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# Long Memory in Volatility: Application of Fractionally Integrated GARCH Model

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**Abstract**

Volatility is an important characteristic of time series. If the volatility of a series at any time epoch is affected by its distant counterpart, then it is known as long memory in volatility. The (FIGARCH) model is useful for addressing the long memory in volatility. In this paper, for empirical illustration, the daily modal spot price of mustard from four markets of Rajasthan namely Khedli (Laxmangarh), Atru, Nimbahera and Anoopgarh, are used. The GARCH, EGARCH, APARCH, GJR-GARCH, and FIGARCH models are fitted to the log return series of the selected datasets. It is seen that the FIGARCH model is the best-fitted model for all the time series and it confirmed the presence of long memory in volatility.

*Key words:* GARCH; Long memory; Nonlinear models; Time series; Volatility.

## 1. Introduction

Time series analysis is used to identify patterns and trends in the dataset, and it helps make predictions about future values. Time series modelling is a crucial aspect for understanding the price behaviour and movement of any economic goods including the prices of agricultural commodities. The major breakthrough in time series modelling was first pioneered by Box and Jenkins (1970) through the introduction of the autoregressive integrated moving average (ARIMA) model. The ARIMA model is based on the assumptions of linearity and stationarity of the dataset and the homoscedasticity of the error variance. Lots of applications of the ARIMA model can be found in the literature (Paul *et al.*, 2014, 2020; Agarwal *et al.*, 2021). Linear models take advantage of their analytical and implementable easiness over the others. But it is irrational to assume a priori about the linear process for time series. Volatility is the nonlinear aspect of time series. It is the degree of unexpected variation of its realizations over a certain period. Engle (1982) introduced the autoregressive conditional heteroscedastic (ARCH) model for capturing the volatility of any time series. Later, its generalization, i.e. generalized ARCH (GARCH) model was proposed by Bollerslev (1986) and Taylor (1986) independent of each other. Applications of the GARCH model can be found in Paul *et al.* (2009, 2015), etc. The GARCH model is symmetric. It does not

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account for the sign of shocks and only takes into consideration the amount of shocks' effects on volatility. Hence, it cannot capture the asymmetric behaviour of price volatility, i.e., reactions to the volatility may differ depending on whether the positive and negative shocks are of the same magnitude. The exponential GARCH (EGARCH) model (Nelson, 1991), Asymmetric Power ARCH (APARCH) model (Ding *et al.*, 1993), and GJR-GARCH model (Glosten *et al.*, 1993) are better alternatives to the GARCH model for addressing asymmetric volatility. Again, the realizations of a time series may have long term dependency. In the presence of long term dependency, the autocorrelation function (ACF) and partial autocorrelation function (PACF) are significant for a long lag. This is known as hyperbolic decay. The long memory process can be present in both linear and nonlinear dynamics of a time series. If long memory is present in the linear model then the autoregressive fractionally integrated moving average (ARFIMA) model (Granger and Joyeux, 1980) is useful. Fractional integration is a generalization of ordinary integration, where the integral is taken to a fractional power. Some applications of the ARFIMA model can be found in Paul (2014) and Rakshit *et al.* (2022). Similarly, the fractionally integrated GARCH (FIGARCH) model (Baillie *et al.*, 1996) is useful for capturing the long memory in volatility. Paul *et al.* (2016) applied the FIGARCH model for modelling long memory in the volatility of the spot price of gram in Delhi, India. In the presence of long memory both in the mean and variance structure, Mitra *et al.* (2018) applied the ARFIMA-FIGARCH models for modelling the potato price of the Agra and Amritsar markets, India.

