Special Proceedings: ISBN #: 978-81-950383-1-2
24th Annual Conference, 23-27 February 2022; pp 149-157

Perishable Stochastic Inventory Models for Two and Multiple Suppliers

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Received: 26 May 2022; Revised: 27 July 2022; Accepted:01 August 2022

Abstract

In order to avoid the relevant losses due to deterioration of perishable items, we need an efficient and effective inventory management. This research work develops a perishable stochastic inventory models for two suppliers and multiple suppliers to determine an optimal ordering policy for allowable shortages. In case of two suppliers, spectral theory is used to derive explicit expression for the transition probabilities of a four-state continuous time Markov chain representing the status of the systems. These probabilities are used to compute the exact form of the average cost expression. We use concepts from renewal reward processes to develop average cost objective function. Optimal solution is obtained using Newton Rapson method in R programming. Finally, sensitivity analysis of the varying parameter on the optimal solution is done. We have extended the case of two suppliers to multiple suppliers and for the multiple suppliers problem, assuming that all the suppliers have similar availability characteristics; we develop a simple model and show that as the suppliers become large, the model reduces to classical EOQ model.

Key words: Future supply uncertainty; Two suppliers; Deteriorating items; EOQ model; Multiple suppliers, Sensitivity analysis.

1. Introduction

Inventory can be defined as the goods or stock hold by a person or a firm in order to use it in future for consumption, production or sale. Inventory management is used to minimize cost required to hold the inventory effectively in such way that there is no gap between demand and supply. The main and foremost reason for maintaining inventory level is to shorten the gap between demand and supply for the commodity under consideration. Any inventory system consists of an input process and output process. The input process refers to supply either by means of production or purchase while the output process refers to demand due to which depletion of inventory occurs. Thus, supply is a replenishment process, whereas demand is a depletion process. Though the inventories are essential and provide an alternative to production or purchase in future, they also mean lock up capital of an enterprise. Maintenance of inventories also costs money by way of expenses on stores, equipment, personnel, insurance *etc*. Thus, excess of inventories is undesirable. This calls for controlling the inventories in the most profitable way. Hence inventory theory deals with the determination of the optimal level of such ideal resources. Some products lose value faster than others, these are known as

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perishable products. Perishable inventory forms a large portion of total inventory and include virtually all foodstuffs, pharmaceuticals, fashion goods, electronic items, digital goods (computer software, video games, DVD), periodicals (magazines/Newspapers), and many more as they lose value with time due to deterioration or obsolescence. Perishable goods can be broadly classified into two main categories based on: (i) Deterioration (ii) Obsolescence. Deterioration refers to damage, spoilage, vaporization, depletion, decay (*e.g.* radioactive substances), degradation (*e.g.* electronic components) and loss of potency (*e.g.* chemicals and pharmaceuticals) of goods. Obsolescence is loss of value of a product due to arrival of new and better product. Perishable goods have continuous or discrete loss of utility and therefore can have either fixed life or random life. Fixed life perishable products have a deterministic, known and definite shelf life and examples of such goods are pharmaceuticals, consumer packed goods and photographic films. On the other hand, random life perishable products have a shelf life that is not known in advance and variable depending on variety factors including storage atmosphere. Items are discarded when they spoil and the time to spoilage is uncertain. For example, fruits, vegetables, dairy products, bakery products *etc.*, have random life.

