measures takes values in [0 1], where zero means tested clusters and ground truth have no similarities and one means all segments are identical.

  - VOI measures the amount of information that is lost or gained in changing from one segment to another.

  - GCE measures the extent to which one segmentation map can be viewed as a refinement of another segmentation. And it is a measure of variation.

SEGMENTATION PERFORMANCE MEASURES

- It is observed that the PRI values of the proposed algorithm for the five images considered for experimentation are more than that of the values from the segmentation algorithm based on finite Gaussian mixture model with K-means. Similarly GCE and VOI values of the proposed algorithm are less than that of finite Gaussian mixture model

IMAGES	METHOD	PERFORMACE MEASURES		
		PRI	GCE	VOI
HORSE	NSMM-K	0.9283	0.1634	1. 8403
	GNSMM-K	0.9374	0.1088	1.8379
	NSMM-H	0.9420	0.1054	1. 8249
	GNSMM-H	0.9596	0.0435	1.7899
MAN	NSMM-K	0.9342	0.1734	1.7875
	GNSMM-K	0.9468	0.1226	1. 7707
	NSMM-H	0.9521	0.0839	1.7366
	GNSMM-H	0.9604	0.0499	1.7254
BIRD	NSMM-K	0.9140	0.1352	1.7259
	GNSMM-K	0.9229	0.1048	1. 6423
	NSMM-H	0.9432	0.0702	1.6373
	GNSMM-H	0.9649	0.0558	1.6321
BOAT	NSMM-K	0.9174	0.6483	1.7542
	GNSMM-K	0.9249	0.2626	1. 7405
	NSMM-H	0.9356	0.1431	1.6980
	GNSMM-H	0.9548	0.1115	1.6587
TOWER	NSMM-K	0.9246	0.0981	1.7988
	GNSMM-K	0.9431	0.0820	1. 7752
	NSMM-H	0.9640	0.0137	1.7539
	GNSMM-H	0.9735	0.0135	1. 7491

COMPARATIVE STUDY OF IMAGE QUALITY METRICS

IMAGE	Quality Metrics	NSMM-K	GNSMM-K	NSMM-H	GNSMM-H	Standard Limits
HORSE	Average Difference	0.4413	0.4089	0.3983	0.3042	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	1.0000	1.0000	1.0000	1.0000	Close to 1
	Mean Square Error	0.4414	0.4090	0.3940	0.2056	Close to 0
	Signal to Noise Ratio	5.9301	6.0957	6.7480	7.6412	As big as possible
	Image Quality Index	1.0000	1.0000	1.0000	1.0000	Close to 1
MAN	Average Difference	0.8002	0.7907	0.7776	0.7002	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	1.0000	1.0000	0.9999	1.0000	Close to 1
	Mean Square Error	0.5079	0.4946	0.4092	0.4079	Close to 0
	Signal to Noise Ratio	5.6251	5.6615	6.4092	6.6251	As big as possible
	Image Quality Index	1.0000	1.0000	1.0000	1.0000	Close to 1
BIRD	Average Difference	0.6573	0.6050	0.6022	0.6017	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	1.0000	1.0000	1.0000	1.0000	Close to 1
	Mean Square Error	0.5050	0.4939	0.4032	0.1880	Close to 0
	Signal to Noise Ratio	4.4842	5.6376	5.6452	6.7799	As big as possible
	Image Quality Index	1.0000	1.0000	1.0000	1.0000	Close to 1
BOAT	Average Difference	0.7217	0.6043	0.5267	0.5067	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	1.0000	1.0000	0.9999	1.0000	Close to 1
	Mean Square Error	0.5070	0.5064	0.5044	0.2117	Close to 0
	Signal to Noise Ratio	5.6573	5.6691	6.2673	6.7371	As big as possible
	Image Quality Index	1.0000	1.0000	1.0000	1.0000	Close to 1
TOWER	Average Difference	0.6640	0.6074	0.5187	0.5153	Close to 0
	Maximum Distance	1.0000	1.0000	1.0000	1.0000	Close to 1
	Image Fidelity	0.9999	0.9999	0.9999	0.9999	Close to 1
	Mean Square Error	0.5076	0.4936	0.4019	0.3945	Close to 0
	Signal to Noise Ratio	4.4647	5.6264	5.6511	5.6833	As big as possible
	Image Quality Index	1.0000	1.0000	1.0000	1.0000	Close to 1

JYOTHIRMAYI, K.SRINIVASA RAO , P.SRINIVASA RAO and CH.SATYANARAYANA (2016a, 2016b) have developed and analysed Image segmentation methods based on mixture of Laplace type distributions.