Agriculture is the backbone of the Indian economy. Around 60% of the Indian population depends on agriculture for their livelihood. As per the Second Advance Estimates of National Income, 2022-23 released by the Ministry of Statistics and Programme Implementation (MoSPI), the share of Gross value added (GVA) of agriculture and allied sectors in the total economy is 18.3% at current prices. The volatility study of the price series of agricultural commodities is an important aspect to social science researchers (Paul and Garai, 2021; Rakshit *et al.*, 2021, 2023; Garai *et al.*, 2023). Mustard is an important oilseed crop in India. It is grown in the rabi (winter) season and is a major source of edible oil for the country. The oilcake from mustard seeds is used as a feed for livestock. In addition to its edible oil, mustard has a number of other uses. The leaves of the mustard plant can be eaten as a vegetable, and the flowers can be used to make mustard seed paste, which is used as a condiment. Mustard seeds also have medicinal properties and have been used traditionally to treat a variety of ailments, including arthritis, rheumatism, and respiratory problems. Mustard cultivation provides livelihood opportunities for a large number of farmers, especially in the states of Rajasthan, Uttar Pradesh, Madhya Pradesh, and Punjab, where it is extensively grown. Earlier, it is seen that the modelling and forecasting of rapeseed and mustard prices helps in improving decision making in Rajasthan (Bhardwaj *et al.*, 2015). In the present study, the modal daily spot price series of mustard for Khedli (Laxmangarh), Atru, Nimbahera and Anoopgarh markets of Rajasthan are used. The GARCH, EGARCH, APARCH, GJR-GARCH and FIGARCH models are applied to the selected time series. Section 2 includes a description of the used models. The empirical illustration is given in Section 3 followed by concluding remarks in Section 4.

## 2. Materials and methods

### 2.1. The ARCH and GARCH models

ARIMA is a linear model that cannot address the nonlinear dynamics of a time series. Homoscedasticity in the error variance is a basic assumption of this model. By relaxing the linear and homoscedasticity assumptions, the ARCH model is introduced by taking into account substantial autocorrelations present in the squared residual series to capture the nonlinear dynamics of a time series. A process $\{\varepsilon_t\}$ is said to follow the ARCH $(q)$ model if the conditional distribution of $\{\varepsilon_t\}$ given the available information $(\psi_{t-1})$ up to $t-1$ time epoch is represented as:

$$\varepsilon_t|\psi_{t-1} \sim \ N\left(0, h_t\right) \ and \ \varepsilon_t = \sqrt{h_t}\nu_t \tag{1}$$

where $\nu_t$ is identically and independently distributed (IID) innovation with zero mean and unit variance. The conditional variance $h_t$ of ARCH $(q)$ model is calculated as

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \ \alpha_0 > 0, \ \alpha_i \geq 0 \ \forall \ i \ and \ \sum_{i=1}^{q} \alpha_i < 1 \tag{2}$$

The GARCH model is a more parsimonious version of the ARCH model where the number of parameters to be estimated is less. Here, the conditional variance is treated as a linear function of its own lags. The GARCH $(p, q)$ model has the following form of conditional variance

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{3}$$

provided $\alpha_0 > 0, \alpha_i \geq 0 \ \forall \ i \ \beta_j \geq 0 \ \forall \ j$

$\alpha_i$ and $\beta_j$ parameters indicate how previous shocks and volatility have influenced current volatility, respectively. The GARCH $(p, q)$ model is said to be weakly stationary if and only if

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \tag{4}$$

The GARCH model only considers the dependencies of volatility on the magnitude of the shocks, and it does not consider the sign of the shocks that influence the degree of volatility. The EGARCH, APARCH, and GJR-GARCH models can be useful to overcome this gap.

