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Integrated Redundant Reliability Model using k out of n Configuration with Integer Programming Approach

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Abstract

Reliability engineering is a branch of systems engineering concerned with the dependability of machinery. Reliability is the degree to which a system or component continues to function as intended over time and under stress. In a 'k' out of 'n' systems, the total efficiency is greater than the efficiency of any individual component. Here, we propose an additional system, An Integrated Reliability Model (IRM) for the 'k' out of 'n' systems to take into account the factors' efficiencies, the number of factors in each stage, and the various constraints in order to maximize the system's efficiency. The authors used the above-cited integrated model for obtaining various components' reliability and efficiency in a Muffle Box Furnace machine by using Lagrangean methods to calculate the price-component, weight-component and volume-component associated with various configurations of the system, all in an effort to maximize overall system performance. To get a real-looking result in an integer space, we adopted the integer programming method and the dynamic programming technique.

Key words: IRM; Lagrangean approach; Stage efficiency; Integer programming; D.P. approach; Structure's efficiency.

AMS Subject Classifications: Primary: 90B25; Secondary: 90C39, 90C59

1. Introduction

Aggarwal *et al.* (1975) present a practical approach for resolving the redundancy-based, multi-criteria optimization issues that typically occur in the dependability design of engineering systems. Because it can take into consideration any combination of redundancy, limitations, and individual cost functions, it might resolve many design issues relating to dependability. Kuo and Prasad (2000) are well-known names when it comes to the secure and efficient creation of technical systems. They gave actual instances to show how multicriteria optimization issues may be utilised to successfully tackle redundancy optimization issues.

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The Structure's reliability can be improved by either placing superfluous units, applying the element of greater reliability or by adopting the two methods at a time and both of them use extra resources. Optimizing structure reliability, and conditions to resource availability viz. price-components, weight-components and volume-components are examined. In general, reliability is tested as an element of price-component; But, when tested with real-world problems, the invisible effect of other restraints such as weight-component, volume-components, etc., has a special effect on improving structural reliability.

A team of researchers under the direction of Sankaraiah *et al.* (2011) set out to look at how numerous restrictions affect system reliability. An integrated redundant reliability system is modelled and solved using a Lagrangian multiplier. This provides a real-valued response on the total number of components, the dependability of all system stages, and the dependability of individual stages. Sridhar *et al.* (2013) created a novel method for optimising a redundant IRM with several limitations. The method accounts for the k-out-of-n configuration system and enables the optimization to discover the unexpected effects of other constraints in addition to the cost constraint. A unique strategy for optimising a redundant IRM was developed by Sasikala *et al.* (2013), however it has several drawbacks. The technique takes into consideration the k-out-of-n configuration system and enables the optimization to find the unexpected impacts of additional constraints aside from the cost constraint.

The specific functionality of the over-reliability model with several limitations to optimize the recommended setup was examined to maximize the recommended setup. The problem examines the unknowns that is, various elements (Y_{cj}), the element reliability (r_{cj}), and the stage reliability (R_{RP}) at a specific point for disposing of multiple restraints to magnify the structure reliability (R_{RS}) that is described as a United Reliability Model (URM). In literature, United Reliability Models are enhanced by applying value restraints where there is a fixed association between price-component and reliability. A unique pattern of planned work is a deliberation of the weight-component and volume-component as supplementary restraints along with price-component to form and improve the superfluous reliability system for 'k' out of 'n' structure composition. The rest of the paper has been organized into five sections. In Section 2, we detail the Lagrangean analysis of the corresponding new mathematical function. In Section 3, we get an overview of the Muffle Box furnace's parts, including its price-component, weight-component, volume-component, stage, component, and structure reliability. Our rounded-off Lagrangean approach results are presented in Section 4. In Sections 5 and 6, we will present the Integer programming, results and make comparisons to the Lagrangean method (both without and with rounding off). Finally, the author concludes and makes some suggestions in Section 7.

