

U-statistics approach to Hollander-Proschan test for NBUE alternatives

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Abstract

In this paper we develop a simple non-parametric test based on U-statistics for testing exponentiality against NBUE alternative. The proposed test is asymptotically equivalent to that of Hollander and Proschan (1975). Since the test is based on U-statistics, the study of asymptotic theory is very simple. The test statistic is shown to be asymptotically normal and consistent against the alternatives under consideration.

Key words: Exponential distribution; Hollander-Proschan test; NBUE class; U-Statistics

1 Introduction

The problem of testing exponentiality against New Better Than Used in Expectation (NBUE) alternatives has received considerable attention during the last four decades. In fact this test procedure enables engineers to develop a better replacement policies for efficient running of several systems. Test for exponentiality against NBUE alternatives was first considered by Hollander and Proschan (1975). Subsequently, various authors used different types of approaches in deriving the test statistics, see Koul (1978), Borges et al. (1984), Fernandez-Ponce et al. (1996), Belzunce et al. (2000) and Belzunce et al. (2001).

Recently, Anis and Mitra (2011) have generalized the Hollander-Proschan approach to propose a family of tests for NBUE alternatives. Both Anis and Mitra (2011) and Hollander and Proschan (1975) have shown that the asymptotic null distribution of their statistics is normal. Anis and Basu (2011) obtained an exact null distribution of the generalized Hollander-Proschan type test developed by Anis and Mitra (2011). Anis and Basu (2014) conducted a Monte-Carlo study to compare the different approaches for testing exponentiality against NBUE alternative.

Motivated from these recent works, we develop a new test procedure for testing exponentiality against NBUE alternatives. We organize the paper as follows. In Section 2, based on U-statistics, we propose a non-parametric test and show that it is asymptotically equivalent to that of Hollander and Proschan (1975). Using U-statistics theory, the asymptotic properties of the test statistic are studied in Section 3. Finally, in Section 4 we give conclusions of our study.

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