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A Survey on Cyclic Solution of Block Designs

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Abstract

Cyclic designs are incomplete block designs based on cyclic development of one or more initial blocks. John *et al.* (1972) described the advantages of cyclic designs as calibration designs and experimental designs and tabulated these designs in the useful range of parameters which was published by National Bureau of Standards, Washington, DC. The cyclic designs may have up to $v/2$ associate classes. The purpose of this survey is to present cyclic solutions of balanced incomplete block designs, group divisible designs, Latin square type designs and cyclic designs, wherever possible, which have at most two associate classes and higher efficiencies.

Key words: Balanced incomplete block (BIB) designs; Semi–regular and regular group divisible designs; Latin square type designs; Cyclic Designs.

1. Introduction

Cyclic designs are incomplete block designs based on cyclic development of one or more initial blocks. Their flexibility, ease in conduct of experiment and natural groupings for one-way elimination of heterogeneity, make them worthy of attention in their own right. All cyclic designs are partially balanced incomplete block (PBIB) designs with up to $v/2$ associate classes. Among the class of cyclic designs, cyclic balanced incomplete block (BIB) designs are obviously best in the sense that all the pair-wise treatment comparisons are measured with same and maximum efficiency. When no cyclic BIB design exists, then we look for cyclic solution of two associate class PBIB design with same (v, b, r, k). These designs are used as calibration designs and experimental designs [see John *et al.* (1972), Clatworthy (1973), John and Williams (1995)]. Cyclic designs were catalogued by John *et al.* (1972). The cyclic solutions of BIB designs were given by Hall (1998), wherever possible. Clatworthy (1973) tabulated two associate classes PBIB designs. The purpose of this paper is to present a survey on cyclic solutions of BIB designs, group divisible designs, Latin square type designs and cyclic designs in the range of $r, k \leq 10$.

The concept of cyclic designs is extended to generalized cyclic designs which are useful as factorial experiments [see Jarrett and Hall (1978), Lamacraft and Hall (1982), Nigam *et al.* (1988), Dean and Lewis (1990), Bailey (1990)].

A *Group divisible design* (*GDD*) is an arrangement of v (= mn; $m, n \geq 2$) treatments into b blocks such that each block contains k ($<v$) distinct treatments, each treatment occurs r times and any pair of distinct treatments which are first associates occur together in λ_1 blocks and in λ_2 blocks if they are second associates. Furthermore, if $r–\lambda_1 = 0$ then the GD design is

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singular (S); if $r–\lambda_1 > 0$ and $rk–v\lambda_2 = 0$ then it is semi–regular (SR); and if $r–\lambda_1 > 0$ and $rk–v\lambda_2 > 0$, the design is regular (R). For definitions and terminologies, we refer to Dey (1986, 2010), Raghavarao (1971), Raghavarao and Padgett (2005).

2. Cyclic Solutions of Block Designs

Table 1: Cyclic Solutions: BIBD/ GD design/ Cyclic Design / Latin square type design

No.	BIBD/ GDD/ CD/ LSD: (*v*, *r*, *k*, *b*); Overall Efficiency	John No.; Overall Efficiency	Cyclic Solutions
1[M]	*SR*1: (4, 2, 2, 4); 0.60	-	*G*: (1, 4) mod 4
2[*]	*C*1: (5, 2, 2, 5); 0.50	-	*C*: (1, 3) mod 5
3[*]	*C*6: (5, 6, 2, 15); 0.61	-	*C*: (1, 3); (1, 3); (1, 2) mod 5
4[*]	*C*7: (5, 8, 2, 20); 0.59	-	*C*: (1, 3); (1, 3); (1, 3); (1, 2) mod 5
5[*]	*C*8: (5, 10, 2, 25); 0.58	-	*C*: (1, 3); (1, 3); (1, 3); (1,3); (1, 2) mod 5
6*	*C*9: (5, 10, 2, 25); 0.62	-	*C*: (1, 3); (1, 3); (1, 3); (1, 2); (1, 2) mod 5
7[M]	*SR*7: (6, 6, 2, 18); 0.56	2×*A*2; 0.55	*G*: (0, 1); (0, 3); (0, 5) mod 6
8[M]	*SR*13: (12, 6, 2, 36); 0.52	2×*A*26; 0.39	*G*: (0,1); (0, 3); (0, 5) mod 12
9[*]	*C*10: (13, 6, 2, 39); 0.50	*A*36; 0.50	(1, 3); (1, 6); (1, 7) mod 13
10[M]	*SR*15: (16, 8, 2, 64); 0.52	*A*57; 0.52	*G*: (0, 1); (0, 3); (0, 5); (0, 7) mod 16
11[*]	*C*11: (17, 8, 2, 68); 0.50	*A*62; 0.51	(1, 4); (1, 6); (1, 7); (1, 8) mod 17
12[M]	*SR*17: (20, 10, 2, 100); 0.51	*A*81; 0.51	*G*: (0, 1); (0, 3); (0, 5); (0, 7); (0, 9) mod 10
13[*]	*C*12: (5, 3, 3, 5); 0.81	-	*C*: (1, 2, 4) mod 5
14[*]	*C*15: (5, 9, 3, 15); 0.83	-	(1, 3, 5); (1, 3, 5); (1, 2, 5) mod 5
15[*]	*R*42: (6, 3, 3, 6); 0.78	*B*1, 0.78	*G*: (1, 2, 4) mod 6
16	*H*1: (7, 3, 3, 7); 0.78	*B*2; 0.78	*B*: (1, 2, 4) mod 7
17[*]	*R*54: (8, 3, 3, 8); 0.75	*B*3; 0.75	*G*: (1, 2, 4) mod 8
18[DN]	*R*55: (8, 6, 3, 16); 0.75	*B*5; 0.75	*G*: (1, 2, 3); (1, 3, 6) mod 8
19[*]	*R*58: (8, 9, 3, 24); 0.76	3× *B*3; 0.75	*G*: (1, 2, 3); (1, 2, 5); (1, 3, 6) mod 8
20[MD]	*SR*23: (9, 3, 3, 9); 0.73	*B*9; 0.72	*G*: (3, 5, 8); (2, 6, 8); (2, 5, 9); 1↔3, 4↔6, 7↔9 (*PC*)
21	*H*2: (9, 4, 3, 12); 0.75	-	Add the blocks: $(1+3x, 2+3x, 3+3x)$; $0 \leq x \leq 2$ to the solution in Serial No. 20
22[M]	*SR*25: (9, 9, 3, 27); 0.73	3× *B*9; 0.72	*G*: (0, 1, 2); (0, 4, 8); (0, 5, 7) mod 9
23[MD]	*R*68: (9, 10, 3, 30); 0.74	-	*G*:(1, 2, 3); (1, 2, 6); (1, 3, 5); (1, 4, 7) mod 9
24	*H*26: (10, 9, 3, 30); 0.74	*B*14; 0.70	*B*: (∞, 0, 5); (0, 1, 4); (0, 2, 3); (0, 2, 7) mod 9
25[*]	*C*16: (13, 3, 3, 13); 0.67	*B*50; 0.67	*C*: (1, 3, 9) mod 13
26	*H*9: (13, 6, 3, 26); 0.72	-	*B*: (1, 3, 9); (2, 5, 6) mod13
27[*]	*C*19: (13, 9, 3, 39); 0.72	*B*54; 0.72	*C*: (1, 12, 13); (3, 10, 13); (4, 9, 13) mod 13
28[*]	*R*80: (14, 9, 3, 42); 0.67	*B*64; 0.67	*G*: (1, 2, 8); (1, 8, 9); (1, 3, 8); (1, 8, 10); (1, 4, 8); (1, 8, 11); 1↔7, 8↔14 (*PC*)
29[*]	*R*81: (15, 6, 3, 30); 0.71	*B*75; 0.71	*G*: (1, 4, 15); (2, 8, 15) mod 15
30[*]	*R*83: (15, 9, 3, 45); 0.71	*B*77; 0.71	*G*: (1, 7, 13); (1, 4, 5); (1, 3, 8) mod 15 *G*: (1, 2, 5); (1, 3, 8); (1, 4, 10) mod15
31	*H*14: (15, 7, 3, 35); 0.71	*B*76; 0.71	*B*: $(1_1, 4_1, 0_2)$; $(2_1, 3_1, 0_2)$; $(1_2, 4_2, 0_3)$; $(2_2, 3_2, 0_3)$; $(1_3, 4_3, 0_1)$; $(2_3, 3_3, 0_1)$;