1.1. Assumptions and notations

1. Each stage's elements are believed to be identical, i.e., all elements have the same level of reliability.

2. All elements are supposed to be statistically independent, meaning that their failure

has no bearing on the performance of other elements in the structure.

$$\begin{aligned}
R_{RS} &= \text{Reliability of structure} \\
R_{RP} &= \text{Reliability of stage } , 0 < R_{RP} < 1 \\
r_{cj} &= \text{Reliability of each component in stage } cj;\ 0 < r_{cj} < 1 \\
Y_{cj} &= \text{Number of components in stage } cj \\
PC_{cj} &= \text{Price component in stage } cj \\
WC_{cj} &= \text{Weight component in stage } cj \\
VC_{cj} &= \text{Volume component in stage } cj \\
P_{c0} &= \text{Greatest allowable complex for price component} \\
W_{c0} &= \text{Greatest allowable complex for weight component} \\
V_{c0} &= \text{Greatest allowable complex for volume component}
\end{aligned}$$

$P_{cj}; \alpha_{cj}; W_{cj}; \beta_{cj}; V_{cj}; \gamma_{cj}$ are constants.

2. Mathematical analysis

The efficiency of the system to the provided price-component function

$$R_{RS} = \sum_{i=1}^{n} B(m,i)(p)^{i}(1-p)^{(m-i)} \tag{1}$$

The following relationship between price-component and efficiency is used to calculate the price-component coefficient of each unit in Stage j.

$$r_{cj} = Cosh^{-1}\left[\frac{PC_{cj}}{P_{cj}}\right]^{\frac{1}{\alpha_{cj}}}$$

Therefore

$$PC_{cj} = P_{cj}Cosh[r_{cj}]^{\alpha_{cj}} \tag{2a}$$
$$WC_{cj} = W_{cj}Cosh[r_{cj}]^{\beta_{cj}} \tag{2b}$$
$$VC_{cj} = V_{cj}Cosh[r_{cj}]^{\gamma_{cj}} \tag{2c}$$

Since price-components are linear in Y_{cj},

$$\sum_{j=1}^{n} PC_{cj}Y_{cj} \leq P_{co} \tag{3a}$$

Similarly, weight-components and volume-components are also linear in Y_{cj}

$$\sum_{j=1}^{n} WC_{cj}Y_{cj} \leq W_{co} \tag{3b}$$
$$\sum_{j=1}^{n} VC_{cj}Y_{cj} \leq V_{co} \tag{3c}$$

Substituting (2a, 2b & 2c) in (3a, 3b & 3c) respectively, we get

$$\sum_{j=1}^{n} P_{cj} Cosh[r_{cj}]^{\alpha_{cj}}.\, Y_{cj} - P_{C0} \leq 0 \tag{4a}$$

$$\sum_{j=1}^{n} W_{cj} Cosh[r_{cj}]^{\beta_{cj}}.\, Y_{cj} - W_{C0} \leq 0 \tag{4b}$$

$$\sum_{j=1}^{n} V_{cj} Cosh[r_{cj}]^{\gamma_{cj}}.\, Y_{cj} - V_{C0} \leq 0 \tag{4c}$$

The transformed

$$Y_{cj} = \frac{lnR_{RP}}{lnr_{cj}} \tag{5}$$

where

$$R_{RP} = \sum_{k=2}^{n} B(Y_{cj}, k)(r_{cj})^k (1 - r_{cj})^{(cj-k)} \tag{6}$$

Subject to the constraints

$$\sum_{j=1}^{n} P_{cj} Cosh[r_{cj}]^{\alpha_{cj}}.\, \frac{lnR_{RP}}{lnr_{cj}} - P_{C0} \leq 0 \tag{7a}$$

$$\sum_{j=1}^{n} W_{cj} Cosh[r_{cj}]^{\beta_{cj}}.\, \frac{lnR_{RP}}{lnr_{cj}} - W_{C0} \leq 0 \tag{7b}$$

$$\sum_{j=1}^{n} V_{cj} Cosh[r_{cj}]^{\gamma_{cj}}.\, \frac{lnR_{RP}}{lnr_{cj}} - V_{C0} \leq 0 \tag{7c}$$

Positivity restrictions $Y_{cj} \geq 0$.