### 2.2. EGARCH model

The EGARCH model is introduced by defining the conditional variance in terms of the logarithm function. The main advantage of this model over the GARCH model, aside from addressing the asymmetric volatility, is that no restriction is imposed on the parameters as the positivity of the conditional variance is always achieved. The conditional variance for the EGARCH model is defined as

$$\ln h_t = \ \alpha_0 + \sum_{j=1}^{p} \beta_j \ln h_{t-j} + \sum_{i=1}^{q} \left( \alpha_i \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \ \gamma_i \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right) \tag{5}$$

where, $\gamma_i$ is the asymmetric factor which explains the asymmetric effect due to external shocks. For EGARCH (1,1) model conditional variance $h_t$ is reduced to

$$\ln h_t = \alpha_0 + \beta_1 \ln h_{t-1} + \left( \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right) \tag{6}$$

## 2.3. APARCH model

The APARCH model considers some asymmetric power to the conditional variance $h_t$. The conditional variance of the APARCH model is defined as

$$h_t^{\frac{\delta}{2}} = \alpha_0 + \sum_{j=1}^{p} \beta_j h_{t-j}^{\frac{\delta}{2}} + \sum_{i=1}^{q} \alpha_i \left( |\varepsilon_{t-i}| - \gamma \varepsilon_{t-i} \right)^{\delta} \tag{7}$$

where $\gamma(-1 < \gamma < 1)$ is the parameter for asymmetry and $\delta(> 0)$ is the power term parameter. The APARCH model is a general framework of models. Different orders of GARCH models can be fitted within the APARCH model by defining specific values to the parameters. For $\delta = 2$ and $\gamma = 0$, the APARCH model is the same as the GARCH model. The conditional variance $h_t$ for APARCH (1,1) model is reduced to

$$h_t^{\frac{\delta}{2}} = \alpha_0 + \beta_1 h_{t-1}^{\frac{\delta}{2}} + \alpha_1 \left( |\varepsilon_{t-1}| - \gamma \varepsilon_{t-1} \right)^{\delta} \tag{8}$$

## 2.4. GJR-GARCH model

The GJR-GARCH model considers the impact of $\varepsilon_{t-1}^2$ on the conditional variance based on the sign of $\varepsilon_{t-1}$. An indicator variable is introduced to capture the sign dependency. The conditional variance of the GJR-GARCH model is defined as

$$h_t = \alpha_0 + \sum_{j=1}^{p} \beta_j h_{t-j} + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \tag{9}$$

where $\gamma(-1 < \gamma < 1)$ is the asymmetric parameter and $I_{t-1}$ is the indicator variable, such that

$$\begin{aligned} I_{t-1} = 1 \quad & if\ \varepsilon_{t-1} < 0 \\ 0 \quad & if\ \varepsilon_{t-1} \geq 0 \end{aligned}$$

For GJR-GARCH (1,1) model conditional variance $h_t$ is reduced to

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma \varepsilon_{t-1}^2 I_{t-1} \tag{10}$$

## 2.5. FIGARCH model

The FIGARCH model is useful when the volatility is symmetric i.e. positive and negative shocks of the same magnitude exhibit the same response to volatility and the volatility exhibits long term persistence. The FIGARCH model is derived by introducing a fractional differencing parameter in the GARCH model after some algebraic operations.

Tayefi and Ramanathan (2012) provided a thorough review of the FIGARCH model. The FIGARCH $(p, d, q)$ model can be expressed as

$$[1-\alpha(L)-\beta(L)](1-L)^d \varepsilon_t^2 = \alpha_0 + [1-\beta(L)] z_t \tag{11}$$

where, $\alpha(L)$ and $\beta(L)$ are polynomials in lag operator and $(1-L)^d$ is the fractional difference operator. Here, $d$ is a fraction and $0 < d < 1$.

## 3. Empirical illustration

### 3.1. Data description

For empirical illustration purposes, the daily modal spot prices (Rs./q) of mustard for four markets in Rajasthan namely Khedli (Laxmangarh), Atru, Nimbahera and Anoopgarh are collected from the Ministry of Agriculture and Farmers' Welfare, Government of India for the study period of 1st January 2010 to 31st May 2023 (total number of observation is 4899). Since the square of return is regarded as the realization of volatility, the analysis is done with the log return series of the selected time series data. For a time series $\{y_t\}$ the log return series $\{r_t\}$ is calculated as

$$r_t = \ln \frac{y_t}{y_{t-1}} \tag{12}$$

The latest 250 realizations of the log return series of each of the selected markets are used as the model validation set, while the remaining previous portion is used as the model building set.