2. Review of Literature

An excellent survey on research in inventory management in a single product, single location inventory environment is provided by Lee and Nahmias (1993). A large number of researchers developed the models in the area of deteriorating inventories. At first Whitin (1957) considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Various types of inventory models for items deteriorating at a constant rate were discussed by Chowdhury and Choudhuri (1983). A complete survey of the published literature in mathematical modelling of deteriorating inventory systems is given by Raafat (1991). Goyal and Giri (2001) developed recent trends of inventory models for deteriorating items. Teng and Chang(2005) determined economic production quantity in an inventory model for deteriorating items. Supply uncertainty can have a drastic impact on firms who fail to protect against it. Supply uncertainty has become a major topic in the field of inventory management in recent years. Supply disruptions can be caused by factors other than major catastrophes. More common incidents such as snow storms, customs delays, fires, strikes, slow shipments, *etc.* can halt production or transportation capability, causing lead time delays that disrupt material flow. Silver (1981) appears to be first author to discuss the need for models that deal with supplier uncertainty. Articles by Parlar and Berkin (1991) consider the supply uncertainty problem, for a class of EOQ model with a single supplier where the availability and unavailability periods constitute an alternating Poisson process. Parlar and Parry (1996) generalized the formulation of Parlar and Berkin (1991) by first assuming that the reorder point r is a non-negative decision variable instead of being equal to zero. Kandpal and Tinani (2009) developed inventory model for deteriorating items with future supply uncertainty under inflation and permissible delay in payment for single supplier. According to Yavari *et al.* (2020), one of the challenges when managing inventories is the inherent perishability of many items, which means their freshness and quality decrease over time and these cannot be sold after their expiration date.
Tirkolaee *et al.* (2017) noted that the inherent perishability widely occurs in food goods organisms and ornamental flowers. These authors also stated that the time window between preparation and sales of perishable items is very significant for producers and purchasers.

In this paper it is assumed that the inventory manager may place his order with any one of two suppliers who are randomly available. Here we assume that the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 whereas state 3 denotes the nonavailability of either of them. State 0 indicates that supplier 1 and supplier 2 both are available. Here it is assumed that one may place order to either one of the two suppliers or partly to both. State 1 represents that supplier 1 is available but supplier 2 is not available. State 2 represents that supplier 1 is not available but supplier 2 is available.

3. Notations, Assumptions and Model

The inventory model here is developed on the basis of following assumptions.

a) Demand rate *d* is deterministic and it is *d* >1.
b) We define *Xi* and *Yi* be the random variables corresponding to the length of ON and OFF period respectively for i^{th} supplier where *i* =1, 2. We specifically assume that $X_i \sim exp\ (\lambda_i)$ and $Y_i \sim exp\ (\mu_i)$. Further *Xi* and *Yi* are independently distributed.
c) Ordering cost is Rs. *k*/order.
d) Holding cost is Rs. *h*/unit/unit time.
e) Shortage cost is Rs. π/unit.
f) θ is the rate of deterioration which is constant fraction of on hand inventory.
g) q_i = order up to level *i* =0, 1, 2.
h) *r* = reorder up to level; *qi* and *r* are decision variables.
i) Time dependent part of the backorder cost is Rs. $\hat{\pi}$/unit/time.
j) Purchase cost is Rs. *c*/unit.
k) T_{00} is the expected cycle time. T_{00} is a decision variable.

The policy we have chosen is denoted by (q_0, q_1, q_2, *r*). An order is placed for q_i units *i* = 0, 1, 2, whenever inventory drops to the reorder point *r* and the state found is *i* = 0, 1, 2. When both suppliers are available, q_0 is the total ordered from either one or both suppliers. If the process is found in state 3 that is both the suppliers are not available nothing can be ordered in which case the buffer stock of *r* units is reduced. If the process stays in state 3 for longer time, then the shortages start accumulating at rate of *d* units/time. When the process leaves state 3 and supplier becomes available, enough units are ordered to increase the inventory to $q_i + r$ units where *i* = 0, 1, 2. The cycle of this process start when the inventory goes up to a level of q_0+r units. Once the cycle is identified, we construct the average cost objective function as a ratio of the expected cost per cycle to the expected cycle length. *i.e.* $AC\ (q_0, q_1, q_2, r) = \frac{C_{00}}{T_{00}}$ where, C_{00} = *E*(cost per cycle) and T_{00} = *E*(length of a cycle). Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, *i, j*=0, 1, 2, 3 for the four state CTMC. The solution is provided in the lemma. (Refer Parlar and Perry [1996]). $A(q_i, r, \theta)$ = cost of ordering+ cost of holding inventory+ cost of items that deteriorate during a single interval that starts with an inventory of q_i units and ends with r units.