			$(0_1, 0_2, 0_3)$ mod 5
32*	*LS*18: (16, 3, 3, 16); 0.63	*C1*; 0.63	*L*: (7, 10, 16); (4, 6, 13); (4, 7, 9); (2, 9, 16); 1↔4, 5↔8, 9↔12, 13↔16 (PC)
33*	*R*86: (16, 6, 3, 32); 0.70	2× *C*1; 0.63	*G*: (1, 2, 11); (1, 3, 6) mod 16
34*	*R*87: (16, 9, 3, 48); 0.71	*C*1; 0.63	*G*: (1, 5, 13); (1, 2, 11); (1, 3, 6) mod 16
35*	*R*89: (18, 9, 3, 54); 0.70	*C*11; 0.61	*G*: (1, 11, 13); (1, 10, 14); (1, 15, 18); (1, 16, 17); (1, 2, 5); (1, 3, 12); 1↔9, 10↔18 (*PC*)
36[F]	*R*89*a*: (18, 10, 3, 60); 0.69	-	*G*: (*A*1, *A*2, *B*1); (*B*1, *B*2, *A*1); (*A*1, *A*8, *B*1); (*B*1, *B*8, *A*4); (*A*1, *A*6, *B*4); (*B*1, *B*6, *A*1); $\frac{1}{3}$\{(*A*1, *A*4, *A*7), (*B*1, *B*4, *B*7)\} mod 9
37*	*R*91: (21, 9, 3, 63); 0.70	3× *C*32; 0.60	*G*: (1, 2, 11); (1, 3, 7); (1, 4, 9) mod 21
38	*H*38: (21, 10, 3, 70); 0.70	-	*B*: $(1_1, 6_1, 0_2)$; $(2_1, 5_1, 0_2)$; $(3_1, 4_1, 0_2)$; $(1_2, 6_2, 0_3)$;$(2_2, 5_2, 0_3)$; $(3_2, 4_2, 0_3)$; $(1_3, 6_3, 0_1)$; $(2_3, 5_3, 0_1)$; $(3_3, 4_3, 0_1)$; $(0_1, 0_2, 0_3)$ mod 7
39*	*R*92: (24, 9, 3, 72); 0.69	3× *C*52; 0.58	*G*: (1, 2, 12); (1, 3, 8); (1, 4, 10) mod 24
40*	*LS*22: (25, 6, 3, 50); 0.67	2× *C*60; 0.57	*L*: (1, 5, 25); (9, 14, 15); (18, 23, 24); (2, 7, 8); (11, 16, 17); (1, 3, 13); (12, 15, 22); (6, 21, 24); (8, 10, 20); (4, 17, 19); 1↔5, 6↔10, 11↔15, 16↔20, 21↔25 (PC)
41*	*C*20: (37, 9, 3, 111); 0.67	-	(1, 10, 26); (1, 31, 34) (1, 11, 37) mod 37
42*	*R*94: (6, 4, 4, 6); 0.89	-	*G*: (1, 2, 4, 6) mod 6
43[DB]	*SR*36: (8, 4, 4, 8); 0.84	*B*6; 0.85	*G*: (2, 3, 4, 5); (1, 6, 7, 8); 1↔4, 5↔8 (*PC*)
44*	*R*98: (8, 8, 4, 16); 0.85	2× *B*6; 0.85	*G*: (1, 2, 3, 5); (1, 2, 4, 6) mod 8
45[MD]	*SR*39: (8, 8, 4, 16); 0.84	2× *B*6; 0.85	*G*: (1, 4, 6, 7); (1, 2, 3, 4) mod 8
46*	*R*104: (9, 4, 4, 9); 0.80	*B*12; 0.83	*G*: (1, 2, 4, 7) mod 9 *J*: (1, 2, 4, 5) mod 9
47*	*R*105: (9, 8, 4, 18); 0.80	2× *B*12; 0.83	*G*: (1, 2, 4, 7); (1, 2, 5, 8) mod 9 2 copies of *J*: (1, 2, 4, 5) mod 9
48	*H*20: (9, 8, 4, 18); 0.84	2× *B*12; 0.83	*B*: (0, 1, 2, 4); (0, 1, 4, 6) mod 9
49[DB]	*R*106: (10, 8, 4, 20); 0.82	2× *B*18; 0.83	*G*: (3, 4, 5, 6); (1, 8, 9, 10); (2, 4, 5, 6); (1, 7, 9, 10); 1↔5, 6↔10 (*PC*)
50*	*R*109: (12, 4, 4, 12); 0.81	-	*G*: (1, 2, 5, 7) mod 12
51[F]	*R*109*a*: (12, 7, 4, 21); 0.82	-	*G*: (*A*1, *A*2, *A*3, *B*4); (*A*1, *A*3, *B*1, *B*6); (*A*1, *A*4, *B*2, *B*6); $\frac{1}{2}$(*B*1, *B*2, *B*4, *B*5) mod 6
52[MD]	*R*110: (12, 8, 4, 24); 0.81	*B*39; 0.82	*G*: (1, 2, 5, 7); (1, 2, 8, 10) mod 12
53[F]	*R*110*b*: (12, 10, 4, 30); 0.81	2× *B*37; 0.81	*G*: (*A*1, *A*2, *A*3, *A*6); (*A*1, *A*3, *B*4, *B*6); (*B*1, *B*2, *B*3, *A*6); (*B*1, *B*3, *A*4, *B*6); (*A*1, *A*2, *B*3, *B*5); (*B*1, *B*2, *A*3, *A*5); (mod 5) and 6 invariant
54	*H*3: (13, 4, 4, 13); 0.81	*B*55; 0.81	*B*: (0, 1, 3, 9) mod 13
55*	*C*21: (13, 8, 4, 26); 0.80	*B*56; 0.81	*C*: (1, 4, 12, 13); (1, 4, 10, 13) mod 13
56*	*R*112: (14, 4, 4, 14); 0.80	*B*65; 0.80	*G*: (1, 2, 5, 7) mod 14
57[MD]	*R*113: (14, 8, 4, 28); 0.80	*B*67; 0.80	*G*: (1, 2, 5, 7); (1, 2, 10, 12) mod 14
58[F]	*R*113*a*: (14, 10, 4, 35); 0.80	*B*69; 0.80	*G*: (*A*1, *A*2, *A*4, *B*7); (*B*1, *B*2, *B*7, *A*7); (*A*1, *A*2, *B*1, *B*2); (*A*1, *A*3, *B*1, *B*3); (*A*1, *A*4, *B*1, *B*4) mod 7
59*	*R*114: (15, 4, 4, 15); 0.80	*B*79; 0.80	*G*: (1, 3, 4, 12) mod 15
60*	*R*115: (15, 8, 4, 30); 0.73	*B*81; 0.80	*G*: (1, 2, 6, 11); (1, 6, 7, 11); (1, 6, 11, 12); (1, 6, 8, 11); (1, 6, 11, 13); (1, 3, 6, 11); 1↔-