A Lagrangean function is defined as

$$\begin{aligned} L_G &= R_{RS} + \omega_1 \left[\sum_{j=1}^{n} P_{cj}.\, Cosh[r_{cj}]^{\alpha_{cj}}.\, \frac{lnR_{RP}}{lnr_{cj}} - P_{c0} \right] + \omega_2 \Big[\sum_{j=1}^{n} P_{cj}.\, Cosh[r_{cj}]^{\beta_{cj}}.\, \frac{lnR_{RP}}{lnr_{cj}} \\ &\quad - W_{c0} \Big] + \omega_3 \left[\sum_{j=1}^{n} V_{cj}.\, Cosh[r_{cj}]^{\gamma_{cj}}.\, \frac{lnR_{RP}}{lnr_{cj}} - V_{c0} \right] \end{aligned} \tag{8}$$

The Lagrangean function can be used to find the ideal point and separating it by R_{RS}, r_{cj}, ω_1, ω_2 and ω_3.

$$\begin{aligned} \frac{\partial L_G}{\partial R_{RS}} &= 1 + \omega_1 \left[\sum_{j=1}^{n} P_{cj}.\, Cosh[r_{cj}]^{\alpha_{cj}}.\, \frac{1}{lnr_{cj}} \frac{1}{R_{RP}} \right] + \omega_2 \left[\sum_{j=1}^{n} W_{cj}.\, Cosh[r_{cj}]^{\beta_{cj}}.\, \frac{1}{lnr_{cj}} \frac{1}{R_{RP}} \right] \\ &\quad + \omega_3 \left[\sum_{j=1}^{n} V_{cj}.\, Cosh[r_{cj}]^{\gamma_{cj}}.\, \frac{1}{lnr_{cj}} \frac{1}{R_{RP}} \right] \end{aligned} \tag{9}$$

$$\begin{aligned}
\frac{\partial L_G}{\partial r_{cj}} &= \omega_1 \left[\sum_{j=1}^{n} P_{cj}.\ Cosh[r_{cj}]^{\alpha_{cj}}.\ \frac{lnR_{RP}}{lnr_{cj}}\right]\left[\alpha_{cj}.\ Tanh(r_{cj}) - \frac{1}{r_{cj}.lnr_{cj}}\right] \\
&+\omega_2 \left[\sum_{j=1}^{n} W_{cj}.\ Cosh[r_{cj}]^{\beta_{cj}}.\ \frac{lnR_{RP}}{lnr_{cj}}\right]\left[\beta_{cj}.\ Tanh(r_{cj}) - \frac{1}{r_{cj}.lnr_{cj}}\right] \\
&+\omega_3 \left[\sum_{j=1}^{n} V_{cj}.\ Cosh[r_{cj}]^{\gamma_{cj}}.\ \frac{lnR_{RP}}{lnr_{cj}}\right]\left[\gamma_{cj}.\ Tanh(r_{cj}) - \frac{1}{r_{cj}.lnr_{cj}}\right] \quad (10)
\end{aligned}$$

$$\frac{\partial L_G}{\partial \omega_1} = \sum_{j=1}^{n} P_{cj}.\ Cosh[r_{cj}]^{\alpha_{cj}}.\ \frac{lnR_{RP}}{lnr_{cj}} - P_{c0} \quad (11)$$

$$\frac{\partial L_G}{\partial \omega_2} = \sum_{j=1}^{n} W_{cj}.\ Cosh[r_{cj}]^{\beta_{cj}}.\ \frac{lnR_{RP}}{lnr_{cj}} - W_{c0} \quad (12)$$

$$\frac{\partial L_G}{\partial \omega_3} = \sum_{j=1}^{n} V_{cj}.\ Cosh[r_{cj}]^{\gamma_{cj}}.\ \frac{lnR_{RP}}{lnr_{cj}} - V_{c0} \quad (13)$$

where ω_1, ω_2 and ω_3 are Lagrangean multipliers.

Using the Lagrangean method, we can calculate the number of elements in each Stage (Y_{cj}), the best reliability of an individual element (r_{cj}), the reliability of an entire Stage ($\mathbf{R_{RP}}$), and the reliability of the entire structure (R_{RS}). When it comes to the price-component, weight-component, and volume-component, this method yields a true (valued) answer.

