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An Economical Study of Two-State Queueing Model with Initial Customers and Bernoulli Schedules

Rimmy Sharma and Indra Rani
Department of Statistics and O.R.,
Kurukshetra University, Kurukshetra (Haryana), INDIA

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Abstract

Present paper studies the behaviour of few clientele available in the system, when the system starts and derives the time dependent probabilities of a single server queueing model with impatient customers ("balking and reneging"), Bernoulli schedule and multiple vacations. "Server accepts a customer with fix probability ρ or commences a vacation of random duration with probability *(*1- ρ*)"*. An arriving customer may balk (do not enter) or renege pursuant to the negative exponential distribution. Time dependent probabilities are computed with the help of recurrence relations and provide us a better understanding of the behaviour of the system. Finally, measureable outcomes are calculated with the help of Maple software.

Key words: M/M/1; Bernoulli schedule; Impatient customers; Multiple vacations; Laplace transform.

1. Introduction

In recent years, various authors have studied queueing models from productive point of view. Many real-life situations occur where clienteles are dejected by longer queue and as a result clienteles have to wait long to get into service upon arrival. In queues, "balking and reneging are common phenomena, as a consequences the customer either decides to join the queue or depart after joining the queue without getting service due to impatience". Queueing systems with impatient units ("balking and reneging") have engross many authors because of their extensive applications in many practical situations such as perishable goods in supermarkets, emergency room in hospitals *etc.* Haight (1957), Haight (1959) obtained probabilities for impatient customers ("balking and reneging") respectively. Anker and Gafarian (1963), Anker and Gafarian (1963) calculated steady state probabilities with impatient clienteles for a finite and an infinite queueing model respectively. Abou-El- Ata (1991) derived steady state probabilities for single-server Markovian queue with state dependent arrivals and impatient clienteles. Seddy *et al.* (2009) obtained time dependent probabilities by using generating function technique for c-servers Markovian queueing model. Bouchentouf and Messabihi (2018) obtained time independent probabilities for a heterogeneous server queueing system with feedback. Sharma and Indra (2020) obtained time dependent probabilities for a two-dimensional state Markovian queueing model with reneging.

From the past few decades, Vacations Queueing system has attracted much attention from numerous researchers. "Vacation: when the server finishes serving a unit and finds the system empty, however, it goes away for a length of time". Cooper (1970) was the first who

Corresponding Author: Indra Rani
E-mail: indra@kuk.ac.in

talked about vacation model and obtained waiting time distribution for M/G/1 model by using Laplace Stieltjes transform. There are different kinds of vacation policies available in literature *i.e.*, single vacation, multiple vacations, Bernoulli schedule, t- policy and *so on*. "In Multiple vacations policy, server keeps on taking vacations until it finds at least one customer waiting in the system at the instant of vacation completion". Bacot and Dshalalow (2001) obtained time independent probabilities for single server bulk queueing system with multiple vacations. Altman and Yechailli (2006) analyzed both single and multiple vacations cases and calculated time independent probabilities by using PGF technique for different markovian model. Banik (2009) obtained time independent probabilities and queue length distribution at various epochs for an infinite-buffer single server queueing model. Another feature that is widely used in queueing models is the Bernoulli schedule. "In Bernoulli schedule the server serves the new customer with probability ρ or takes a vacation with probability $(1-\rho)$". Keilson and Servi (1987) introduced the concept of Bernoulli schedule and obtained steady state probabilities for M/G/1 queueing model. Khedhairi and Tadj (2007) studied bulk service queueing system for both discrete and continuous time by using semi regenerative technique. The combined effects of impatience customers and multiple vacations were studied by numerous researchers such as: Ramaswamy and Servi (1988) calculated joint distribution for busy period of M/G/1 model. Madan *et al.* (2003) calculated steady state probabilities by using generating function technique for a queueing system with two parallel servers. Yue *et al.* (2003) derived the closed-form expressions for the system sizes for a queueing model with variant of multiple vacations. Choudhary *et al.* (2007) calculated steady state behaviour in terms of recursive solutions of batch arrival queue with two phases of heterogeneous service. Ammar (2015) obtained transient probabilities in terms of modified Bessel function by employing PGF technique.

All aforesaid authors have worked on the concept that, there is no clienteles available in the system when the system starts. Thus, the main aim of the paper is to make a model that is more applicable in day-to-day life activities such as railway booking counters, banks, doctor clinics, *etc*. In call centre: Calls arriving to a call centre are managed by agent to answer the calls. Primary calls are automatically answered by machines (*i.e.* initially a few clienteles are always present). The behaviour of the call may depend on several circumstances including waiting time and others. Each individual call may decide to balk or wait for some time and it may happen that clienteles abandon their call when their patience time expires. Server after completing all the clienteles (calls) in the system can go for vacation and after coming back from vacation if there are no clienteles available (calls) in the system server can go on vacation again.

To obtain the time dependent solution by taking together all the above mentioned parameters is very interesting. As transient probability obtained by recursive technique does not involve heavy algebraic manipulations. "The two dimensional concept helps us to understand the probability of exactly *a*- arrivals and *b*- services occurs over a time interval of length *t*". Validation of the model in form of tables is also done with the previous existing results. Graphical analysis shows the impact of parameters on measuring outcomes. Finally an expected cost model is discussed.