			5, 6↔10, 11↔15 (*PC*) *J*: (1, 2, 5, 6); (1, 3, 9, 11) mod 15
61^{MD}	*R*116: (15, 8, 4, 30); 0.80	*B*81; 0.80	*G*: (0, 1, 3, 7); (1, 3, 4, 12) mod15
62^{*}	*R*117: (15, 8, 4, 30); 0.80	*B*81; 0.80	*G*: (1, 3, 11, 15); (1, 5, 7, 15) mod15
63^{*}	*LS*38: (16, 8, 4, 32); 0.80	2× *C*2; 0.79	*L*: (5, 6, 8, 11); (1, 5, 9, 13); (1, 4, 10, 15); (7, 13, 14, 16); (9, 10, 12, 15); (1, 2, 7, 12); (1, 5, 9, 13); (1, 3, 6, 16); 1↔4, 5↔8, 9↔12, 13↔16 (PC)
64^{*}	*C*22: (17, 8, 4, 34); 0.79	2× *C*7; 0.78	*C*: (2, 9, 11, 17); (1, 4, 5, 17) mod 17
65^{A}	*C*22*A*: (17, 10, 5, 34); 0.85	2× *C*8; 0.84	*C*: (0, 5, 12, 14, 3); (0, 7, 10, 11, 6) mod 17
66^{F}	*R*123*a*: (18, 10, 4, 45); 0.79	-	*G*: (*A*1, *A*2, *A*3, *B*4); (*A*1, *A*3, *B*5, *C*4); $\frac{1}{2}$(*A*1, *A*4, *B*2, *B*5) perm *A*, *B*, *C* mod 6
67	*SR*46: (20, 5, 4, 25); 0.78	-	By deleting the treatments 21, 22, 23, 24, 25 from *SR*60
68^{F}	*R*124*a*: (22, 8, 4, 44); 0.77	2× *C*41; 0.76	*G*: (*A*1, *A*3, *A*4, *B*1); (*A*1, *A*7, *B*1, *B*8) perm *A*, *B* and mod 11
69^{F}	*R*126*a*: (24, 9, 4, 54); 0.77	-	*G*: (*A*1, *A*2, *A*9, *B*1); (*B*1, *B*2, *B*9, *A*1); (*A*1, *A*4, *B*1, *B*11); (*B*1, *B*4, *A*1, *A*11); $\frac{1}{2}$(*A*1, *A*7, *B*1, *B*7) mod 12
70	*H*22: (25, 8, 4, 50); 0.78	2× *C*61; 0.75	*B*: [(0, 0); (1, 0); (0, 1); (4, 4)] mod (5, 5); [(0, 0); (2, 0); (0, 2); (3, 3)] mod (5, 5)
71^{*}	*R*128: (26, 8, 4, 52); 0.78	2× *C*68; 0.74	*G*: (2, 4, 10, 14); (1, 16, 19, 20); (3, 6, 7, 14); (1, 15, 17, 23); 1↔13, 14↔26 (*PC*)
72^{F}	*R*128*a*: (26, 10, 4, 65); 0.76	-	*G*: (*A*1, *A*6, *A*8, *B*1); (*B*1, *B*6, *B*8, *A*1); (*A*1, *A*2, *B*1, *B*4); (*B*1, *B*2, *A*1, *A*4); (*A*1, *A*5, *B*1, *B*5) mod 13
73^{*}	*R*132: (30, 10, 4, 75); 0.78	-	*G*: (1, 3, 15, 20); (5, 16, 18, 30); (1, 5, 11, 17); (2, 16, 20, 26); (1, 9, 16, 24); 1↔15, 16↔30 (*PC*)
74^{*}	*R*133: (8, 5, 5, 8); 0.90	-	*G*: (1, 2, 3, 5, 7) mod 8
75^{*}	*R*134: (8, 5, 5, 8); 0.91	-	*G*: (1, 3, 4, 5, 6) mod 8
76^{DN}	*R*136: (8, 10, 5, 16); 0.91	-	*G*: (1, 5, 6, 7, 8); (1, 3, 5, 6, 8) mod 8
77^{*}	*R*137: (9, 5, 5, 9); 0.89	-	*G*: (1, 3, 4, 6, 7) mod 9
78^{*}	*R*138: (9, 10, 5, 18); 0.89	-	*G*: (1, 3, 4, 6, 7); (1, 3, 4, 6, 9) mod 9
79^{*}	*R*139: (10, 5, 5, 10); 0.88	*B*21; 0.88	*G*: (1, 2, 3, 6, 8) mod 10
80^{*}	*R*141: (10, 10, 5, 20); 0.89	2× *B*21; 0.88	*G*: (1, 2, 3, 4, 7); (1, 2, 4, 6, 8) mod 10
81	*H*5: (11, 5, 5, 11); 0.88	*B*27; 0.88	*B*: (1, 3, 4, 5, 9) mod 11
82^{*}	*R*143: (12, 5, 5, 12); 0.81	*B*43; 0.87	*G*: (1, 2, 4, 7, 10) mod 12 *J*: (1, 2, 3, 5, 8) mod 12
83^{*}	*R*144: (12, 5, 5, 12); 0.87	*B*43; 0.87	*G*: (1, 2, 4, 9, 12) mod 12
84^{*}	*R*145: (12, 5, 5, 12); 0.87	*B*43; 0.87	*G*: (1, 2, 4, 6, 7) mod 12
85^{*}	*R*146: (12, 10, 5, 24); 0.81	2× *B*43; 0.87	*G*: (1, 2, 4, 7, 10); (1, 3, 4, 7, 10) mod 12 *J*: 2 copies of (1, 2, 3, 5, 8) mod 12
86^{MD}	*R*147: (12, 10, 5, 24); 0.87	2× *B*43; 0.87	*G*: (0, 1, 2, 4, 9); (0, 1, 2, 5, 10) mod 12 *J*: 2 copies of (1, 2, 3, 5, 8) mod 12
87^{*}	*R*148: (12, 10, 5, 24); 0.87	2× *B*43; 0.87	*G*: (1, 2, 3, 6, 12); (1, 3, 6, 8, 12) mod 12 *J*: 2 copies of (1, 2, 3, 5, 8) mod 12
88^{*}	*R*149: (15, 10, 5, 30); 0.82	2× *B*82; 0.85	*G*: (1, 2, 6, 7, 11); (1, 3, 6, 8, 11) mod 15