3. Case problem

To derive the multiple parameters of a given mechanical system using optimization techniques, where all the assumptions like price-component, weight-component, and volume-component are directly proportional to system reliability has been considered in this research work. The same logic may not be true in the case of electronic systems. Hence, the optimal element accuracy (r_{cj}), Stage reliability (R_{RP}), Number of elements in each Stage (Y_{cj}), and structure accuracy (R_{RS}) can be evaluated in any given mechanical system. In this work, an attempt has been made to evaluate the Structure accuracy of a special purpose of Muffle Box Furnace machine that is utilized in the laboratories for testing different materials.

The machine is used for the assembly of many components. But our case steady, we are considering 3 or 4 important components on the base of Muffle Box Furnace machine.

The muffle furnace is an essential laboratory testing instrument used for a wide variety of materials. The instrument is useful in the study of materials' properties and can be found in most laboratory settings. The tool is put to use in numerous heat-treating metallurgical procedures. Altering the molecular structure of a material is a common application of heat treatment. The machine's approximate price was Rs. 5000, which is considered a structure price, the weight of the machine is 156 kg, which is the volume of the structure, and the space occupied by the machine is $100cm^3$, which is the volume of the structure. To attract the authors from different cross sections, the authors attempted to use hypothetical numbers, which can be changed according to the environment. The Lagrangean approach of modelling and solving produces real-valued solutions for the number of elements, element reliability,

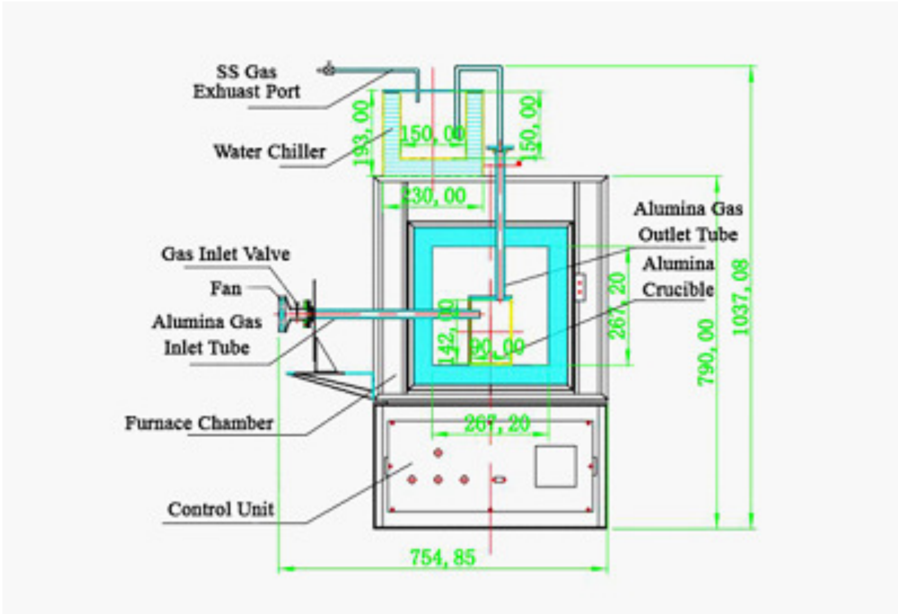

Figure 1: Schematic diagram of muffle box furnace

moment reliability, and therefore structure reliability, as shown by Sasikala *et al.* (2020).

3.1. Constants

The data required for the constants for the case problem are provided hereunder.

Table 1: The details of price-component, weight-component and volume-component for case problem

Phase	Price constants		Weight constants		Volume constants	
	P_{cj}	α_{cj}	W_{cj}	β_{cj}	V_{cj}	γ_{cj}
1	600	0.85	110	0.92	95	0.94
2	650	0.88	90	0.88	85	0.89
3	700	0.91	70	091	75	0.86

The efficiency of each factor, phase and number of factors in each stage, as well as the structural efficiency, are shown in the tables below.

3.2. The details of price-component, weight-component and volume-component

The price-components, weight-components, and volume-components related efficiency design and total price of the components is described in the table below.