K.SRINIVASA RAO and SRINIVAS Y-(2007a, 2007b, 2010) developed image segmentation methods based on Generalized Gaussian mixture model.

The Generalized Gaussian Distribution was used by Sharif .K et al (1995) for modeling the atmospheric noise sub band encoding of Audio and Video Signals, Choi . S et al (2000) have used this distribution for impulsive noise direction of arrival and independent component analysis.

The probability density function is

$$f(z \mid \mu, \sigma, P) = \frac{1}{2\Gamma(1+\frac{1}{P})A(P,\sigma)} e^{-\left|\frac{(Z_i-\mu_i)}{A(P,\sigma)}\right|^P}$$

$$-\infty < z_i < \infty$$

$$-\infty < \mu_i < \infty$$

$$0 < \sigma; \quad 0 < P < \infty$$

-----(4.3.1)

where

.

$$A(P,\sigma) = \left[\frac{\sigma^2\Gamma(1/P)}{\Gamma(3/P)}\right]^{\frac{1}{2}}$$

----(4.3.2)

Using EM algorithm the updated estimators are:

$$\mu_k^{(l+1)} = \frac{\sum_{s=1}^{N} t_k(z_s, \theta^{(l)}) z_s}{\sum_{s=1}^{N} t_k(z_s, \theta^{(l)})}$$

$$\sigma_k^{(l+1)} = \left[\frac{\sum_{s=1}^{N} t_k(z_s, \theta^{(l)}) \left(\frac{\Gamma(3/P_k)}{P_k \Gamma(1/P_k)} \right) \left| z_s - \mu_k^{(l)} \right|^{\frac{1}{P_k}}}{\sum_{s=1}^{N} t_k(z_s, \theta^{(l)})} \right]^{\frac{1}{P_k}}$$

$$\alpha_k^{(l+1)} = \frac{1}{N} \sum_{s=1}^{N} \left(\frac{\alpha_k^{(l)} f_k(z_s, \theta^{(l)})}{\sum_{i=1}^{K} f_k(z\ ,\theta^{(l)})} \right)$$

- Segmentation Algorithm

$$L_i = \max_i \left\{ \frac{\exp^{\left| \frac{z_i - \mu_i^{EM}}{A(P_i, \sigma_i^{EM})} \right|^{P_i}}}{2\Gamma(1 + \frac{1}{P_i}) A(P_i, \sigma_i)} \right\}$$

- Experimental Results

Performance Evaluation

Estimation of Initial Parameters						Estimation of Final Parameters by EM Algorithm				
Number of Image Regions/Segments 'K' =5						Number of Image Regions/Segments 'K' =5				
	P=0.5	P=0.6	P=0.7	P=0.72	P=0.75	P=0.5	P=0.6	P=0.7	P=0.72	P=0.75
Region Weights α_i	0.1311	0.1321	0.441	0.1177	0.20	0.1132	0.3216	0.4220	0.1103	0.0444
Region Means μ_i	3346.27	-5674.30	3412.234	0.3421	3111	3112.22	3121.02	-3112.12	3112.2	3123.1
Region Variance σ_i	10001.7	10879.37	34272.73	92314.1	10028.1	10210.1	10112.15	3212.2	98862.1	12121.2

Estimation of Initial Parameters				Estimation of Final Parameters by EM Algorithm		
Number of Image Regions/Segments 'K' =3				Number of Image Regions/Segments 'K' =3		
	P=0.53	P=0.65	P=0.75	P=0.53	P=0.65	P=0.75
Region Weights α_i	0.2139	41150	0.36950	0.2136	0.4220	0.3644
Region Means μ_i	1121.21	-3124.32	1612.23	3216.21	-2114.32	-3412.211
Region Variances σ_i	3221.71	31129.37	32272.73	32531.71	31219.37	34212.73