$$A(q_i\,, r, \theta) = k + \frac{1}{2}\frac{hq_i^2}{(d+\theta)} + \frac{hrq_i}{(d+\theta)} + \frac{\theta c q_i}{(d+\theta)} \qquad i = 0,1,2.$$

Lemma 3.1: Define C_{i0} = *E*(cost incurred to the beginning of the next cycle from the time when inventory drops to *r* at state *i* = 0, 1, 2, 3 and q_i units are ordered if *i* = 0, 1 or 2). Then,

C_{i0} is given by

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right)A(q_i, r, \theta) + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[A(q_i, r, \theta) + C_{j0}\right] \quad i=0,1,2. \tag{1}$$

$$C_{30} = \bar{C} + \sum_{i=1}^{2}\rho_i C_{i0} \qquad \text{where } \rho_i = \frac{\mu_i}{\delta} \ \text{ with } \delta = \mu_1 + \mu_2 \tag{2}$$

$$\bar{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}(\delta r - (d+\theta)) + (\pi\delta d + h(d+\theta) + \hat{\pi}) - \theta c\delta\right] + \frac{\theta c}{\delta} \tag{3}$$

Proof: First consider $i = 0$. Conditioning on the state of the supplier availability process when inventory drops to r, we obtain

$$C_{00} = P_{00}\left(\frac{q_0}{d+\theta}\right)A(q_0, r, \theta) + \sum_{j=1}^{3} P_{0j}\left(\frac{q_0}{d+\theta}\right)\left[A(q_0, r, \theta) + C_{j0}\right] \tag{4}$$

The equation follows because $q_0 + r$ being the initial inventory, when q_0 units are used up we either observe state 0, 1, 2 or 3 with probabilities $P_{00}\left(\frac{q_0}{d+\theta}\right)$, $P_{01}\left(\frac{q_0}{d+\theta}\right)$, $P_{02}\left(\frac{q_0}{d+\theta}\right)$ and $P_{03}\left(\frac{q_0}{d+\theta}\right)$ respectively. If we are in state 0 when r is reached, we must have incurred a cost of $A(q_0, r, \theta)$. On the other hand, if state $j = 1, 2, 3$ is observed when inventory drops to r, then the expected cost will be $A(q_0, r, \theta) + C_{j0}$ with probability $P_{0j}\left(\frac{q_0}{d+\theta}\right)$. The equation relating C_{10} and C_{20} are very similar but C_{30} is obtained as

$$C_{30} = [\bar{C} + C_{10}]\frac{\mu_1}{\mu_1+\mu_2} + [\bar{C} + C_{20}]\frac{\mu_2}{\mu_1+\mu_2} \tag{5}$$

Here, $\bar{C}$ is defined as the expected cost from the time inventory drops to r until either of the suppliers becomes available and it is computed as follows:
Now, note that the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of next cycle is equal to

$$\frac{1}{2}hy^2(d+\theta) + hy[r - y(d+\theta)] + \theta cy \qquad y < \frac{r}{d+\theta}$$

$$\frac{1}{2}\frac{hr^2}{(d+\theta)} + \pi\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta cr}{(d+\theta)} \qquad y \geq \frac{r}{d+\theta}$$

Hence,

$$\bar{C} = \int_0^{r/(d+\theta)} \left\{\frac{1}{2}hy^2(d+\theta) + hy(r - y(d+\theta) + \theta cy\right\}\delta e^{-\delta y}$$

$$+ \int_{r/(d+\theta)}^{\infty} \left\{\frac{1}{2}\frac{hr^2}{(d+\theta)} + \pi\left[y - \frac{r}{(d+\theta)}\right]d + \frac{\hat{\pi}}{2}\left[y - \frac{r}{(d+\theta)}\right]^2 + \frac{\theta cr}{(d+\theta)}\right\}\delta e^{-\delta y}$$

$$\bar{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}(\delta r - (d+\theta)) + (\pi\delta d + h(d+\theta) + \hat{\pi}) - \theta c\delta\right] + \frac{\theta c}{\delta}$$

with $\delta = \mu_1 + \mu_2$ as the rate of departure from state 3. This follows because if supplier availability process is in state 3 (OFF for both suppliers) when inventory drops to r, then the expected holding and backorder costs are equal to $\bar{C}$. If the process makes a transition to state 1, the total expected cost would then be $\bar{C} + C_{10}$. The probability of a transition from state 3 to state 1 is $P(Y_1 < Y_2) = \int_0^{\infty} P(Y_1 < Y_2 / Y_2 = t)\,\mu_2 e^{-\mu_2 t} dt = \frac{\mu_1}{\mu_1+\mu_2}$.
Multiplying this probability with the expected cost term above gives the first term of (5). The second term is obtained in a similar manner. Combining the results proves the lemma.