			J: 2 copies of (1, 2, 3, 5, 11) mod 15
89*	*R*150: (15, 10, 5, 30); 0.86	*B*82; 0.85	*G*: (1, 2, 3, 5, 8); (1, 2, 5, 9, 11) mod 15
90*S*	*R*150*a*: (15, 10, 5, 30); 0.84	*B*82; 0.85	*G*: (1, 2, 4, 7, 11); (1, 2, 4, 10, 13) mod 15
91*	*R*152: (20, 10, 5, 40); 0.74	-	*G*: (1, 2, 6, 11, 16); (1, 6, 7, 11, 16); (1, 6, 11, 12, 16); (1, 6, 11, 16, 17); (1, 6, 8, 11, 16); (1, 6, 11, 13, 16); (1, 6, 11, 16, 18); (1, 3, 6, 11, 16); 1↔5, 6↔10, 11↔15, 16↔20 (*PC*)
92	*H*7: (21, 5, 5, 21); 0.84	*C*34; 0.84	*B*: (3, 6, 7, 12, 14) mod 21
93*F*	*R*152*a*: (22, 10, 5, 44); 0.84		*G*: (A1, *A*2, *A*3, *A*6, *B*9) (*A*1, *A*3, *A*8, *B*2, *B*10); perm *A*, *B* and mod 11
94*	*R*153: (24, 5, 5, 24); 0.83	-	*G*: (1, 2, 5, 10, 12) mod 24
95*MD*	*R*154: (24, 10, 5, 48); 0.83	2× *C*54; 0.83	*G*: (1, 2, 5, 10, 12); (1, 2, 4, 12, 21) mod 24
96	*SR*60: (25, 5, 5, 25); 0.83	*C*62; 0.83	*G*: (1, 6, 11, 16, 21); (1, 7, 13, 19, 25); (1, 10, 14, 18, 22); (1, 9, 12, 20, 23); (1, 8, 15, 17, 24); 1↔5, 6↔10, 11↔15, 16↔20, 21 ↔25 (*PC*)
97	*H*11: (25, 6, 5, 30); 0.83	-	Add the blocks: (1+5*x*, 2+5*x*, 3+5*x*, 4+5*x*, 5+5*x*); 0≤ *x* ≤4 to the solution in Serial No. 96
98*	*R*159: (35, 10, 5, 70); 0.82	-	*G*: (2, 5, 6, 11, 21); (7, 10, 11, 16, 26); (12, 15, 16, 21, 31); (1, 17, 20, 21, 26); (6, 22, 25, 26, 31); (1, 11, 27, 30, 31); (1, 6, 16, 32, 35); (3, 4, 6, 11, 21); (8, 9, 11, 16, 26); (13, 14, 16, 21, 31); (1, 18, 19, 21, 26); (6, 23, 24, 26, 31); (1, 11, 28, 29, 31); (1, 6, 16, 33, 34); 1↔5, 6↔10, 11↔15 16↔20, 21↔25, 26↔30, 31↔35 (*PC*)
99*	*R*160: (39, 10, 5, 78); 0.82	-	*G*: (2, 4, 10, 14, 27); (1, 15, 17, 23, 27); (1, 14, 28, 30, 36); (1, 14, 29, 32, 33); (1, 16, 19, 20, 27); (3, 6, 7, 14, 27); 1↔13, 14↔26, 27↔39 (*PC*)
100	*H*42: (41, 10, 5, 82); 0.82	-	*B*: (1, 10, 16, 18, 37); (5, 8, 9, 21, 39) mod 41
101*	*R*166: (10, 6, 6, 10); 0.90	-	*G*: (1, 2, 3, 5, 7, 9) mod 10
102*F*	*R*167*a*: (12, 9, 6, 18); 0.91	3× *D*5; 0.89	*G*: (*A*1, *A*2, *A*4, *A*6, *B*2, *B*3); (*B*1, *B*2, *B*4, *B*6, *A*2, *A*3); (*A*1, *A*2, *A*4, *B*1, *B*2, *B*4) mod 6
103*	*C*23: (13, 6, 6, 13); 0.90	-	*C*: (1, 2, 4, 7, 9, 13) mod 13
104*	*R*168: (15, 6, 6, 15); 0.82	-	*G*: (1, 2, 4, 7, 10, 13) mod 15
105	*H*10: (16, 6, 6, 16); 0.89	-	*B*: (1, 0, 0, 0); (0, 1, 0, 0); (0, 0, 1, 0); (0, 0, 0, 1); (1, 1, 0, 0); (0, 0, 1, 1) mod (2, 2, 2, 2)
106	*SR*72: (18, 6, 6, 18); 0.90	*C*14; 0.88	*G*: (1, 4, 7, 10, 13, 16); (1, 4, 8, 11, 15, 18); (1, 6, 8, 12, 14, 16); (1, 6, 9, 11, 13, 17); (1, 5, 7, 12, 15, 17); (1, 5, 9, 10, 14, 18); 1↔3, 4↔6, 7↔9, 10↔12, 13↔15, 16↔18,19↔21 (*PC*)
107*MD*	*R*170: (27, 6, 6, 27); 0.86	*C*77; 0.86	*G*: (0, 9, 12, 13, 16, 18) mod 27
108*MD*	*R*171: (28, 6, 6, 28); 0.86	*C*85; 0.86	*G*: (0, 1, 4, 15, 20, 22) mod 28
109	*SR*76: (30, 10, 6, 50);	5× *D*59; 0.55	By deleting the treatments 31, 32, 33, 34, 35