### 3.2. Descriptive statistics

The descriptive statistics of the selected price series are given in Table 1. The Khedli market has the highest mean price, while the Nimbahera market has the lowest mean price. The Atru market has the highest median price and the Nimbahera market is the lowest one. Regarding the minimum price, the Khedli market minimum price is significantly lower than the others. The Khedli market has the highest maximum price and the Nimbahera has the lowest maximum price. All the selected price series are positively skewed and leptokurtic. Figure 1 shows the time plots of the selected price series. The time plots of all the price series show a similar pattern of price variation.

### 3.3. Test for stationarity

The stationarity of the time series is a prior assumption for the GARCH modelling. Using the Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979), Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski *et al.*, 1992), and the Phillips-Perron (PP) test (Phillips and Perron, 1988), the stationarity of the log return series and the squared log return series are tested (Table 2). For ADF and PP tests, the null hypothesis is that the unit root is present in the time series. For the KPSS test, the null hypothesis is that the unit root is not present in the time series. All three tests terminate the possibility of the

**Table 1: Descriptive statistics of selected price series**

| Statistics | Khedli | Atru | Nimbahera | Anoopgarh |
|---|---|---|---|---|
| Mean (Rs./q) | 4040.02 | 3883.61 | 3748.34 | 3863.27 |
| Median (Rs./q) | 3523.35 | 3589.03 | 3475.82 | 3588.00 |
| Minimum (Rs./q) | 1055.00 | 2026.00 | 2000.00 | 2108.00 |
| Maximum (Rs./q) | 8300.00 | 8091.00 | 7715.00 | 8031.00 |
| S.D. (Rs./q) | 1215.30 | 1232.00 | 1175.12 | 1234.29 |
| CV (%) | 30.08 | 31.72 | 31.35 | 31.95 |
| Skewness | 1.45 | 1.15 | 1.00 | 1.11 |
| Kurtosis | 1.32 | 0.84 | 0.39 | 0.75 |