2. Assumptions and Notations

i. Inter-arrival time, Service times, vacation times and reneging times are exponentially distributed with parameter λ, μ, w, and ξ respectively.

ii. On arrival a customer either decides to join the queue with probability β or balk with probability $(1-\beta)$.
iii. Initially there are 'n' customers present at time $t=0$ *i.e.* $P_{n,0}(n,0) = 1$.
iv. When a customer has just been served and other customers are present, the server accepts a customer with fix probability ρ or commences a vacation of random duration with probability $(1-\rho)$
v. The system state is given by (a, b), where a is the number of arrivals and b is the number of departures up to time t, *i.e.*

$$P(n,0) = \sum_{b=0}^{\infty} P_{n+b,b,V}(n,0) = 1 \tag{1}$$

3. Model

Define
$P_{a,b,B}(c,t)$ = The probability of exactly a arrivals, b departures and c- customers remain in the system by time t and the server is busy corresponding to the queue; $b< a$
$P_{a,b,V}(c,t)$ = The probability of exactly a arrivals, b departures and c- customers remain in the system by time t and the server is on vacation; $b \le a$
$P_{a,b}(c,t)$ = The probability that there are exactly a arrivals and b departures and c- customers remain in the system by time t; $b \le a$

3.1. Equations of the system

$$\frac{d}{dt}P_{a,b,V}(c,t) = -(\lambda\beta + w)P_{a,b,V}(c,t) + (\lambda\beta)P_{a-1,b,V}(c-1,t) + \mu(1-\rho)P_{a,b-1,B}(c+1,t)\left(1-\delta_{b,0}\right)$$

$$0 \le b < a\,, c \ge 1 \tag{2}$$

$$\frac{d}{dt}P_{a,a,V}(0,t) = -(\lambda\beta)P_{a,a,V}(0,t) + \mu P_{a,a-1,B}(1,t)\left(1-\delta_{a,0}\right), \qquad a \ge 0 \tag{3}$$

$$\frac{d}{dt}P_{a,b,B}(c,t) = -(\lambda\beta + \mu + (c-1)\xi)P_{a,b,B}(c,t) + \lambda\beta P_{a-1,b,B}(c-1,t)\left(1-\delta_{b,a-1}\right) + wP_{a,b,V}(c,t) + (\mu\rho + c\xi)P_{a,b-1,B}(c+1,t)$$

$$0 \le b < a, c \ge 1 \tag{4}$$

Clearly,

$$P_{a,b}(c,t) = P_{a,b,V}(c,t) + P_{a,b,B}(c,t)(1-\delta_{(a,b)})\ a \ge b \ge 0 \tag{5}$$

3.2. Findings of equations

Solving above equation recursively with the help of Laplace transform:

$$\overline{P}_{a,b,V}(0,s)=\frac{1}{(s+\lambda\beta)}\delta_{(n,0)}P_{0,0,V}(0,0),\ a=0=b \tag{6}$$

$$\bar{P}_{a,0,V}(a,s)=(\lambda\beta)^{a}\bar{H}^{\lambda\beta+w,\lambda\beta,0}_{a,1,0}(s)\delta_{(n,0)}\,P_{0,0,V}(0,0)+\sum_{d=1}^{\infty}(\lambda\beta)^{a-d}\bar{H}^{\lambda\beta+w,0,0}_{a-d+1,0,0}(s)\delta_{(n,d)}P_{d,0,V}(d,0),a>0 \tag{7}$$

$$\bar{P}_{a,0,B}(a,s)=w.\sum_{f=1}^{\infty}\frac{(\lambda\beta)^{a-f}}{\prod_{d=0}^{a-f}\{s+\lambda\beta+\mu+(a-d-1).\xi\}}\ \bar{P}_{f,0,V}(f,s),\quad a>0 \tag{8}$$

$$\bar{P}_{a,b,V}(c,s)=\sum_{e=0}^{a-b}(\lambda\beta)^{a-e-b}\,\mu(1-\rho)^{(1-\delta_{(e,0)})}\,\bar{H}^{\lambda\beta+w,\lambda\beta,0}_{a-b+1-e-\delta_{(e,0)},\delta_{(e,0)},0}(s)\bar{P}_{b+e,b-1,B}(e+1,s)+(\lambda\beta)^{a-b}\bar{H}^{\lambda\beta+w,\lambda\beta,0}_{a-b,1,0}(s)\,\delta_{(n,0)}P_{b,b,V}(0,0)+\sum_{d=b+1}^{\infty}(\lambda\beta)^{a-d}\,\bar{H}^{\lambda\beta+w,0,0}_{a-d+1,0,0}(s)\,\delta_{(n,d-b)}P_{d,b,V}(d-b,0),\quad a>b>0 \tag{9}$$

$$\bar{P}_{a,b,B}(c,s)=\sum_{f=b+1}^{\infty}\frac{(\lambda\beta)^{a-f}.\{\mu\rho+(f-b).\xi\}}{\prod_{d=0}^{a-f}\{s+\lambda\beta+\mu+(a-d-b-1).\xi\}}\ \bar{P}_{f,b-1,B}(f-b+1,s)+w.\sum_{f=b+1}^{\infty}\frac{(\lambda\beta)^{a-f}.}{\prod_{d=0}^{a-f}\{s+\lambda\beta+\mu+(a-d-b-1).\xi\}}\ \bar{P}_{f,b,V}(f-b,s),\quad a>b>0 \tag{10}$$

$$\bar{P}_{a,a,V}(0,s)=\frac{\mu}{(s+\lambda\beta)}\bar{P}_{a,a-1,B}(1,s)(1-\delta_{(a,0)})+\frac{1}{(s+\lambda\beta)}\delta_{(n,0)}P_{a,a,V}(0,0),\quad a>0 \tag{11}$$

3. Substantiations

The Laplace Transform $\bar{P}_{a.}(c,s)$of the probability $P_{a.}(c,t)$ that exactly *a* unit arrives by the time *t*":

a) $\bar{P}_{a.}(s)=\sum_{b=0}^{a}\left[(\bar{P}_{a,b,V}(c,s)+\bar{P}_{a,b,B}(c,s)(1-\delta_{a,b})\right]$

$$=\sum_{b=0}^{a}\bar{P}_{a,b}(c,s)=\frac{(\lambda\beta)^{a}}{(s+\lambda\beta)^{a+1}}$$