	0.86		from *SR*86*a*
110	*H*12: (31, 6, 6, 31); 0.86	-	*B*: (1, 5, 11, 24, 25, 27) mod 31
111	*SR*77: (42, 7, 6, 49); 0.85	-	By deleting the treatments 43, 44, 45, 46, 47, 48, 49 from *SR*87
112*	*LS*82: (49, 6, 6, 49); 0.84	-	*L*: (9, 19, 28, 32, 38, 43); (2, 13, 19, 24, 39, 49); (7, 9, 20, 26, 31, 49); (7, 15, 25, 34, 38, 44);(2, 12, 21, 25, 31, 36); (7, 13, 18, 33, 36, 45);(3, 8, 23, 33, 42, 46); 1↔7, 8↔14, 15↔21, 22↔28, 23↔29, 29↔35, 36↔42, 43↔49 (PC)
113*	*R*172: (9, 7, 7, 9); 0.96	-	*G*: (1, 2, 3, 5, 6, 8, 9) mod 9
114*	*R*173: (12, 7, 7, 12); 0.90	-	*G*: (1, 2, 3, 5, 7, 9, 11) mod 12
115*	*R*174: (12, 7, 7, 12); 0.92	-	*G*: (1, 2, 4, 5, 7, 8, 11) mod 12
116*	*R*175: (12, 7, 7, 12); 0.93	-	*G*: (1, 2, 3, 4, 6, 7, 11) mod 12
117*	*R*176: (12, 7, 7, 12); 0.93	-	*G*: (1, 4, 5, 6, 7, 8, 11) mod 12
118*MD*	*R*177: (14, 7, 7, 14); 0.92	-	*G*: (0, 1, 2, 5, 7, 8, 12) mod 14
119	*H*16: (15, 7, 7, 15); 0.92	*B*188; 0.92	*B*: (0, 1, 2, 4, 5, 8, 10) mod 15
120*	*LS*83: (16, 7, 7, 16); 0.91	*C*16; 0.92	*L*: (4, 8, 12, 13, 14, 15, 16); (4, 8, 9, 10, 11, 12, 16); (4, 5, 6, 7, 8, 12, 16); (1, 2, 3, 4, 8, 12, 16); 1↔4, 5↔8, 9↔12, 13↔16 (PC)
121*	*R*178: (18, 7, 7, 18); 0.82	*C*15; 0.90	*G*: (1, 2, 4, 7, 10, 13, 16) mod 18 *J*: (1, 2, 3, 4, 6, 9, 13) mod 18
122*MD*	*R*179: (20, 7, 7, 20); 0.90	*C*29; 0.90	*G*: (0, 1, 2, 4, 8, 11, 16) mod 20
123*DN*	*R*180*a*: (21, 7, 7, 21); 0.90	*C*36; 0.90	*G*: (1, 2, 5, 7, 11, 12, 14) mod 24
124*F*	*R*180*b*: (24, 7, 7, 24); 0.89	*C*56; 0.89	*G*: (*A*1, *A*2, *A*4, *A*5, *B*6, *B*8, *C*7) perm *A*, *B*, *C* and mod 8
125*	*C*25: (29, 7, 7, 29); 0.88	-	*C*: (1, 7, 16, 20, 23, 24, 25) mod 29
126*MD*	*R*182: (33, 7, 7, 33); 0.88	-	*G*: (2, 4, 5, 6, 10, 12, 23); (1, 13, 15, 16, 17, 21, 23); (1, 12, 24, 26, 27, 28, 32); 1↔11, 12↔22, 23↔33 (*PC*)
127*F*	*R*182*a*: (35, 7, 7, 35); 0.87	-	*G*: (*A*1, *A*2, *A*4, *B*7, *C*7, *D*7, *E*7) perm *A*, *B*, *C*, *D*, *E* and mod 7
128	*SR*86*a*: (35, 10, 7, 50); 0.88	-	By deleting the treatments 36, 37, 38, 39, 40 from *SR*95*a*
129*	*R*183: (48, 7, 7, 48); 0.87	-	*G*: (1, 2, 5, 11, 31, 36, 38) mod48
130	*SR*87: (49, 7, 7, 49); 0.87	-	*G*: (1, 8, 15, 22, 29, 36, 43); (1, 9, 17, 25, 33, 41, 49); (1, 14, 20, 26, 32, 38, 44); (1, 13, 18, 23, 35, 40, 45); (1, 12, 16, 27, 31, 42, 46); (1, 11, 21, 24, 34, 37, 47); (1, 10, 19, 28, 30, 39, 48); 1↔7, 8↔14, 15↔21, 22↔28, 29↔35, 36↔42, 43 ↔49 (*PC*)
131	*H*24: (49, 8, 7, 56); 0.87	-	Add the blocks: (1+7*x*, 2+7*x*, 3+7*x*, 4+7*x*, 5+7*x*, 6+7*x*, 7+7*x*); 0≤ *x* ≤6 to the solution in Serial No. 130
132*	*R*186: (12, 8, 8, 12); 0.95	2× *D*7; 0.95	*G*: (1, 3, 4, 5, 6, 7, 10, 11) mod 12
133*	*R*187: (14, 8, 8, 14); 0.90	2× *D*11; 0.94	*G*: (1, 2, 3, 5, 7, 9, 11, 13) mod 14 *J*: 2 copies of (1, 2, 3, 5, 8, 9, 10, 12) mod 14
134*	*C*26: (17, 8, 8, 17); 0.93		*C*: (1, 2, 4, 8, 9, 13, 15, 16) mod 17