The following lemma provides a simpler means of expressing C_{00} in an exact manner. To simplify the notation, we let $A_i = A(q_i, r, \theta)$, i = 0, 1, 2 and $P_{ij} = P_{ij}\left(\frac{q_i}{d+\theta}\right)$ i, j = 0, 1, 2, 3.

Lemma 3.2: The exact expression for C_{00} is

$$C_{00} = A_0 + P_{01}C_{10} + P_{02}C_{20} + P_{03}(\bar{C} + \rho_1 C_{10} + \rho_2 C_{20}) \quad (6)$$

where the pair [C_{10}, C_{20}] solves the system

$$\begin{bmatrix} 1 - P_{11} - P_{13}\rho_1 & -(P_{12} + P_{13}\rho_2) \\ -(P_{21} + P_{23}\rho_1) & (1 - P_{22} - P_{23}\rho_2) \end{bmatrix} \begin{bmatrix} C_{10} \\ C_{20} \end{bmatrix} = \begin{bmatrix} A_1 + P_{13}\bar{C} \\ A_2 + P_{23}\bar{C} \end{bmatrix} \quad (7)$$

Proof: Rearranging the linear system of four equations in lemma (3.1) in matrix form gives

$$\begin{bmatrix} 1 & -P_{01} & -P_{02} & -P_{03} \\ 0 & 1 - P_{11} & -P_{12} & -P_{13} \\ 0 & -P_{21} & 1 - P_{22} & -P_{23} \\ 0 & -\rho_1 & -\rho_2 & 1 \end{bmatrix} \begin{bmatrix} C_{00} \\ C_{10} \\ C_{20} \\ C_{30} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \bar{C} \end{bmatrix} \quad (8)$$

We have $C_{30} = \bar{C} + \rho_1 C_{10} + \rho_2 C_{20}$ from the last row of the system. Substituting this result in rows two and three and rearranging gives the system in (7), with (C_{10}, C_{20}). From the first row of (8) we obtain $C_{00} = A_0 + \sum_{j=1}^{3} P_{0j} C_{j0}$.
Hence above lemma is proved.

Lemma 3.3: Define, T_{i0} = E[Time to the beginning of the next cycle from the time when inventory drops to r at state i = 0, 1, 2, 3 and q_i units are ordered if i = 0, 1, 2]. Then, expected cycle length is given by

$$T_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right)\frac{q_i}{d+\theta} + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[\frac{q_i}{d+\theta} + T_{j0}\right] \qquad i = 0, 1, 2.$$

$$T_{30} = \bar{T} + \sum_{j=1}^{2} \rho_i T_{i0}$$

where $\bar{T} = \frac{1}{\mu_1 + \mu_2}$ is the expected time from the time inventory drops to r until either supplier 1 or 2 becomes available.

Lemma 3.4: The exact expression for T_{00} is

$$T_{00} = \frac{q_0}{d+\theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\bar{T} + \rho_1 T_{10} + \rho_2 T_{20})$$

where the pair $[T_{10}, T_{20}]$ solves the system.

$$\begin{bmatrix} 1 - P_{11} - P_{13}\rho_1 & -(P_{12} + P_{13}\rho_2) \\ -(P_{21} + P_{23}\rho_1) & (1 - P_{22} - P_{23}\rho_2) \end{bmatrix} \begin{bmatrix} T_{10} \\ T_{20} \end{bmatrix} = \begin{bmatrix} q_1 + P_{13}\bar{T} \\ q_2 + P_{23}\bar{T} \end{bmatrix}$$

The proof of the above two lemmas (3.3) and (3.4) are very similar to lemma (3.1) and (3.2).