And its Inverse Laplace transform is $P_{a.}(c,t)=\frac{e^{-\lambda\beta t}(\lambda\beta t)^{a}}{a!}$

b) $\sum_{a=0}^{\infty}\sum_{b=0}^{a}\{\bar{P}_{a,b,V}(c,s)+\bar{P}_{(a,b,B}(c,s)(1-\delta_{(a,b)})\}=\frac{1}{s}$

$$\sum_{a=0}^{\infty}\sum_{b=0}^{a}\{P_{a,b,V}(c,t)+P_{a,b,B}(c,t)(1-\delta_{(a,b)})\}=1$$

5. Analytical Results

i. Pr {a arrivals in (*0, t*)} = $\frac{e^{-\lambda t}(\lambda t)^a}{a!} = \sum_{b=0}^{a} P_{a,b}(c,t) = P_{a,.}(c,t)$

Table 1: For Exactly *a* customers served by time *t*

λ	μ	w	β	ξ	T	a	$\frac{e^{-\lambda t} * (\lambda t)^a}{a!}$	$\sum_{b=0}^{a} P_{a,b}(t)$
1	2	1	1	1	3	1	0.149361	0.1493612051
1	2	1	1	1	3	3	0.224042	0.2240418076
1	2	1	1	1	3	5	0.100819	0.1008188135
2	2	1	1	1	3	1	0.014873	0.01487251306
2	2	1	1	1	3	3	0.089235	0.08923507838
2	2	1	1	1	3	5	0.160623	0.1606231410
1	2	1	1	1	4	1	0.073263	0.07326255556
1	2	1	1	1	4	3	0.195367	0.1953668148
1	2	1	1	1	4	5	0.156293	0.1562934518
2	2	1	1	1	4	1	0.002684	0.002683701023
2	2	1	1	1	4	3	0.028626	0.02862614425
2	2	1	1	1	4	5	0.091604	0.09160366157
2	4	1	1	1	4	5	0.091604	0.09160366160
1	2	1	1	1	4	4	0.195367	0.1953668148
1	2	1	1	1	3	6	0.050409	0.05040940672
3	2	1	1	1	3	1	0.0011106	0.001110688237
3	2	1	1	1	3	3	0.0149942	0.01499429120
3	2	1	1	1	3	5	0.0607268	0.06072687936

The last Column of Table-1 completely matches with Table-1 of Pegden and Rosenshine (1982).

ii. The probability that exactly b number of customers have been served.

Server is on vacation *i.e.* $\sum_{a=b}^{\infty} P_{a,b,V}(c,t)$

Server is busy *i.e.* $\sum_{a=b}^{\infty} P_{a,b,B}(c,t)$ are based on the following relationship $P_{.,b}(c,t) = \sum_{a=b}^{\infty} P_{a,b}(c,t)$ where $P_{a,b}(c,t)$ is defined in equation (5)".

Table 2: For exactly *b* customers served by time *t*

$\lambda = 1,\ \mu = 4,\ w = 1,\ n = 0,\ \beta = 1,\ \xi = 1,\ \rho = 1,\ b = 0$ to 6 $P_{.b}(c,t) = P_{.b,B}(c,t) + P_{.b,V}(c,t)$				
$t = 1$	$t = 3$	$t = 5$	$t = 7$	$t = 10$
.483485	.392200	.338348	.222880	.0739809
.333343	.293382	.232925	.136406	.0375456
.13382	.156248	.112178	.0580149	.0132014
.03866	.0738856	.048044	.0216532	.0040095
.00875	.032712	.019831	.0075688	.0010965
.00161	.013342	.007936	.0024681	.0002598
.000252	.0047242	.002920	.0007192	.0000477
.99992	**.966494**	**.762184**	**.44971**	**.130142**

Table-2 Coincides with table I of Hubbard *et al.* (1986)

iii. $P_N(t) = P(Exactly\ N\ customers\ in\ the\ system\ by\ time\ t)$. $P_N(t)$ can be expressed in terms of $P_{a,b}(c,t)$ and is based on the relationship, we have

$$P_N(t) = \sum_{b=0}^{\infty} P_{b+N,b}(N,t) \quad \& \quad P_N(t) = P_B(N,t) + P_V(N,t)$$

where, $P_B(N,t) = \sum_{b=0}^{\infty} P_{b+N,b,B}(N,t)$, $P_V(N,t) = \sum_{b=0}^{\infty} P_{b+N,b,V}(N,t)$

Table 3: Exactly *N* customers in the system

$\lambda = 1, \mu = 2, w = 1, n = 1, \rho = 0.2, \beta = 0.6, \xi = 0.7, N = 0$ to 6		
$t = 1$	$t = 3$	$t = 5$
.245353	.367102	.362711
.462100	.323646	.303897
.221013	.181321	.158741
.058932	.079292	.063808
.010870	.028463	.020800
.001528	.008188	.005287
.000166	.001609	.000837
.999963	**.989623**	**.916082**

iv. The server's utilization time, server's vacation time *i.e.* the fraction of time the server is busy and the fraction of time server is on vacation until time *t* can also be expressed in terms of $P_{a,b}(c,t)$

Server's utilization time: $U(t) = \sum_{a=0}^{\infty} \sum_{b=0}^{a} P_{a,b,B}(c,t)$

Server's vacation time: $V(t) = \sum_{a=0}^{\infty}\sum_{b=0}^{a} P_{a,b,V}(c,t)$

Table 4: Server's utilization time and Server's vacation time

λ =1.7, μ = 2.5, w = 1.5, n = 1, ρ = 0.6, β = 0.7, ξ = 1.1			
	$\sum_{a=0}^{\infty}\sum_{b=0}^{a} P_{a,b,V}(c,t)$	$\sum_{a=0}^{\infty}\sum_{b=0}^{a} P_{a,b,B}(c,t)$	Total
t = 1	0.650353	0.342156	**0.992509**
t = 2	0.621177	0.285446	**0.906623**
t = 3	0.513311	0.198857	**0.712168**
t = 4	0.364730	0.118833	**0.483563**
t = 5	0.228494	0.0633109	**0.291805**