135*	*R*188: (21, 8, 8, 21); 0.82	*C*37; 0.92	*G*: (1, 3, 6, 9, 12, 15, 18, 21) mod 21 *J*: (1, 2, 3, 5, 6, 9, 15, 17) mod 21
136	*R*189: (24, 8, 8, 24); 0.91	*C*57, 0.91	*G*: (2, 3, 4, 5, 6, 7, 13, 19); (1, 8, 9, 10, 11, 12, 13, 19); (1, 7, 14, 15, 16, 17, 18, 19); (1, 7, 13, 20, 21, 22, 23, 24); 1↔6, 7↔12, 13↔18, 19 ↔24 (*PC*)
137*	*LS*101: (25, 8, 8, 25); 0.91	*C*65; 0.91	*L*: (1, 6, 11, 16, 22, 23, 24, 25); (1, 6, 11, 17, 18, 19, 20, 21); (1, 6, 12, 13, 14, 15, 16, 21); (1, 7, 8, 9, 10, 11, 16, 21); (2, 3, 4, 5, 6, 11, 16, 21); 1↔5, 6↔10, 11↔15 16↔20, 21↔25 (*PC*)
138*	*C*27: (29, 8, 8, 29); 0.90	-	*C*: (1, 2, 8, 17, 21, 24, 25, 26) mod 29
139	*SR*95: (32, 8, 8, 32); 0.90	2× *D*66; 0.88	*G*: (1, 5, 9, 13, 17, 21, 25, 29); (1, 8, 11, 13, 18, 23, 26, 32); (1, 7, 9, 14, 19, 22, 28, 32); (1, 5, 10, 15, 18, 24, 28, 31); (1, 6, 11, 14, 20, 24, 27, 29); (1, 7, 10, 16, 20, 23, 25, 30); (1, 6, 12, 16, 19, 21, 26, 31); (1, 8, 12, 15, 17, 22, 27, 30); 1↔4, 5↔8, 9↔12, 13↔16, 17↔20, 21↔24, 25↔28, 29↔32 (*PC*)
140	*SR*95*a*: (40, 10, 8, 50); 0.89	5× *D*84; 0.62	By deleting the treatments 41, 42, 43, 44, 45 from *SR*103*a*
141*F*	*R*189*a*: (42, 8, 8, 42); 0.88	2× *D*89; 0.89	*G*: (*A*1, *A*2, *A*4, *B*7, *C*7, *D*7, *E*7, *F*7) perm *A*, *B*, *C*, *D*, *E*, *F* and mod 7
142	*H*25: (57, 8, 8, 57); 0.89	-	*B*: (1, 6, 7, 9, 19, 38, 42, 49) mod 57
143*	*R*191: (63, 8, 8, 63); 0.89	-	*G*: (1, 6, 8, 14, 38, 48, 49, 52) mod 63
144*	*R*193: (12, 9, 9, 12); 0.97	3× *D*8; 0.97	*G*: (1, 2, 3, 5, 6, 8, 9, 11, 12) mod 12
145*	*R*194: (15, 9, 9, 15); 0.94	3× *D*14; 0.94	*G*: (1, 2, 4, 5, 7, 8, 11, 13, 14) mod 15
146*	*R*195: (16, 9, 9, 16); 0.90	-	*G*: (1, 2, 4, 6, 8, 10, 12, 14, 16) mod 16
147	*H*30: (19, 9, 9, 19); 0.94	*C*24; 0.94	*B*: (1, 4, 5, 6, 7, 9, 11, 16, 17) mod 19
148*DN*	*R*197*a*: (20, 9, 9, 20); 0.93	*C*30; 0.93	*G*: (1, 2, 3, 4, 6, 10, 15, 17, 18) mod 20
149*	*R*198: (24, 9, 9, 24); 0.82	*C*58; 0.93	*G*: (1, 2, 4, 7, 10, 13, 16, 19, 22) mod 24 *J*: (1, 2, 3, 4, 7, 12, 15, 19, 21) mod 24
150*	*LS*117: (25, 9, 9, 25); 0.92	-	*L*: (1, 2, 3, 4, 5, 6, 11, 16, 21); (1, 6, 7, 8, 9, 10, 11, 16, 21); (1, 6, 11, 12, 13, 14, 15, 16, 21); (1, 6, 11, 16, 17, 18, 19, 20, 21); (1, 6, 11, 16, 21, 22, 23, 24, 25); 1↔5, 6↔10, 11↔15 16↔20, 21↔25 (PC)
151	*SR*102: (27, 9, 9, 27); 0.92	3× *D*52; 0.89	*G*: (1, 4, 7, 10, 13, 16, 19, 22, 25); (1, 6, 8, 10, 15, 17, 19, 24, 26); (1, 5, 9, 10, 14, 18, 19, 23, 27); (1, 4, 7, 12, 15, 18, 20, 23, 26); (1, 6, 8, 12, 14, 16, 20, 22, 27); (1, 5, 9, 12, 13, 17, 20, 24, 25); (1, 4, 7, 11, 14, 17, 21, 24, 27); (1, 6, 8, 11, 13, 18, 21, 23, 25); (1, 5, 9, 11, 15, 16, 21, 22, 26); 1↔3, 4↔6, 7↔9, 10↔12, 13↔15 16↔18, 19↔21, 22↔24, 25↔27 (*PC*)
152	*R*200: (28, 9, 9, 28); 0.91	*C*87; 0.92	*G*: (2, 3, 4, 5, 6, 7, 8, 15, 22); (1, 9, 10, 11, 12, 13, 14, 15, 22);