Structure efficiency $= R_{CR} = 0.9664$

Table 2: The details of price-component, weight-component and volume-component by using Lagrangean approach

Phase	P_{cj}	α_{cj}	r_{cj}	$Log\ r_{cj}$	R_{RP}	$Log\ R_{RP}$	Y_{cj}	$PC_{cj} = P_{cj}Cosh[r_{cj}]^{\alpha_{cj}}$	$Y_{cj}.PC_{cj}$
1	600	0.85	0.8741	-0.0584	0.6777	-0.169	2.89	801.00	2314.89
2	650	0.88	0.8445	-0.0734	0.6487	-0.188	2.56	861.30	2204.93
3	700	0.91	0.8456	-0.0728	0.5461	-0.2627	3.61	937.60	3384.74
Total price of the components									7904.55
Phase	W_{cj}	β_{cj}	r_{cj}	$Log\ r_{cj}$	R_{RP}	$Log\ R_{RP}$	Y_{cj}	$WC_{cj} = W_{cj}Cosh[r_{cj}]^{\beta_{cj}}$	$Y_{cj}.WC_{cj}$
1	110	0.92	0.8741	-0.0584	0.6777	-0.169	2.89	150.48	434.89
2	90	0.88	0.8445	-0.0734	0.6487	-0.188	2.56	119.34	305.51
3	70	0.91	0.8456	-0.0.728	0.5461	-0.2627	3.61	93.73	338.37
Total weight of the components									1078.76
Phase	V_{cj}	γ_{cj}	r_{cj}	$Log\ r_{cj}$	R_{RP}	$Log\ R_{RP}$	Y_{cj}	$VC_{cj} = V_{cj}Cosh[r_{cj}]^{\gamma_{cj}}$	$Y_{cj}.VC_{cj}$
1	95	0.94	0.8741	-0.0584	0.6777	-0.169	2.89	130.8	378.01
2	85	0.89	0.8445	-0.0734	0.6487	-0.188	2.56	113.8	291.33
3	75	0.86	0.8456	-0.0728	0.5461	-0.2627	3.61	98.85	356.85
Total volume of the components									1026.19

4. Efficiency design with e_j rounding off

The acceptable outcomes for the price-component, weight-component, and volume components are listed in the tables, and the Y_{cj} values are summarised as integers (rounding the value of cj to the nearest integer) in the efficiency design. The information you seek can be determined by calculating the variance caused by the price-component, the weight-component, and the volume-component of the building's capacity (both before and after rounding Y_{cj} to the nearest integer).

4.1. Efficiency design concerning price-component, weight-component and volume-component with rounding off

Table 3: The details of price-component, weight-component and volume-component analysis by using rounding off approach

Phase	r_{cj}	R_{RP}	Y_{cj}	P_{cj}	$Y_{cj}.P_{cj}$	W_{cj}	$Y_{cj}.W_{cj}$	V_{cj}	$Y_{cj}.V_{cj}$
1	0.8741	0.6777	3	801	2403	150	450	131	393
2	0.8445	0.6487	3	861	2583	119	357	114	342
3	0.8456	0.5461	4	938	3752	94	376	99	396
Total price of the components					8738		1183		1131
Structure efficiency (R_{CR})					0.9687				

4.1.1. Variation in total price-component

$$= \frac{\text{Total price-component with rounding off - Total price-component without rounding off}}{\text{Total price without rounding off}} = 10.54\%$$

4.1.2. Variation in total weight-component

$$= \frac{\text{Total weight-component with rounding off - Total weight-component without rounding off}}{\text{Total weight-component without rounding off}} = 09.66\%$$

4.1.3. Variation in total price-component

$$= \frac{\text{Total volume-component with rounding off - Total volume-component without rounding off}}{\text{Total volume-component without rounding off}} = 10.21\%$$

4.1.4. Variation in structure efficiency

$$= \frac{\text{Structure efficiency with rounding off - Structure efficiency without rounding off}}{\text{Structure efficiency without rounding off}} = 01.24\%$$

Instead of using complex algorithms, the Lagrangian multiplier method provides a way to quickly find the best possible design. Naturally, this assumes that the component count at each Stage (Y_{cj}) is real. When Y_{cj} is rounded off to the nearest integer, it has a domino effect on all the other numbers in the reliability design, including the values of the reliability at each Stage ($\mathbf{R_{RP}}$) the reliability of the system as a whole ($\mathbf{R_{RS}}$), the total Price of each Stage, and the Price of the system as a whole. It is demonstrated in the examples how changing the way Y_{cj} is rounded can affect the reliability of a design. Integer programming can be used to counter this shortcoming.