Theorem 3.5: The Average cost objective function for deteriorating items in case of two suppliers is given by $AC = \frac{C_{00}}{T_{00}}$,

$$AC = \frac{C_{00}}{T_{00}} = \frac{A(q_0\,, r, \theta) + P_{01}C_{10} + P_{02}C_{20} + P_{03}(\bar{C} + \rho_1 C_{10} + \rho_2 C_{20})}{\frac{q_0}{d+\theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\bar{T} + \rho_1 T_{10} + \rho_2 T_{20})}$$

Proof: Proof follows using Renewal reward theorem (RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

4. Numerical Analysis

Here, we assume that k = Rs. 5/order, c = Rs.5/unit, d = 20/units, θ = 5, h = Rs. 5/unit/time, π = Rs. 250/unit, $\hat{\pi}$ = Rs.25/unit/time, $\lambda_1 = 0.25$, $\lambda_2 = 1$, $\mu_1 = 2.5$, $\mu_2 = 0.5$.
With these parameters the long run probabilities are obtained as $p_0 = 0.303$, $p_1 = 0.606$, $p_2 = 0.030$ and $p_3 = 0.061$. The optimal solution is obtained as
$q_0 = 1.86448$, $q_1 = 10.490$, $q_2 = 15.44333$, $r = 20.4988$ and $AC = \frac{C_{00}}{T_{00}} = 197.81$.

5. Sensitivity Analysis

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r and AC are shown in Table1.

Table 1: Sensitivity Analysis by varying the parameter values of μ_1

μ_1	q_0	q_1	q_2	r	AC
2.4	0.7827	10.6468	15.3941	20.4989	234.57
2.5	1.86448	10.4905	15.4433	20.4988	197.81
2.6	3.2545	10.3081	15.497	20.4987	186.73
2.7	5.03811	10.0839	15.5592	20.4986	181.83
2.8	7.3501	9.7962	15.6347	20.4984	179.52

We see that as μ_1 increases *i.e.*, expected length of OFF period for 1st supplier decreases the value of q_0, q_2 increases, q_1 decreases and r remain almost constant which result in decrease in average cost.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r and AC are shown in Table 2.

Table 2: Sensitivity Analysis by varying the parameter values of λ_1

λ_1	q_0	q_1	q_2	r	AC
0.25	1.86448	10.4905	15.4433	20.4988	197.81
0.28	2.8761	10.3761	15.6391	20.4987	199.95
0.3	3.5782	10.3275	15.7820	20.4987	205.17
0.35	4.1852	10.2792	15.8861	20.4986	213.63
0.37	5.8807	9.8391	15.9840	20.4986	224.87

We see that as λ_1 increase *i.e.*, expected length of ON period for 1st supplier decreases the value of q_0, q_2 increases, q_1 decreases and r remain almost constant which result in increase in average cost.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value θ and keeping other parameter values fixed. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r and AC are shown in Table 3.

Table 3: Sensitivity Analysis by varying the parameter values of θ

θ	q_0	q_1	q_2	r	AC
3	16.7227	8.5525	16.0268	20.4979	169.89
3.5	9.4464	9.4878	15.7431	20.4982	170.74
4	5.5986	9.8872	15.5901	20.4985	175.58
4.5	3.3168	10.2917	15.5017	20.4986	183.73
5	1.86448	10.4905	15.4433	20.4988	197.81

We see that as θ increases it results in decrease in q_0, increase in q_1 but q_2 decreases and r remains almost constant. This results in increase in average cost.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2, r and AC are shown in Table 4.

Table 4: Sensitivity analysis by varying the parameter values of h

h	q_0	q_1	q_2	r	AC
4	2.0571	10.469	15.4496	20.4985	170.55
5	1.86448	10.4905	15.4433	20.4988	197.81
6	1.67464	10.5106	15.4374	20.499	225.58
7	1.48785	10.5295	15.4318	20.4992	254.01
8	1.30409	10.5471	15.4267	20.4993	283.38

We see that as h increases it results in decrease in q_0, increase in q_1 but q_2 decreases and r remains almost constant. This results in increase in average cost.