			(1, 8, 16, 17, 18, 19, 20, 21, 22); (1, 8, 15, 23, 24, 25, 26, 27, 28) 1↔7, 8↔14, 15↔21, 22 ↔28 (PC)
153	*H*34: (37, 9, 9, 37); 0.91	-	*B*: (1, 7, 9, 10, 12, 16, 26, 33, 34) mod 37
154[DN]	*R*200*c*: (40, 9, 9, 40); 0.91	-	*G*: (1, 3, 4, 6, 10, 17, 18, 22, 35) mod 40
155	*SR*103*a*: (45, 10, 9, 50), 0.91	-	By deleting the treatments 46, 47, 48, 49, 50 from *SR*109*a*
156[F]	*R*200*e*: (49, 9, 9, 49); 0.89	-	*G*: (*A*1, *A*2, *A*4, *B*7, *C*7, *D*7, *E*7, *F*7, *G*7); perm *A*, *B*, *C*, *D*, *E*, *F*, *G* and mod 7
157	*H*37: (73, 9, 9, 73); 0.90	-	*B*: (1, 2, 4, 8, 16, 32, 37, 55, 64) mod 73
158*	*R*202: (80, 9, 9, 80); 0.90	-	*G*: (1, 3, 6, 10, 22, 44, 57, 58, 75) mod 80
159*	*LS*134: (100, 9, 9, 100); 0.89	-	*L*: (63, 95, 59, 11, 42, 78, 87, 24, 36); (90, 29, 77, 43, 51, 8, 34, 92, 15); (37, 85, 16, 51, 69, 23, 42, 10, 98); (4, 62, 47, 21, 99, 13, 78, 85, 60); (93, 18, 31, 6, 77, 60, 24, 69, 45); (55, 39, 21, 68, 86, 93, 7, 12, 80); (72, 7, 100, 84, 11, 35, 69, 43, 26); (1, 26, 49, 68, 77, 32, 85, 14, 53); (16, 57, 84, 32, 8, 45, 99, 80, 63); (47, 74, 6, 98, 22, 70, 53, 35, 89); 1↔10, 11↔20, 21↔30, …, 91↔100
160*	*R*203: (12, 10, 10, 12); 0.98	-	*G*: (1, 2, 3, 4, 6, 7, 8, 10, 11, 12) mod 12
161*	*R*204: (14, 10, 10, 14); 0.97	2× *D*12; 0.97	*G*: (1, 2, 3, 4, 6, 7, 8, 10, 12, 14) mod 14
162[MD]	*R*205: (14, 10, 10, 14); 0.97	2× *D*12; 0.97	*G*: (0, 1, 3, 5, 6, 7, 8, 9, 10, 11) mod 14
163*	*R*206: (18, 10, 10, 18); 0.90	2× *D*26; 0.95	*G*: (1, 2, 4, 6, 8, 10, 12, 14, 16, 18) mod 18 *J*: 2 copies of (1, 2, 3, 4, 7, 10, 11, 12, 13, 16) mod 18
164[F]	*R*206*a*: (21, 10, 10, 21); 0.94	*C*39; 0.94	*G*: (*A*1, *A*2, *A*4, *A*7, *B*1, *B*2, *B*4, *C*1, *C*2, *C*4); perm *A*, *B*, *C* and mod 7
165[MD]	*R*206*b*: (21, 10, 10, 21); 0.93	*C*39; 0.94	*G*: (0, 1, 3, 4, 6, 9, 10, 12, 15, 18) mod 21
166*	*R*207: (27, 10, 10, 27); 0.82	*C*81; 0.93	*G*: (1, 2, 4, 7, 10, 13, 16, 19, 22, 25) mod 27 *J*: (1, 2, 3, 4, 5, 8, 13, 17, 21, 23) mod 27
167[F]	*R*207*a*: (28, 10, 10, 28); 0.93	*C*88; 0.93	*G*: (*A*1, *A*2, *A*4, *B*1, *B*2, *B*4, *C*1, *C*2, *C*4, *D*7); perm *A*, *B*, *C*, *D* and mod 7
168	*R*208: (32, 10, 10, 32); 0.92	2× *D*67; 0.92	*G*: (2, 3, 4, 5, 6, 7, 8, 9, 17, 25); (1, 10, 11, 12, 13, 14, 15, 16, 17, 25); (1, 9, 18, 19, 20, 21, 22, 23, 24, 25); (1, 9, 17, 26, 27, 28, 29, 30, 31, 32) 1↔8, 9↔16, 17↔24, 25 ↔32 (*PC*)
169*	*LS*136: (36, 10, 10, 36); 0.92	2× *D*79; 0.92	*L*: (2, 3, 4, 5, 6, 7, 13, 19, 25, 31); (1, 8, 9, 10, 11, 12, 13, 19, 25, 31); (1, 7, 14, 15, 16, 17, 18, 19, 25, 31); (1, 7, 13, 20, 21, 22, 23, 24, 25, 31); (1, 7, 13, 19, 26, 27, 28, 29, 30, 31); (1, 7, 13, 19, 25, 32, 33, 34, 35, 36); 1↔6, 7↔12, 13↔18, 19↔24, 25↔30, 31↔36