5. Integer programming

To determine the number of components in each stage, stage reliabilities, and system reliability, integer programming requires the component reliabilities as input. The fundamental disadvantage of integer programming is that it cannot be utilised directly, i.e., without the input of the component reliabilities, even if it is beneficial for creating integrated reliability models. Therefore, integer programming may take the component reliabilities from the previous approach, the Lagrangian method, as input and output the stage reliabilities, system reliabilities, system reliabilities, stage reliabilities, and stage reliabilities. Integer programming allows you some flexibility to select the number of components in each step, the dependability of each stage, and the reliability of the entire system within the limits that are provided.

Pavankumar *et al.* (2020) look into how the many constraints listed above affect how the integrated reliability and optimization is formulated. Statistics are utilised with IRRCCS (Integrated Redundant Reliable Coherent Configuration system is considered). Sridhar *et al.* (2021), have provided a thorough examination, design, analysis, and optimization of a coherent redundant reliability design.The work of Sridhar *et al.* (2022) is applied to parallel-series systems where both technologies include parallel factors. A parallel approach can only function if all of its components are active at all times.To determine the optimum solution, Srinivasa Rao *et al.* (2022) recommended utilising an appropriate method based on Heuristic approach and included the redundancy strategy as a new decision variable.

Integer programming methodology as a whole has grown rapidly over the past half century, but linear integer programming has been its main focus. But there have been some promising theoretical and methodological developments in nonlinear integer programming in recent years. Because of these changes, nonlinear integer programming is now used in many areas of scientific computing. For example, it is a key criterion for choosing portfolios and managing risks.

The author used the LINGO Programme (created by Lindo Corporation, USA) to find a decimal solution to the problem. Lingo is an all-inclusive Programme that streamlines the process of creating and solving linear, nonlinear, and integer optimization models. Lingo is an all-inclusive package that features a robust language for expressing optimization models, a comprehensive environment for creating and editing problems, and a collection of fast, in-built solvers.

An integrated reliability model for redundant systems with multiple constraints is established and optimized using integer programming for the considered function. By using the values of component dependability's (r_{cj}) and the number of components in each stage (Y_{cj}) as inputs for the application of integer programming, we can optimize the design in light of the case problem discussed in the previous section for the respective mathematical function (refer to equation 1). Since the values of Y_{cj} are integers, this method helps optimise the design and makes it easier to use in the real world.

6. Results

6.1. The details of price-component, weight component and volume-component constraint by using integer programming approach

The value-related efficiency design is described in the Table 4.

Table 4: The details of price-component, weight-component and volume-component constraint by using integer programming approach

Phase	P_{cj}	α_{cj}	r_{cj}	$Log\ r_{cj}$	R_{RP}	$Log\ R_{RP}$	Y_{cj}	$PC_{cj} = P_{cj}Cosh[r_{cj}]$	$Y_{cj}.PC_{cj}$
1	600	0.85	0.9982	-0.0008	0.9945	-0.0024	3	866	2598
2	650	0.88	0.9736	-0.0116	0.9229	-0.0348	3	936	2808
3	700	0.91	0.9891	-0.0048	0.9571	-0.0190	4	1031	4124
Final price									9530
Phase	W_{cj}	β_{cj}	r_{cj}	$Log\ r_{cj}$	R_{RP}	$Log\ R_{RP}$	Y_{cj}	$WC_{cj} = W_{cj}Cosh[r_{cj}]$	$Y_{cj}.WC_{cj}$
1	110	0.92	0.9982	-0.0008	0.9945	-0.0024	3	164	492
2	90	0.88	0.9736	-0.0116	0.9229	-0.0348	3	130	390
3	70	0.91	0.9891	-0.0048	0.9571	-0.0190	4	103	412
Final weight									1294
Phase	V_{cj}	γ_{cj}	r_{cj}	$Log\ r_{cj}$	R_{RP}	$Log\ R_{RP}$	Y_{cj}	$VC_{cj} = V_{cj}Cosh[r_{cj}]$	$Y_{cj}.VC_{cj}$
1	95	0.94	0.9982	-0.0008	0.9945	-0.0024	3	143	429
2	85	0.89	0.9736	-0.0116	0.9229	-0.0348	3	123	369
3	75	0.86	0.9891	-0.0048	0.9571	-0.0190	4	108	412
Final volume									1230
Structure efficiency (R_{SR})									0.9998