6. Multiple Suppliers

We have generalized the model and consider the case where there are M suppliers, and at any time suppliers may be available or not available which we represent as ON or OFF state. The stochastic process representing the supplier availabilities would have 2^M states: 0, 1, 2, ...,2^M-1. State 0 would correspond to the situation where all the suppliers being ON, state 1 would correspond to only the M^{th} supplier being OFF *etc.* and finally state 2^M-1 would correspond to all being OFF. The transition probabilities $P_{ij}(t)$, $i, j = 0, 1, 2, ..., 2^M-1$, decision variables q_i and costs C_{i0}, $i = 0, 1, 2, ..., 2^M-1$ are defined in a manner similar to two suppliers. The system of equations for C_{i0} is obtained as

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right) A(q_i, r, \theta) + \sum_{j=1}^{2^M-1} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[A(q_i, r, \theta) + C_{j0}\right], \quad i = 0,1,\ldots,2^M-2 .$$

$C_{2^M-1,0} = \overline{C} + \sum_{i=1}^{M} \rho_i\, C_{i0}$ where, $\rho_i = \frac{\mu_i}{\sum_{j=1}^{M} \mu_j}$

$$\bar{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}(\delta r - (d+\theta)) + (\pi\delta d + h(d+\theta) + \hat{\pi}) - \theta c\delta\right] + \frac{\theta c}{\delta}, \quad \delta = \sum_{j=i}^{M} \mu_j$$

Equations for T_{i0} are written in a similar way as in Lemma (3.3).

Solving the above equations require the exact solution for the transient probabilities $P_{ij}(t)$ of the CTMC with the 2^M states which appears to be a formidable task, because we would first need the exact solution for the transient probabilities $P_{ij}(t)$ of the CTMC with the 2^M states. It would also be necessary to solve explicitly for the quantities C_{00} and T_{00} using the system of 2^M equations in 2^M unknowns. As the number of suppliers is very large, that is we have a situation approximating a free market, we can develop a much simpler model by assuming that if an order needs to be placed and at least one of the suppliers is available, then the order quantity will be q units regardless of which supplier is available. We combine the first 2^M-1 states where at least one supplier is available and define a super state denoted by o. The last state denoted by I, is the state where all the suppliers are OFF. We also assume that for any supplier the ON and OFF periods are exponential with parameters λ and μ, respectively. With these assumptions the expected cost and the expected length of a cycle are obtained as

$$C_{00} = A(q, r, \theta) + P_{0\,\mathrm{I}}\left(\frac{q}{d+\theta}\right) C_{\mathrm{I}0}(r)$$

$$T_{00} = \frac{q}{(d+\theta)} + \frac{P_{0\mathrm{I}}\left(\frac{q}{d+\theta}\right)}{M_\mu}$$

Therefore, the average cost function is given by

$$AC = \frac{C_{00}}{T_{00}}$$

where, A(q, r, θ), $P_{0\,I}\left(\frac{q}{(d+\theta)}\right)$ and $C\, Io(r)$ have the same meaning as in single supplier case.

Thus, when the number of suppliers become large, the objective function of multiple suppliers problem reduces to that of classical EOQ model. This can be shown by arguing that as the length of stay in state I is exponential with parameter M_μ it becomes a degenerate random variable with mass at 0; that is the process never visits or stays in state I.

7. Discussion and Conclusions

In this paper, we have analysed order quantity and reorder point of perishable stochastic inventory models with two and multiple suppliers. We have assumed that suppliers may be ON available or not available (OFF) at a given time and duration of these periods are exponential with specified parameters. Using concepts from renewal reward processes we have constructed the average cost objective function for the case of two and multiple suppliers. When the number of suppliers become large, the objective function of multiple suppliers problem reduces to that of classical EOQ model.

Acknowledgements

The authors express their gratefulness to the anonymous reviewer for giving some useful suggestions that led to considerable improvement in the presentation of the results. The authors also express their thanks to the Chief Editor for his suggestions on restructuring the contents for better expression.

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