6. Performance Indices

(a) The expected number of customers in the system *E(L)* is given by

$$E(L) = \sum_{N=1}^{\infty} N \ \ [P_B(N,t) + P_V(N,t)]$$

(b) The expected number of customers in the queue is given by

$$E(L_q) = \sum_{N=1}^{\infty} (N-1)\ [P_B(N,t) + P_V(N,t)]$$

(c) The throughput is

$$T(P) = \sum_{N=1}^{\infty} \mu[\, P_B(N,t) + P_V(N,t)]$$

(d) Mean balking rate is given by

$$B.R. = \sum_{N=1}^{\infty} \lambda(1-\beta)\,\{P_B(N,t) + P_V(N,t)\}$$

(e) Mean reneging rate is given by

$$R.R. = \sum_{N=1}^{\infty} \xi(N-1)\ \ [P_B(N,t) + P_V(N,t)]$$

(f) Average rate of customer loss *(L.R.)* is given by

$$L.R. = B.R. + R.R.$$

7. Cost Model

We make a expected cost function for the given system, considering cost per some unit of time of server for all the parameters considered above.

Let

$C1$ = Vacation Cost

$C2$ = Busy cost

$C3$ = Idle Cost.

$C4$ = unit is waiting for service.

$C5$ = unit joins the system and is served.

$C6$ = customer renege or balks.

Mean cost function per some unit time:

$$C = C1*P_{VAC}+C2*P_{BUSY}+C3*P_{IDLE}+C4*E(Lq)+C5*[E(L)-E(Lq)]+C6*L.R.$$

We fix cost elements $C1 = 100$, $C2 = 110$, $C3 = 120$, $C4 = 150$, $C5 = 130$, $C6 = 140$.

8. Graphical Presentations

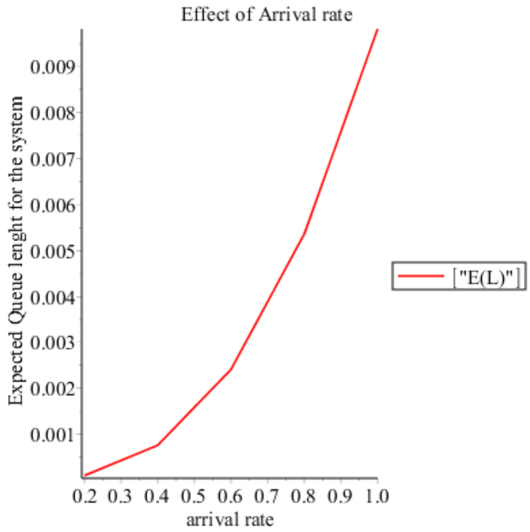

Figure 1: Arrival rate on *E(L)*

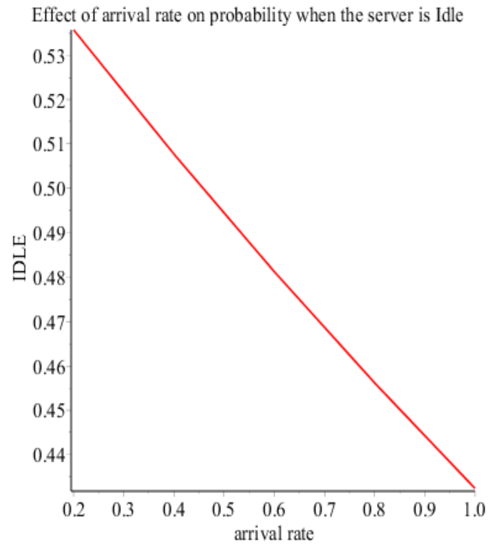

Figure 2: Arrival rate on probability of server remains idle

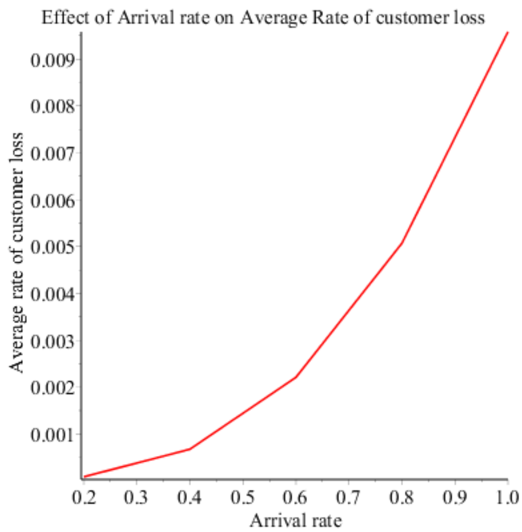

Figure 3: Arrival rate on *L.R.*

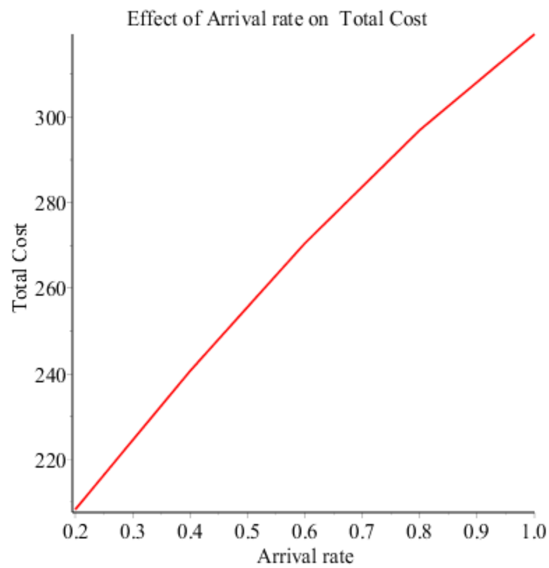

Figure 4: Arrival rate on Cost function

In Figures 1 to 4 we fix $w = 1.5$, $n = 1$, $\mu = 2.5$, $\beta = 0.4$, $\rho = 0.6$, $\xi = 1.1$, $t = 1$ and vary the values of λ. These graphs show that the expected number of customers in the system, expected cost and average rate of customer loss increase as arrival rate increases but probability of server remains idle decreases as λ increases