6.2. Comparison of optimization of integrated redundant reliability k out of n systems - LMM with rounding off and integer programming approach for price-component, weight-component and volume-component

7. Conclusion

This work proposes an integrated reliability model for a k out of n configuration system with many efficiency criteria. When the data are discovered to be in reals, the Lagrangean multiplier approach is used to compute the number of components (cj), component

Table 5: Results correlated LMM with rounding off approach and integer programming approach for price-component, weight-component and volume-component

		With rounding off				Integer programming			
Phase	Y_{cj}	r_{cj}	R_{RP}	PC_{cj}	$Y_{cj}.PC_{cj}$	r_{cj}	R_{RP}	PC_{cj}	$Y_{cj}.PC_{cj}$
1	3	0.8741	0.6777	801	2403	0.9982	0.9945	866	2598
2	3	0.8445	0.6487	861	2583	0.9736	0.9229	936	2808
3	4	0.8456	0.5461	938	3752	0.9891	0.9571	1031	4124
Total price		8738				9530			
Structure efficiency		Using with rounding off approach (R_{SR})			0.9987	Using integer programming approach (R_{SR})			0.9999
		With rounding off				Integer programming			
Phase	Y_{cj}	r_{cj}	R_{RP}	WC_{cj}	$Y_{cj}.WC_{cj}$	r_{cj}	R_{RP}	WC_{cj}	$Y_{cj}.WC_{cj}$
1	3	0.8741	0.6777	150	450	0.9982	0.9945	164	492
2	3	0.8445	0.6487	119	357	0.9736	0.9229	130	390
3	4	0.8456	0.5761	94	376	0.9891	0.9571	103	412
Total weight		1183				1294			
Structure efficiency		Using with rounding off approach (R_{SR})			0.9987	Using integer programming approach (R_{SR})			0.9999
		With rounding off				Integer programming			
Phase	Y_{cj}	r_{cj}	R_{RP}	VC_{cj}	$Y_{cj}.VC_{cj}$	r_{cj}	R_{RP}	VC_{cj}	$Y_{cj}.VC_{cj}$
1	3	0.8741	0.6777	131	393	0.9982	0.9945	143	429
2	3	0.8445	0.6487	114	342	0.9736	0.9229	123	369
3	4	0.8456	0.5461	99	396	0.9891	0.9571	108	432
Total volume		1131				1230			
Structure efficiency		Using with rounding off approach (R_{SR})			0.9664	Using Integer programming approach (R_{SR})			0.9998

efficiencies ($r_{cj} = 0.8741, 0.8445, 0.8456$), stage efficiencies ($R_{RP} = 0.6777, 0.6487, 0.5461$), and structure efficiency ($R_{RS} = 0.9664$). To obtain practical applicability, an integer way of programming approach is employed to construct an integer solution whereas component efficiencies ($r_{cj} = 0.9982, 0.9736, 0.9891$), stage efficiencies ($R_{RP} = 0.9945, 0.9229, 0.9571$), and structure efficiency ($R_{RS} = 0.9998$). using the inputs from the Lagrangean method. Finally, we observed that the price, weight, and volume components changed slightly, but the reliability of the stage and structure increased, resulting in increased system reliability.

The IRM model generated in this manner is quite valuable, particularly in real-world settings when a k from n configuration IRM with reliability engineer redundancy is required. In circumstances where the system value is low, the proposed model is especially valuable for the dependability design engineer to build high-quality and efficient materials.

In future study, the authors recommend utilizing a unique approach that limits the minimum and maximum component reliability values while maximizing system dependability using any of the current heuristic processes to build similar IRMs with redundancy.

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