MAN IMAGE

Estimation of Initial Parameters						Estimation of Final Parameters by EM Algorithm				
Number of Image Regions/Segments 'K' =5						Number of Image Regions/Segments 'K' =5				
	P=0.63	P=0.73	P=0.77	P=0. 82	P=0.85	P=0.63	P=0.73	P=0.77	P=0. 82	P= 0. 85
Region Weights α_i	0.3323	0.1137	0.263	0.1211	0.1702	0.1217	0.3113	0.3212	0.1210	0. 12 48
Region Means μ_i	3325.61	-5421.31	3112.21	3031.2	3121.2	3221.21	-4212.3	-2411.2	2921.2	29 12 .1
Region Variance σ_i	43241.7	2111.37	3111.2	21349.2	0.311	43401.71	29819.3	34272.	21121.7	34 42 12 .1

HILLS IMAGE

Estimation of Initial Parameters					Estimation of Final Parameters by EM Algorithm			
Number of Image Regions/Segments 'K' =4					Number of Image Regions/Segments 'K' =4			
	P=0.56	P=0.66	P=0.73	P=0.79	P=0.56	P=0.66	P=0.73	P=0.79
Region Weights α_i	0.1438	0.1322	0.4323	0.2917	0.13295	0.32227	0.42555	0.1188
Region Means μ_i	3113.21	-5324.37	3022.234	3112.1	3216.21	-521232	-3412.234	22454.2
Region Variances σ_i	11311.71	11319.60	10122.1	10112.1	11001.71	10181.30	34272.09	13121.32

WOMAN IMAGE

Estimation of Initial Parameters					Estimation of Final Parameters by EM Algorithm			
Number of Image Regions/Segments 'K' =4					Number of Image Regions/Segments 'K' =4			
	P=0.58	P=0.67	P=0.76	P=0.86	P=0.58	P=0.67	P=0.76	P=0.86
Region Weights α_i	0.2524	0.2143	0.234	0.2993	0.2132	0.3324	0.4323	0.21221
Region Means μ_i	3216.21	-5121.21	3212.23	3022.1	3109.79	-4312.2	-3001.43	3121.3
Region Variances σ_i	21241.7	22221.37	22342.7	20092.4	20901.71	19821.37	20972.32	20122.7

LOTUS IMAGE

Estimation of Initial Parameters						Estimation of Final Parameters by EM Algorithm				
Number of Image Regions/Segments 'K' =5						Number of Image Regions/Segments 'K' =5				
	P=0.67	P=0.69	P=0.79	P=0.82	P=0.85	P=0.67	P=0.69	P=0.79	P=0.82	P=0.85
Region Weights α_i	0.1098	0.1143	0.345	0.3234	0.1075	0.1032	0.232	0.3212	0.1103	0.2333
Region Means μ_i	3276.21	36323.55	3322.23	3211.7	3112.	33216.2	5232.32	3412.234	3121.1	3211.2
Region Variances σ_i	10100.7	110133	32272.7	12117.1	12131.2	132101.71	224355.54	31232.7	13112.1	12111.3

SRINIVAS Y and K.SRINIVASA RAO (2007, 2010) have developed image segmentation methods based on finite doubly truncated Gaussian mixture model.

JYOTIRMAYI and K. SRINIVASA RAO (2016,2017) have developed image segmentation methods using mixture of Truncated Laplace type distributions.

- This implies the pdf of the pixel intensity in each image region is

$$g(z) = \frac{f(z)}{B-A}, \; z_L < z < z_M$$

- Where f(z) is given as earlier

$$A = \int_{-\infty}^{Z_L} \frac{e^{\left(\frac{-1}{2}\left(\frac{(z-\mu)}{\sigma}\right)^2\right)}}{\sqrt{2\pi}\,\sigma} dz \;\; and \;\; B = \int_{-\infty}^{Z_M} \frac{e^{\left(\frac{-1}{2}\left(\frac{(z-\mu)}{\sigma}\right)^2\right)}}{\sqrt{2\pi}\,\sigma} dz$$

The lower and upper truncation points are ZL and ZM respectively. The degrees of truncation are (A) and (1- B).

If ZL is replaced by -∞, or ZM by ∞, the distribution is singly truncated from above, or below, respectively.