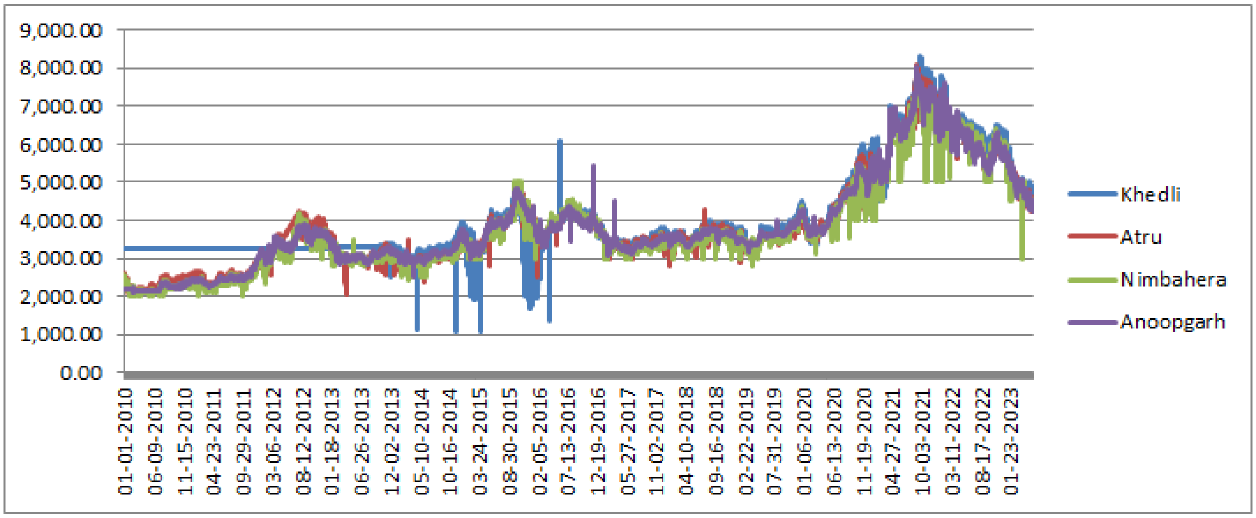


**Figure 1: Time plots of the daily price series**

presence of a unit root in the log return series and the squared log return series ($p$-values are given in parenthesis).

**Table 2: Test for stationarity**

| Market | Khedli | Atru | Nimbahera | Anoopgarh |
|---|---|---|---|---|
| Series | Log return | Log return | Log return | Log return |
| ADF | -23.44 | -18.5 | -18.89 | -17.97 |
| | (0.01) | (0.01) | (0.01) | (0.01) |
| KPSS | 0.01 | 0.04 | 0.03 | 0.10 |
| | (0.10) | (0.10) | (0.10) | (0.10) |
| PP | -5674.1 | -6079.5 | -5850.8 | -5724.6 |
| | (0.01) | (0.01) | (0.01) | (0.01) |
| Series | Squared log return | Squared log return | Squared log return | Squared log return |
| ADF | -15.15 | -14.81 | -14.98 | -15.85 |
| | (0.01) | (0.01) | (0.01) | (0.01) |
| KPSS | 0.39 | 0.25 | 1.22 | 0.41 |
| | (0.08) | (0.10) | (0.10) | (0.07) |
| PP | -2356.8 | -2527 | -2644 | -2781.9 |
| | (0.01) | (0.01) | (0.01) | (0.01) |

### 3.4. Test for long memory

The GPH test (Geweke and Porter-Hudak, 1983) is used to check the presence of long memory in the log return series and the squared log return series (Table 3). It is seen that the fractional differencing parameters for the log return series are not significant. But, they are significant for their corresponding squared log return series. It implies that the long memory is present in the squared log return series but not in the log return series.

**Table 3: GPH test**

| Market | Log return | | | Squared log return | | |
|---|---|---|---|---|---|---|
| | $d$ | s.e. | Z | $d$ | s.e. | Z |
| Khedli | -0.088 | 0.081 | -1.085 | 0.211 | 0.073 | 2.884 |
| Atru | 0.157 | 0.088 | 1.776 | 0.128 | 0.065 | 1.976 |
| Nimbahera | -0.137 | 0.097 | -1.403 | 0.24 | 0.106 | 2.259 |
| Anoopgarh | -0.095 | 0.096 | -0.987 | 0.224 | 0.086 | 2.596 |

### 3.5. ACF and PACF plots

The ACF and PACF plots help to examine the statistical relationships between the realizations of a time series through visualization. Figure 2 depicts the ACF and PACF plots of the selected log return series and the ACF plots of the squared log return series. The ACF and PACF plots of the log return series are decaying at exponential rates. It implies the absence of long memory in the mean model. But, hyperbolic decay is visible in the ACF plots of the squared log return series. It implies the presence of long memory in volatility. The GPH test's results also support the same conclusions.

### 3.6. Fitting of models

In the first step, the AR (1) model is fitted as the mean model in all the log return series. After that, the residuals are obtained and tested for the presence of conditional heteroscedasticity using the ARCH-LM test. The null hypothesis for this test is the absence of the ARCH effect in the residual series. It is seen that the ARCH-LM test is significant for all residual series and the presence of ARCH effect in all the residual series is confirmed. After that, the GARCH, EGARCH, APARCH, GJR-GARCH, and FIGARCH models are fitted to the residual series. The parameters are estimated using the maximum likelihood estimation procedure. The best fitted model for all the time series is chosen based on the degree of fitting in terms of three popularly used error functions namely Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) in the model building set. These error functions are calculated as