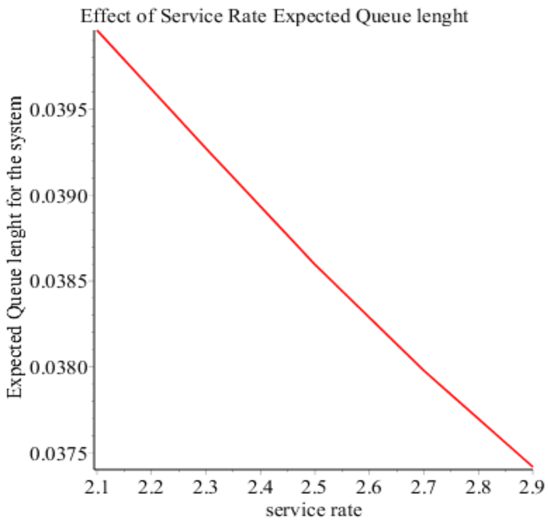

Figure 5: Service rate on *E(L)*

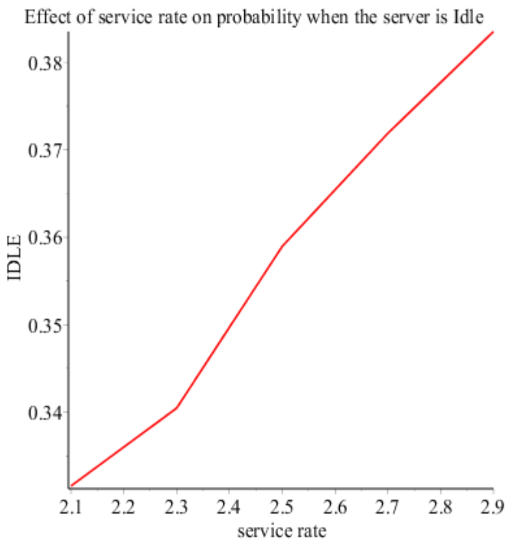

Figure 6: Service rate on throughput

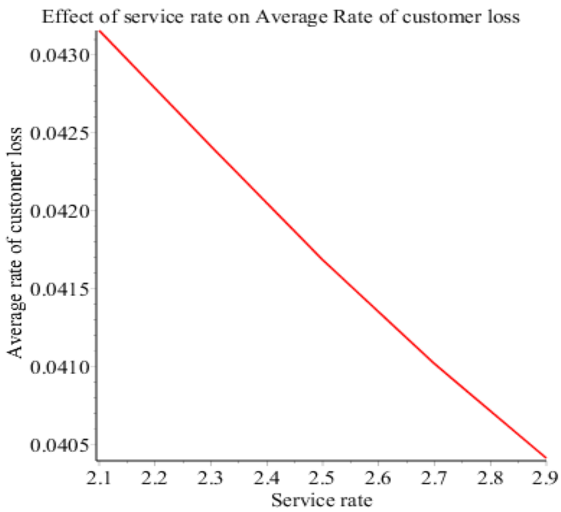

Figure 7: Service rate on *L.R*

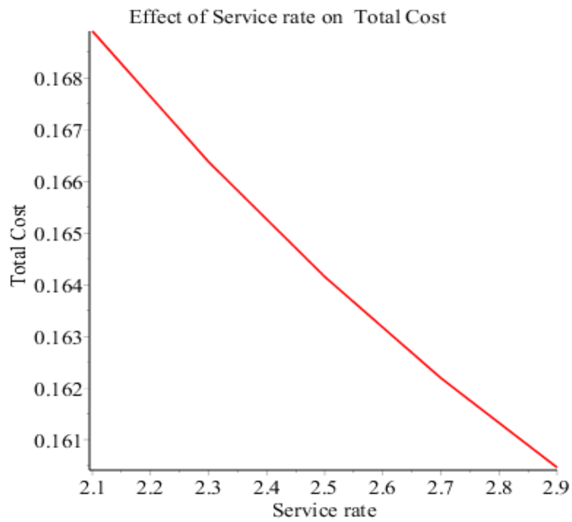

Figure 8: Service rate on cost function

In Figures 5 to 8 we fix $\lambda = 1.7$, $w = 1.5$, $n = 1$, $\beta = 0.4$, $\xi = 1.1$, $\rho = 0.6$ and vary the values of μ. These graphs show that the expected number of customers in the system, expected cost and average rate of customer loss decrease as μ increases but probability of server remains idle increases as μ increases".