- The Mean pixel intensity is given by'

$$E(z) = \mu_i + \frac{\sigma_i^{2}\left[f(Z_L) - f(Z_M) \right]}{B - A}$$

The variance is given by

$$V(Z) = \left[1 + \frac{\left[\left(\frac{Z_L - \mu_i}{\sigma_i} \right) Z_L - \left(\frac{Z_L - \mu_i}{\sigma_i} \right) Z_M \right]}{B - A} \right] \sigma_i^2$$

- The updated equations of the estimates are:

$$\mu_k^{(l+1)} = \left[\mu_k^{(l)} + 2\sigma_k^{2(l)} \left(\frac{f(z_M) - f(z_L)}{B - A} \right) \right]$$

$$\sigma_k^{2(l+1)} = \frac{1}{D} \left\{ \alpha_k^{(l)} \mu_k^{2(l)} + \left(\frac{f_i(z_m, \theta^{(l)}) - f_i(z_l, \theta^{(l)})}{F(z_m, \theta^{(l)}) - F(z_l, \theta^{(l)})} \right) (\alpha_k^{(l)} \sigma_k^{2(l)} - \alpha_k^{(l)} \mu_k^{(l)} \sigma_k^{2(l)}) \right.$$

$$- \alpha_i^{(l)} \sigma_i^{2(l)} \left(\frac{z_m f_k(z_m, \theta^{(l)}) - z_l f_k(z_l, \theta^{(l)})}{F(z_m, \theta^{(l)}) - F(z_l, \theta^{(l)})} \right)$$

$$\left. -2\mu_k \left(\frac{\alpha_k^{(l)} \mu_k^{(l+1)}}{F(z_m, \theta^{(l)}) - F(z_l, \theta^{(l)})} - \frac{\alpha_k^{(l)} \sigma_k^{2(l)}}{F(z_m, \theta^{(l)}) - F(z_l, \theta^{(l)})} \right) + \mu_k^{2(l+1)} \right\}$$

$$\alpha_k^{(l+1)} = \frac{1}{N} \sum_{s=1}^{N} \frac{\alpha_k^{(l)}}{H(z_M, \theta^{(l)}) - H(z_L, \theta^{(l)})}$$

where,

$$H(z_M, \theta^{(l)}) = \int_{-\infty}^{z_M} \alpha_i g_i(z_s, \theta^{(l)}) dz$$

$$H(z_L, \theta^{(l)}) = \int_{-\infty}^{z_L} \alpha_i g_i(z_s, \theta^{(l)}) dz$$

Segmentation criterion is based on conponent maximum likelihood under Bay's frame

$$L_j = \max_i \left\{ \frac{\exp \frac{(z_i - \mu_i^{EM})^2}{2(\sigma_i^{EM})^2}}{\sigma_i^{EM}(B - A)} \right\}$$

where

$$B = \int_{-\infty}^{z_m} \frac{1}{\sqrt{2\pi}\,\sigma_i^{EM}} e^{-\frac{1}{2}\frac{(t-\mu_i^{EM})^2}{(\sigma_i^{EM})^2}} dt,$$

$$A = \int_{-\infty}^{z_l} \frac{1}{\sqrt{2\pi}\,\sigma_i^{EM}} e^{-\frac{1}{2}\frac{(t-\mu_i^{EM})^2}{(\sigma_i^{EM})^2}} dt$$