$$RMSE = \left[\frac{1}{k}\sum_{t=1}^{k}(y_t - \hat{y}_t)^2\right]^{\frac{1}{2}} \tag{13}$$

$$MAE = \frac{1}{k}\sum_{t=1}^{k}|y_t - \hat{y}_t| \tag{14}$$

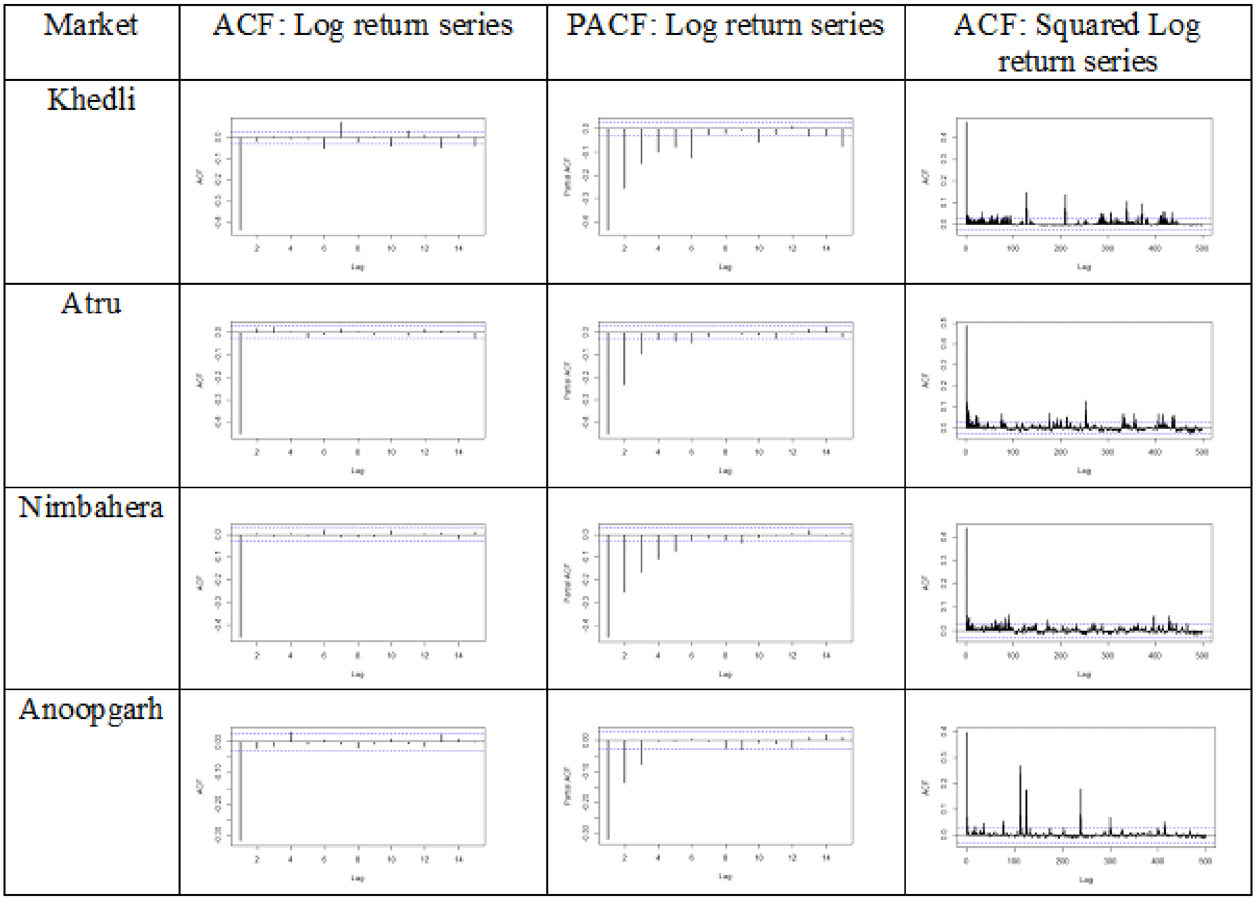


**Figure 2: ACF and PACF plots**

$$MAPE = \frac{1}{k}\sum_{t=1}^{k}\frac{|y_t - \hat{y}_t|}{y_t} \times 100 \tag{15}$$