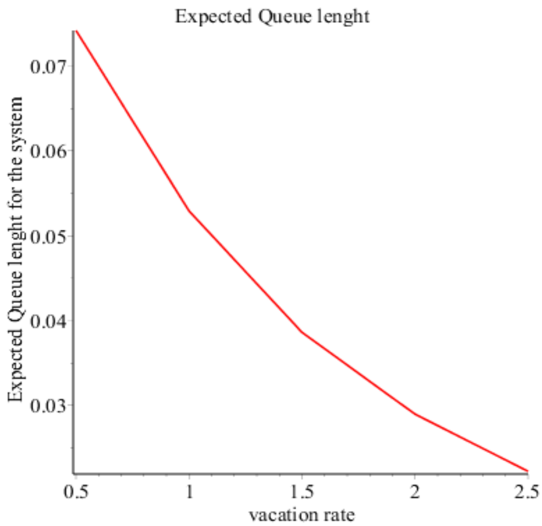

Figure 9: Vacation Rate (w) on *E(L)*

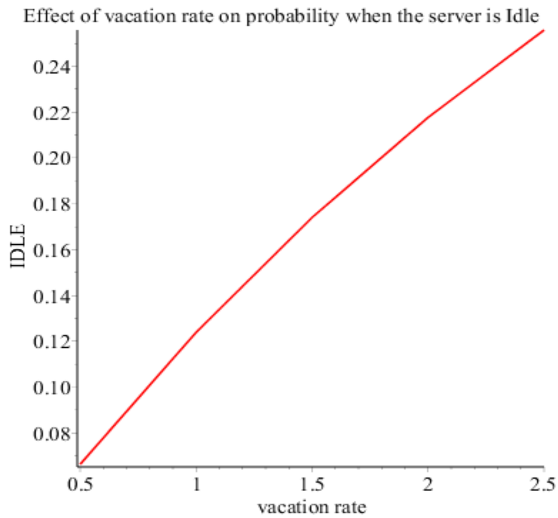

Figure 10: Vacation Rate (w) on Probability of server remains idle

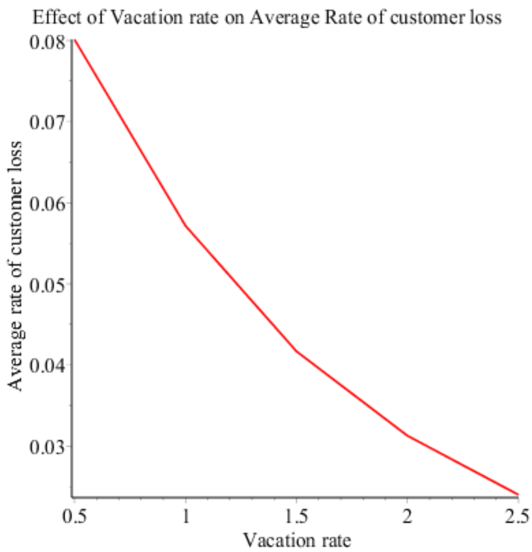

Figure 11: Vacation Rate (w) on *L.R.*

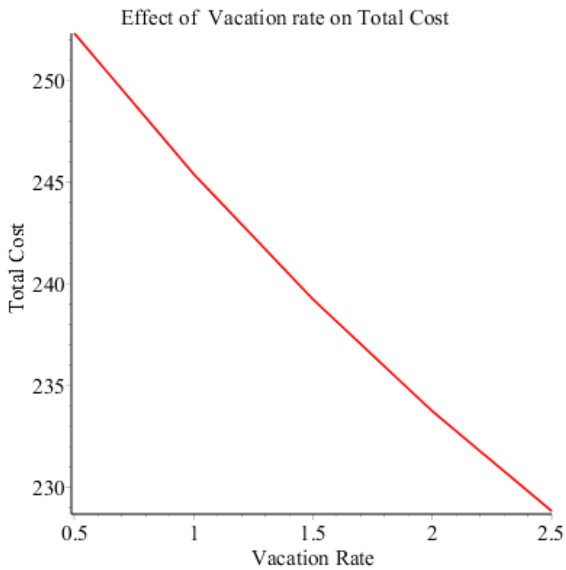

Figure 12: Vacation Rate (w) on Cost Function

In Figures 9 to 12 we fix λ=1.7, μ= 2.5, n=1, β = 0.4, ρ = 0.6, ξ=1.1 and vary the values of w. “These graphs show that the expected number of customers in the system, expected cost, expected queue length and average rate of customer loss decrease as w increases but probability of server remains idle increases as vacation rate increases”.