Original & Reconstructed Images

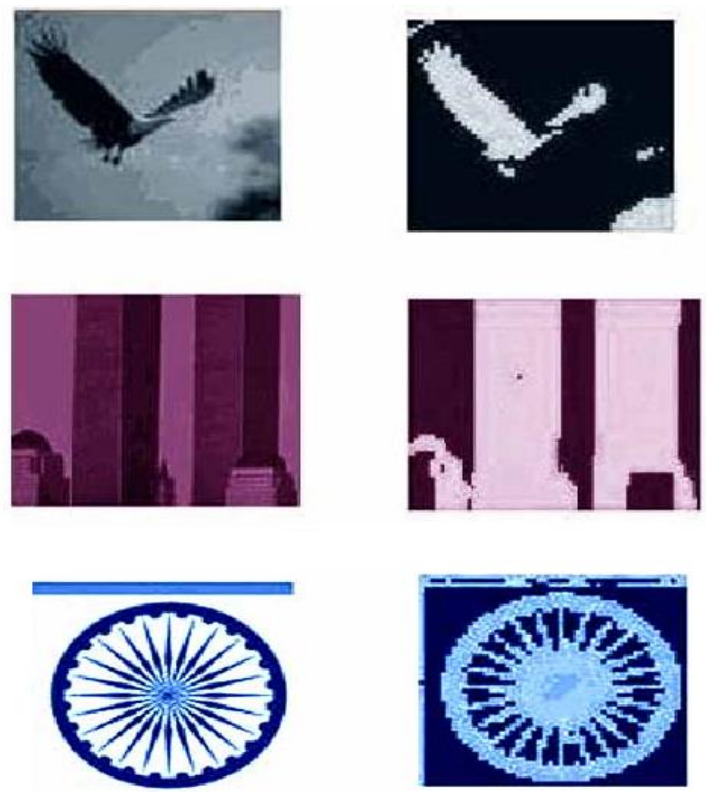


IMAGE	Quality Metric	Finite Gaussian Mixture Model Classifier using K-Means Algorithm	Finite Truncated Gaussian Mixture Model Classifier using K-Means Algorithm
BIRD	Average Difference	0.68763	0.927343
	Maximum Distance	0.21108	0.92378
	Image Fidelity	1.22208	0.001
	Mean Square Error	0.98983	0.6813
	Signal to Noise Ratio	23.3404	45.757
	Image Quality Index	0.2354	0.587
LENA	Average Difference	0.3783	0.681747
	Maximum Distance	2.3222	1.0407
	Image Fidelity	0.2344	0.818
	Mean Square Error	0.1232	0.0285
	Signal to Noise Ratio	12.342	32.434
	Image Quality Index	0.023	0.9107
FISH	Average Difference	0.3793	0.91680
	Maximum Distance	0.3452	1.3378
	Image Fidelity	1.2444	0.744
	Mean Square Error	0.7432	0.5971
	Signal to Noise Ratio	15.342	25.728
	Image Quality Index	0.1233	1.090

P.CHANDRA SEKHAR, K.SRINIVASA RAO and P.SRINIVASA RAO (2014a, 2014b, 2014c) developed and analyzed Image Segmentation Algorithm for Images having Asymmetrically Distributed Image Regions using Finite Mixture of Pearsonian Distributions.

$$b_{0.} + b_1 X + b_2 X^2 = 0$$

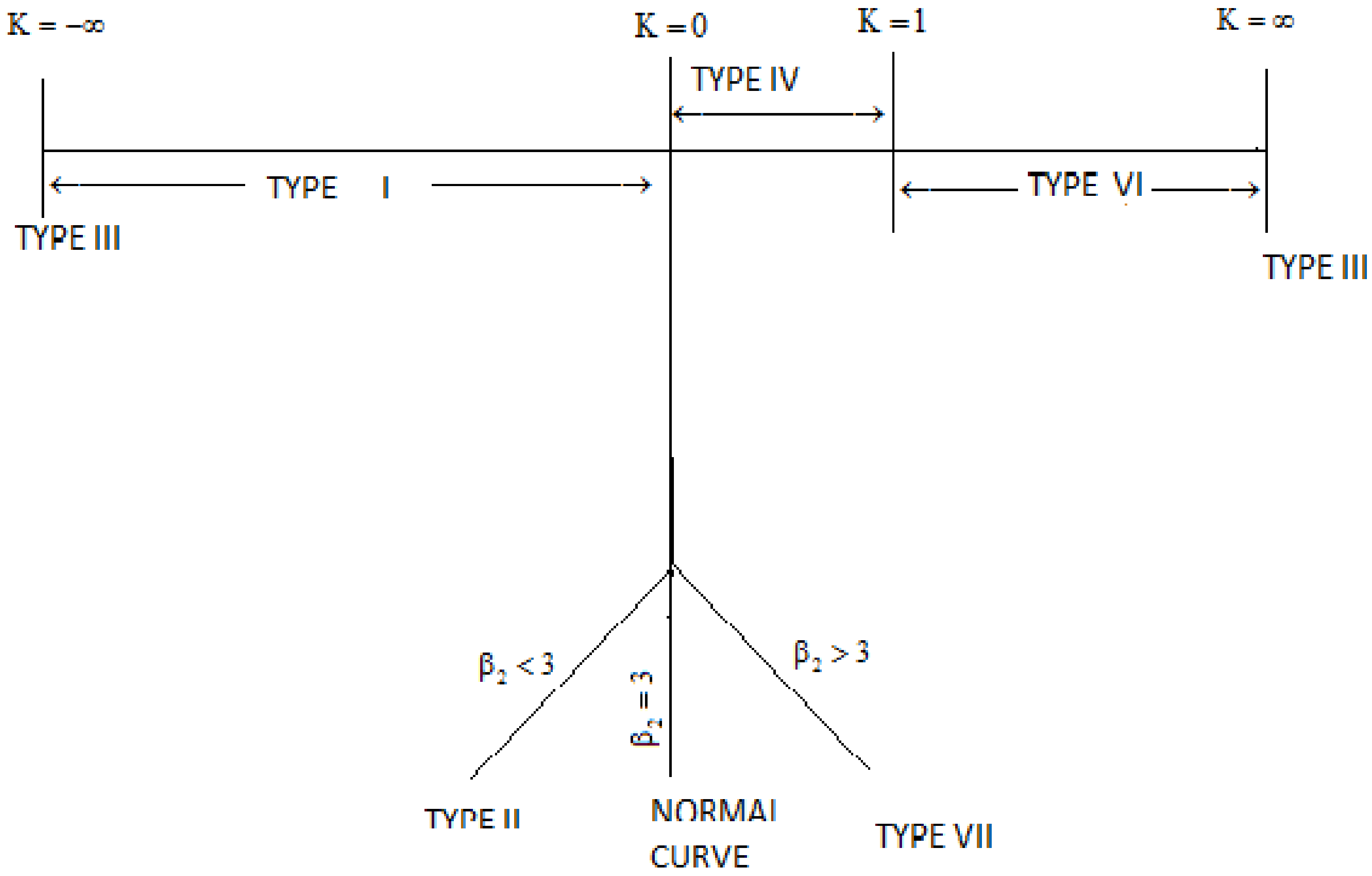
$$b_0 = -\frac{\mu_2(4\mu_2\mu_4 + 3\mu_3^2)}{2(5\mu_2\mu_4 - 9\mu_2^3 - 6\mu_3^2)}$$