where $k$ denotes the number of realizations used, $y_t$ is the observed value and $\hat{y}_t$ is the corresponding predicted value.

The estimated parameters of the best-fitted models are given in Table 4. It is seen that the FIGARCH model is the best-fitted model for all the markets. For all the series the parameters $\alpha_1$, $\beta_1$ and $d$ are highly significant. This implies that the current volatility significantly depends on previous volatility as well as previous shock. The presence of long memory is also significant for all cases. Fitting performances of the used models in the model building set in terms of RMSE, MAE and MAPE are given in Table 5. It is seen that for all the markets the best fitted model is the AR (1)-FIGARCH (1, $d$, 1) model. The ACF and PACF plots of the residual series, after fitting the AR (1)-FIGARCH (1, $d$, 1) model, for all the markets, do not exhibit any systematic trend and almost all the correlations lie within the 95% confidence interval.

In the model validation set, the rolling window forecast for 50 days, 100 days, 150 days, 200 days and 250 days are obtained and they are given in Table 6. It can be seen that for Khedli and Anoopgarh markets the forecasting performance is improving by increasing

**Table 4: Estimate of parameters of the best-fitted models**

| Market | Khedli | Atru | Nimbahera | Anoopgarh |
|---|---|---|---|---|
| parameter | AR(1) - FIGARCH (1, $d$, 1) | AR(1) - FIGARCH (1, $d$, 1) | AR(1) - FIGARCH (1, $d$, 1) | AR(1) - FIGARCH (1, $d$, 1) |
| Mean Model | | | | |
| Constant | 0.000 (0.000)*** | -0.001 (0.000)*** | -0.000 (0.000) | 0.000 (0.000) |
| AR(1) | -0.550 (0.000)*** | -0.327 (0.016)*** | -0.410 (0.016)*** | -0.230 (0.019)*** |
| Variance Model | | | | |
| Constant | 0.000 (0.000) | 0.000 (0.000)*** | 0.000 (0.000) | 0.000 (0.000)*** |
| $\alpha_1$ | 0.152 (0.000)*** | 0.961 (0.004)*** | 0.820 (0.194)*** | 0.311 (0.026)*** |
| $\beta_1$ | 0.649 (0.000)*** | 0.942 (0.003)*** | 0.797 (0.219)*** | 0.824 (0.012)*** |
| $d$ | 0.727 (0.000)*** | 0.577 (0.013)*** | 0.463 (0.050)*** | 0.631 (0.033)*** |