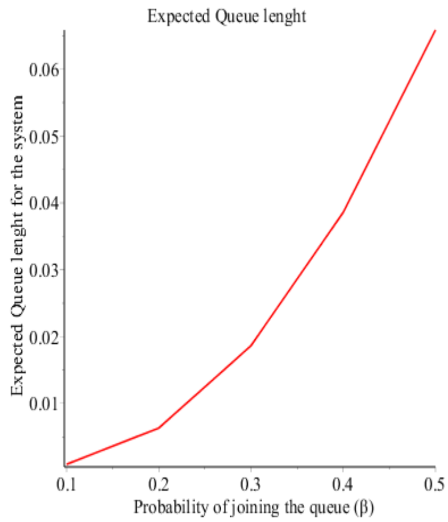

Figure 13: *β* on *E(L)*

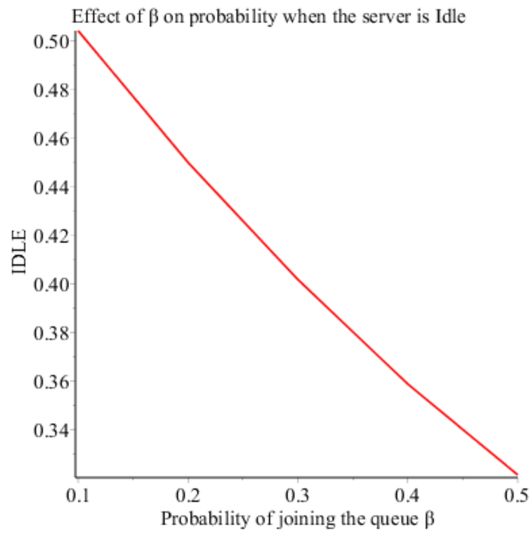

Figure 14: *β* on Probability of server remains idle

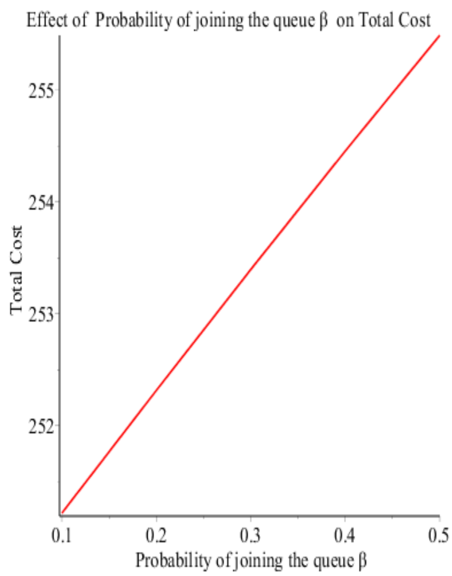

Figure 15: Impact of *β* on Total Cost

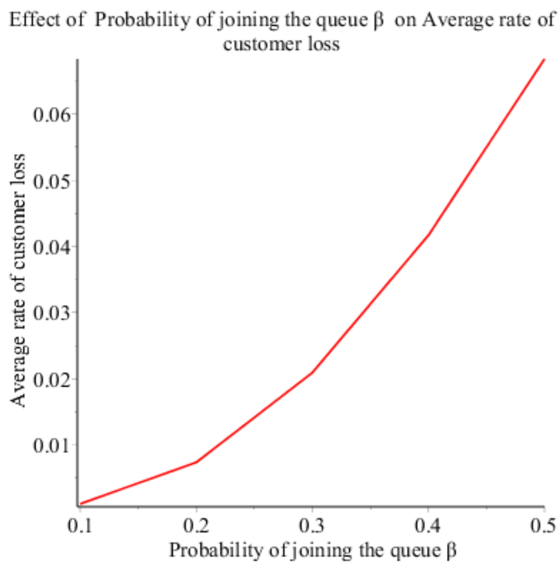

Figure 16: Impact of *β* on *L.R.*

In Figures 13 to 16 we fix λ=1.7, μ=2.5, n=1,w = 1.5, ξ=1.1, ρ = 0.6 and vary the values of β. "These graphs show that the expected number of customers in the system, average rate of customer loss, expected cost increase as β increases but probability of server remains idle decreases as β increases".

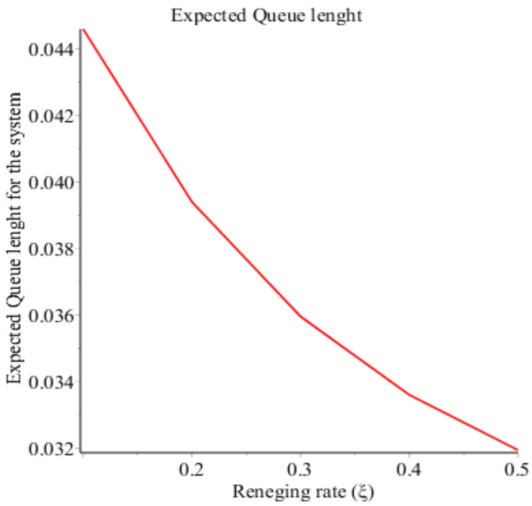

Figure 17: Reneging rate on *E(L)*

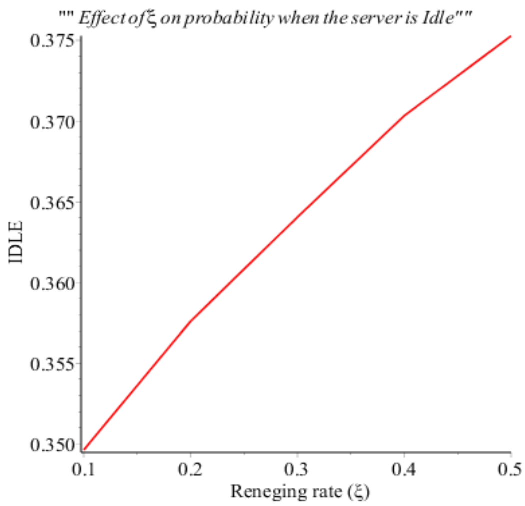

Figure 18: Impact of ξ on probability that server remains Idle

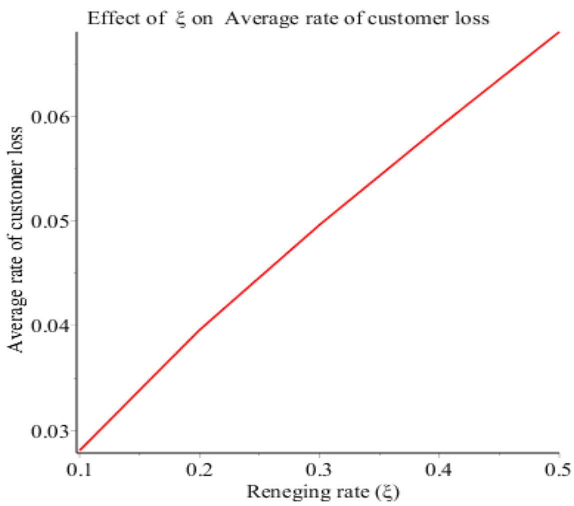

Figure 19: Impact of ξ on *L.R.*

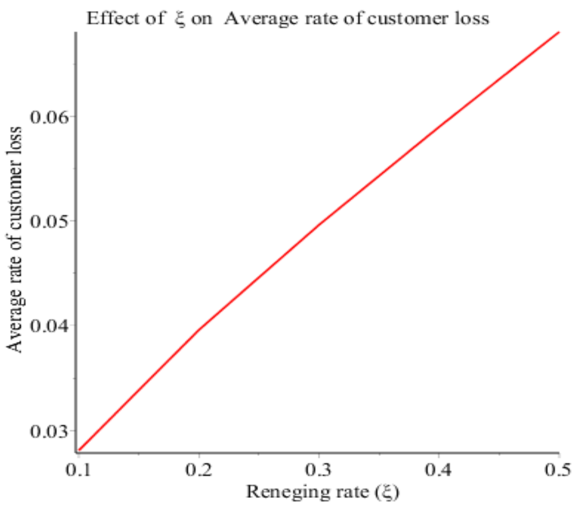

Figure 20: Impact of ξ on cost function

In Figures 17 to 20 we fix $\lambda = 1.7$, $w = 1.5$, $n = 1$, $\mu = 2.5$, $\beta = 0.4$, $\rho = 0.6$, $t = 1$ and vary the values of ξ. These graphs show that expected cost, probability of server remains idle and average rate of customer loss increase as ξ increases but expected queue length decreases as ξ increases.