170[A]	*C*30: (37, 10, 10, 37); 0.92	-	*C*: (0, 1, 16, 34, 26, 9, 33, 10, 12, 7) mod 37
171	*SR*109*a*: (50, 10, 10, 50); 0.92	5×*D*108;0.67	*G*: (1, 10, 12, 17, 25, 26, 33, 39, 44, 48);(1, 9, 15, 19, 21, 28, 32, 40, 42, 48); (1, 8, 13, 16, 22, 29, 35, 40, 44, 47); (1, 7, 11, 18, 23, 29, 32, 39, 45, 50); (1, 6, 14, 20, 24, 28, 33, 37, 45, 47); (1, 6, 11, 16, 21, 26, 31, 36, 41, 46); (1, 8, 15, 17, 24, 27, 34, 36, 43, 50); (1, 10, 14, 18, 22, 30, 34, 38, 42, 46); (1, 7, 13, 19, 25, 30, 31, 37, 43, 49); (1, 9, 12, 20, 23, 27, 35, 38, 41, 49) 1↔5, 6↔10, 11↔15, 16↔20, 21↔25, 26↔30, 31↔35, 36↔40, 41↔45, 46↔50 (PC)
172[F]	*R*208*a*: (56, 10, 10, 56); 0.89	2× *D*125; 0.9	*G*: (*A*1, *A*2, *A*4, *B*7, *C*7, *D*7, *E*7, *F*7, *G*7, *H*7); perm *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H* and mod 7
173	*H*46: (91, 10, 10, 91); 0.91	-	*B*: (0, 1, 3, 9, 27, 49, 56, 61, 77, 81) mod 91

[*]The cyclic solutions are reported in Clatworthy (1973). John numbers are from John *et al.* (1972). A part cycle, such as $\frac{1}{2}$(*B*1, *B*2, *B*4, *B*5) for *R*109*a*, means that only half the six blocks are needed, since the same treatments would then recur [see Freeman (1976)]. m × No. *X* denotes design obtained by taking m copies of the design No. *X*.

PC: the initial blocks are developed in partial cycles, *B*: BIBD, *G*: Group divisible design, *C*: the cyclic design from Clatworthy and Agrawal (1987), *L*: Latin square type designs; *J*: the cyclic design from John *et al.* (1972).

HX numbers are from Hall (1998) and *SRX*, *RX*, *LSX* and *CX* (in the second column of the table) numbers are from Clatworthy (1973) and Agrawal (1987).

The abbreviations *A*, *M*, *F*, *MD*, *S*, *DN* and *DB* stand for Agrawal (1987), Mukerjee *et al.* (1987), Freeman (1976), Midha and Dey (1995), Sinha (1989), Dey and Nigam (1985) and Dey and Balasubramanian (1991) respectively.

The overall efficiency of a partially balanced design is defined as the ratio of the average variance of a treatment comparison to the variance in a randomized block experiment with the same replication, assuming that the standard errors of individual plots are the same. The overall efficiency of a BIB design is obtained using $\lambda v/rk$ and the overall efficiency E of a two associate class PBIB design is calculated as [see Clatworthy (1973)]:

$$E = \frac{(k-1)(v-1)}{n_1(k-c_1)+n_2(k-c_2)}$$

where the computational constants c_1, c_2 are obtained by means of the following relations:

$$k^2 \triangle = (rk-r+\lambda_1)(rk-r+\lambda_2)+(\lambda_1-\lambda_2)\{(r(k-1)(p^1_{12}-p^2_{12})+\lambda_2 p^1_{12}-\lambda_1 p^2_{12}\},$$
$$k \triangle c_1 = \lambda_1(rk-r+\lambda_2)+(\lambda_1-\lambda_2)(\lambda_2 p^1_{12}-\lambda_1 p^2_{12})$$
$$k \triangle c_2 = \lambda_2(rk-r+\lambda_1)+(\lambda_1-\lambda_2)(\lambda_2 p^1_{12}-\lambda_1 p^2_{12}).$$

In case of GD designs, the expression for overall efficiency is given as [see Freeman (1976)]:

$$E = \frac{v(v-1)\lambda_2\{\lambda_1 + (m-1)\lambda_2\}}{rk\{(m-1)\lambda_1 + (mv-2m+1)\lambda_2\}}.$$

The cyclic solutions of BIB designs: $H2$, $H11$, $H24$ and GD designs: $SR60$, $SR72$, $SR87$, $SR95$, $SR102$, $R189$, $R200$, $R208$ are new. Clatworthy (1973) reported six, seven and eight initial blocks for the cyclic solution of $R189$, $R200$ and $R208$ respectively whereas we have used four initial blocks only. For the design $R68$: the first three initial blocks give 9 blocks each and the fourth initial block gives three distinct blocks. Clatworthy (1973) did not report the solutions in cyclic form for Latin square designs except four. We have reported cyclic solutions for such designs. The cyclic block designs which are *m* – multiple of smaller block designs and are with non – repeated initial blocks are included in the above table. For each of these designs, Clatworthy (1973) reported a solution that is obtained by repeating the blocks of a smaller block design *m* times. A resolvable solution of $SR109a$ may be found in Saurabh and Sinha (2022).

The GD scheme for the cyclic semi – regular GD designs: $SR60$, $SR72$, $SR87$, $SR95$, $SR102$, $SR109a$ and cyclic regular GD designs: $R189$, $R200$, $R208$ is given as:

$$\begin{matrix} 1 & 2 & 3 & \cdots & n \\ n+1 & n+2 & n+3 & \cdots & 2n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (m-1)n+1 & (m-1)n+2 & (m-1)n+3 & \cdots & mn \end{matrix}$$

for suitable choices of *m* and *n*.

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