**Table 5: Fitting performance of the selected models in the model building set**

| Market | Model | RMSE | MAE | MAPE (%) |
|---|---|---|---|---|
| Khedli | AR(1)-GARCH (1,1) | 103.165 | 35.684 | 0.930 |
| | AR(1)-EGARCH(1,1) | 107.785 | 36.921 | 0.962 |
| | AR(1)-APARCH (1,1) | 99.267 | 36.163 | 0.702 |
| | AR(1)-GJRGARCH (1,1) | 98.542 | 36.408 | 0.688 |
| | AR(1)-FIGARCH (1, $d$, 1) | 67.986 | 23.223 | 0.575 |
| Atru | AR(1)-GARCH (1,1) | 39.918 | 20.478 | 0.520 |
| | AR(1)-EGARCH(1,1) | 39.871 | 18.858 | 0.478 |
| | AR(1)-APARCH (1,1) | 38.233 | 19.554 | 0.496 |
| | AR(1)-GJRGARCH (1,1) | 39.344 | 20.167 | 0.512 |
| | AR(1)-FIGARCH (1, $d$, 1) | 32.608 | 16.317 | 0.410 |
| Nimbahera | AR(1)-GARCH (1,1) | 56.147 | 28.414 | 0.755 |
| | AR(1)-EGARCH(1,1) | 50.373 | 25.513 | 0.678 |
| | AR(1)-APARCH (1,1) | 56.484 | 28.583 | 0.760 |
| | AR(1)-GJRGARCH (1,1) | 59.615 | 30.157 | 0.802 |
| | AR(1)-FIGARCH (1, $d$, 1) | 48.295 | 24.481 | 0.651 |
| Anoopgarh | AR(1)-GARCH (1,1) | 19.260 | 9.822 | 0.240 |
| | AR(1)-EGARCH(1,1) | 16.133 | 8.229 | 0.218 |
| | AR(1)-APARCH (1,1) | 16.999 | 8.670 | 0.212 |
| | AR(1)-GJRGARCH (1,1) | 18.385 | 9.376 | 0.229 |
| | AR(1)-FIGARCH (1, $d$, 1) | 14.963 | 7.652 | 0.202 |

the forecast horizon. For the Atru market, the numerical values of these three error functions first increase and then decrease. For the Nimbahera market, they decrease, then increase and again then decrease. All these are because of the presence of long memory in volatility. Long memory in volatility plays a crucial role in increasing the forecast efficiency while increasing the forecast horizon at different levels.

**Table 6: Rolling window forecasting performance of best-fitted models in the model validation set**

| Market | Model | Horizon | RMSE | MAE | MAPE (%) |
|---|---|---|---|---|---|
| Khedli | AR (1) - FIGARCH (1, $d$ , 1) | 50 | 138.469 | 89.017 | 1.595 |
| | | 100 | 131.694 | 87.371 | 1.529 |
| | | 150 | 129.799 | 86.693 | 1.488 |
| | | 200 | 124.836 | 85.922 | 1.472 |
| | | 250 | 117.413 | 81.497 | 1.427 |
| Atru | AR (1) - FIGARCH (1, $d$ , 1) | 50 | 75.871 | 61.047 | 1.054 |
| | | 100 | 84.717 | 66.929 | 1.145 |
| | | 150 | 89.824 | 68.346 | 1.207 |
| | | 200 | 88.569 | 65.951 | 1.206 |
| | | 250 | 87.919 | 65.031 | 1.202 |
| Nimbahera | AR (1) - FIGARCH (1, $d$ , 1) | 50 | 202.693 | 116.820 | 2.039 |
| | | 100 | 166.421 | 102.061 | 1.748 |
| | | 150 | 185.467 | 113.154 | 2.072 |
| | | 200 | 241.414 | 114.319 | 2.249 |
| | | 250 | 218.968 | 101.744 | 2.024 |
| Anoopgarh | AR (1) - FIGARCH (1, $d$ , 1) | 50 | 141.472 | 106.313 | 1.874 |
| | | 100 | 131.277 | 98.890 | 1.719 |
| | | 150 | 126.541 | 96.082 | 1.710 |
| | | 200 | 122.185 | 94.336 | 1.709 |
| | | 250 | 118.971 | 91.240 | 1.707 |

## 4. Conclusions

In this article, the mustard price volatility of four markets from the state of Rajasthan is studied. The presence of long memory in volatility for these series is confirmed using the GPH test. The GARCH, EGARCH, APARCH, GJR-GARCH and FIGARCH models are fitted to the log return series of the selected markets and it is seen that the FIGARCH model is the best fitted model for all the markets. The presence of long memory in volatility helps increase the model forecasting efficiency on a larger horizon. A better understanding of price volatility in the presence of long memory in volatility can help improve decision scenarios.

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