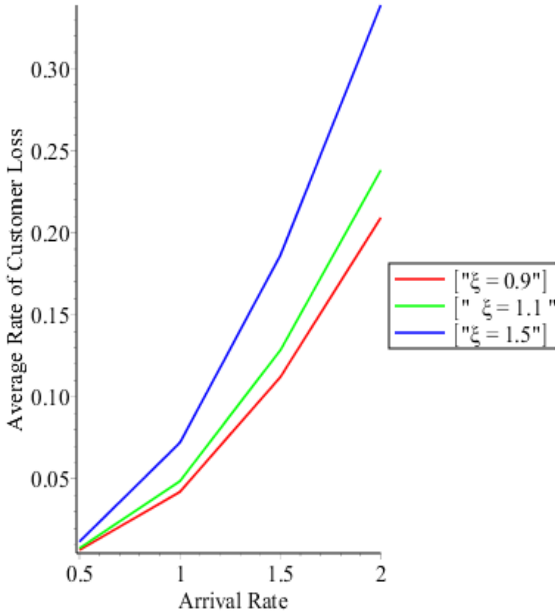

Figure 21: Impact of *λ* on *E(L)*

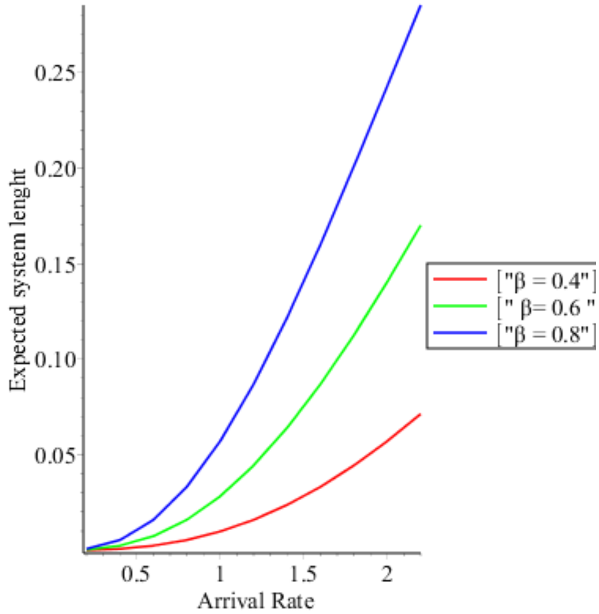

Figure 22: Impact of *λ* on *L.R.*

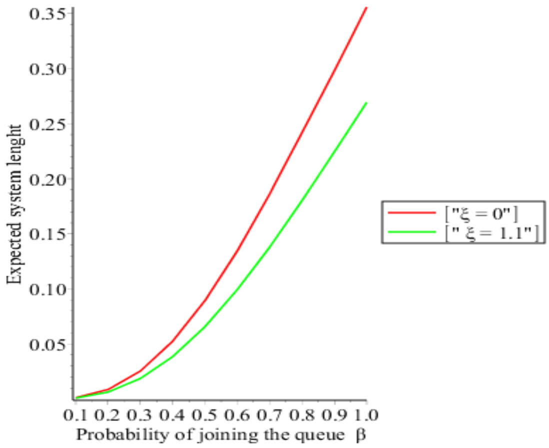

Figure 23: Impact of *β* on *E(L)*

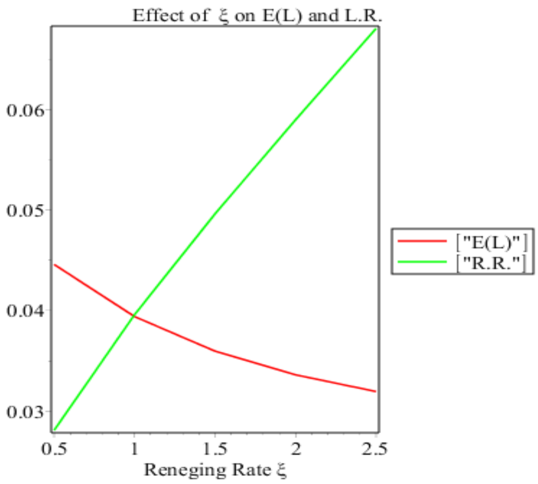

Figure 24: Impact of *λ* on *E(L)* and *L.R.*

Figures 23 and 24 give the effect of β and λ on mean system length and average loss of customer. As probability of joining the queue increases *E [L]* increases and as reneging rate increases *E[L]* decreases and average rate of customer loss increases

9. Conclusions

This paper considers two-dimensional state Markovian queueing model with Bernoulli Schedule, multiple vacations and impatience customers in which the state of the system is given by (*a, b*). The concept of few clienteles (say "*n*") available in the system makes this model different from the previous models available in literature. The governing systems of equations are solved by using the Laplace transform and different measures of effectiveness (Expected system length, throughput of system, mean balking rate, mean reneging rate, *etc.*) are calculated that provide better perception of a queueing system. Finally, an expected cost function is discussed, and it shows that if we increase service rate then the probability that customers may balk or renege from the system is reduced which minimize expected cost for the system. Different firms can utilize this model to model their system accordingly and can have an idea about the minimum cost that system will generate.

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