$$b_1 = -\frac{\mu_3(\mu_4 + 3\mu_2^2)}{2(5\mu_2\mu_4 - 9\mu_2^3 - 6\mu_3^2)}$$

$$b_2 = -\frac{2(\mu_2\mu_4 - 3\mu_3^2 - 6\mu_2^3)}{2(5\mu_2\mu_4 - 9\mu_2^3 - 6\mu_3^2)}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \qquad \beta_2 = \frac{\mu_4}{\mu_2^2} \qquad b_0 + b_1 x + b_2 x^2 = 0$$

$$k = \frac{b_1^2}{(4 b_0 b_2)}$$


$K = -\infty$
$K = 0$
$K = 1$
$K = \infty$
TYPE IV
TYPE I
TYPE VI
TYPE III
TYPE III
$\beta_2 < 3$
$\beta_2 = 3$
$\beta_2 > 3$
TYPE II
NORMAL CURVE
TYPE VII

COLOUR IMAGE SEGMENTATION

GVS RAJKUMAR, K.SRINIVASA RAO, and P.SRINIVASA RAO (2011a, 2011b, 2011c, 2011d, 2017) Image Segmentation and Retrievals based on Finite Doubly Truncated Bivariate Gaussian Mixture Model.

JAGADESH, K.SRINIVASA RAO, SATYANARAYANA and RAJKUMAR (2012, 13,15, 17) have developed methods for Skin color segmentation using finite mixture of bivariate Pearsonian distributions.

DYNAMIC IMAGE SEGMENTATION

VIZIANANDA ROW, K. SRINIVASA RAO and P. SRINIVASA RAO (2015, 2016, 2017) have developed Image Segmentation methods using compound Normal with Gamma mixture models

IMAGE TEXTURE SEGMENTATION

K. NAVEEN KUMAR, K.SRINIVASA RAO, Y.SRINIVAS, Ch. SATYANARAYANA (2015a, 2015b, 2016a, 2016b) have developed Texture Segmentation of images using multivariate generalized Gaussian mixture model under DCT, log DCT Domain, DCT + LBP and log DCT +LBP.

SCOPE FOR FURTHER WORK

- It is possible to develop and analyze image segmentation algorithms based on feature vector which has more than one feature like brightness, hue angle, and saturation etc, using multivariate mixture distributions.

- The image segmentation methods can also be extended to image compression, image filtering, content retrials, image reconstruction with missing regions.

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THANK YOU

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