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Reliability Assessment of Two-Component Series System Shock Model

S. B. Munoli and Abhijeet Jadhav
Department of Statistics, Karnatak University, Dharwad - 580003

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Abstract

Configuration of sub-assemblies in series is recommended in some environments. Reliability assessment of two-component series system receiving shocks from a single source is studied. Shocks are of two types: damage shocks and fatal shocks. The component fails either due to exceedance of damage to its threshold or when it experiences a fatal shock. The two cases of fixed and random thresholds are considered. Computation and comparison of estimators of two models is done through simulation.

Key words: Series system; Damage shock; Fatal shock; Threshold; Reliability Assessment.

AMS Subject Classifications: 62N02, 62N05

1. Introduction

Configuration of sub-assemblies of a system in different ways is attempted (explored) to meet certain requirements. The two fundamental configurations of subassemblies are series and parallel. These have been studied extensively by reliability engineers, economists, life science and social science researchers. One can quote several examples, wherein series configurations of sub-assemblies is considered such as water heaters, lamps in a circuit, water pumps, freezers and refrigerators, solar panels, etc. Series configuration is needed when the same current must flow through all the sub-assemblies, easy overheating of components is to be avoided, voltage is to be increased to meet the minimum operating requirements of the inverter in solar appliances. In this paper, an attempt is made to study the reliability of two component series system receiving shocks from single source wherein the shocks are of two types: damage shocks and fatal shocks. The pioneering work on shock models is by Esary and Marshall (1973). A-hameed and Proschan (1973),A-Hameed and Proschan (1975) have considered non-stationary shock models and shock models with underlying birth process. Ross (1981) has studied generalized Poisson shock models. Survival under the pure birth shock model was studied by Klefsjö (1981). Shanthikumar and Sumita (1983) have discussed on general shock models with correlated renewal sequences. Semi-Markov shock models with additive damages is studied by Posner and Zuckerman (1986). Anderson (1987) has proposed limit theorems for general shock models. Some multivariate distributions were

Corresponding Author: Abhijeet Jadhav
Email: abhijadhav10292@gmail.com

derived from non-fatal shock models by Savits (1988). Gut (1990) and Skoulakis (2000) have contributed to the literature on general shock models for reliability system and cumulative shock models respectively. Mallor and Santos (2003) have dealt with classification of shock models in system reliability. Applications of Poisson shock models in insurance and credit risk is studied by Lindskog and McNeil (2003). Inference for reliability of shock models is studied by Chikkagoudar and Palaniappan (1981), Kunchur and Munoli (1993), Munoli and Suranagi (2007),Munoli and Suranagi (2009) and Munoli and Bhat (2011).

Several researchers have contributed to the fields of modelling system reliability, its optimization, bounds on system reliability and inference for system reliability. Here are few references of contributions to these fields: Rutemiller (1966), Zacks and Even (1966), Chung (1995), Råde (1976), Nakagawa and Rosenfeld (1979), Weier (1981), Necsulescu and Krausz (1986), Fujii and Sandoh (1984), Wani and Kabe (1971), Hanagal (1996), Munoli and Mutkekar (2011a), Munoli and Mutkekar (2011b). In the present study, a two-component series system is subjected to a sequence of shocks occurring randomly in time as events of Poisson process. Shocks are occurring with intensity $\lambda,\ \lambda > 0$. Shocks are of two types; damage shocks and fatal shocks. Any shock will be a damage shock with probability 'p' and fatal shock with probability $(1-p)$. Every damage shock causes some amounts of damage to both components. Damages are non-accumulating. The component fails whenever the damage exceeds threshold (u) of the component. If not, the component functions as good as new one. On the other hand, the component may also fail when it experiences a fatal shock. The two components function independently. The system fails when either of the two components fail (series system). Let X and Y denote respectively the amount of damages to first failing component of the system and surviving component of the system. X and Y are assumed to be exponential random variables *(r.v.'s)* with parameter $\theta,\ \theta > 0$. The system reliability at time 't' is given by

$$S_1(t) = \sum_{k=0}^{\infty} \frac{e^{-p\lambda t}(p\lambda t)^k}{k!}\ e^{-(1-p)\lambda t}\ \overline{P}_k \tag{1}$$

The above expression represents the following:

The first term $\frac{e^{-p\lambda t}(p\lambda t)^k}{k!}$ is the expression for the probability that system has experienced 'k' number of damage shocks during $(0,\ t)$, $e^{-(1-p)\lambda t}$ represents the probability that the system did not experience a fatal shock during $(0,\ t)$. $\overline{P}_k$ is the probability that the system survives with k number of shocks that it has experienced during $(0,\ t)$. The system may experience during $(0, t)$ no shock or one shock or two shocks,...; hence the summation with $k = 0, 1, 2, \ldots$. $\overline{P}_k$ is given by

$$\begin{aligned}
\overline{P}_k &= P(\text{Both components survive with k number of damage shocks}) \\
&= P\left(X_1 < u,\ \ldots,\ X_k < u\right) . \ P\left(Y_1 < u, \ldots, Y_k < u\right) \\
&= \left(1 - e^{-u\theta}\right)^k\ .\ \left(1 - e^{-u\theta}\right)^k \\
&= \left(1 - e^{-u\theta}\right)^{2k} \qquad (2)
\end{aligned}$$

Now, substituting the value of $\overline{P}_k$ from expression (2) in the expression for $S_1(t)$ (expression

(1)) and simplifying, we get the expression for reliability of series system as

$$S_1(t) = e^{-\lambda t[1-p\left(1-e^{-u\theta}\right)^2]} \tag{3}$$

The real-life examples from health science and finance for a shock model with damage shocks and fatal shocks are:

Example 1: Heart disease is the leading cause of death worldwide. The common heart diseases are heart attack and cardiac arrest. Heart attacks occur when blood flow to the heart muscle is temporarily blocked, starving the muscle tissue of oxygen which causes scarring and damage to heart muscle (damage shock with amount of damage tolerable and the person survives with this heart attack). For a heart attack to lead to death the damage to the heart needs to be large enough resulting in irregular heart beat and stop eventually (failure due to damage exceeding threshold). Cardiac arrest is an abrupt loss of heart function, breathing and consciousness. It results from an electric disturbance in the heart that disrupts its pumping action, stopping blood flow to the different organs and can lead to death (fatal shock).

Example 2: While lending loans to customers, financial institutes choose customers who fetch the institute high profit. If a loanee defaults (fatal shock), it will be a loss to financial institute. On the other hand, the loanee may do some partial repayments, close the loan account by paying off the loan early. In this case the lender will lose a proportion of the interest. Here partial repayments are damages due to shocks and due to closing the loan account early is failure due to damage exceeding the shock.

These examples are discussed in detail in Munoli and Suhas (2019).

The rest of the paper is organized as follows: Life testing experiment is explained in Section 2. MLE's of the parameters of the model and their asymptotic distribution are obtained in Section 3. Computation of estimators is dealt with in Section 4. Section 5 deals with the case of thresholds of the components being *r.v's*. Comparison of two cases of fixed and random thresholds is made in Section 6, conclusions are also outlined in the same section.

2. Life testing experiment

Suppose, 'r' two-component systems having life distribution $(1 - S_1(t))$ are subjected to a life testing experiment, and the experiment continues until all the systems fail. For the i^{th} system, let the first failure (of two components) occur at the m_i^{th} shock, $i = 1, 2, \ldots, r$, which coincides with system failure (also known as a series system). Out of 'r' number of first failing components, 'r_1' components fail due to damage exceeding threshold 'u', and $r_2 (= r - r_1)$ components fail due to experiencing a fatal shock. Let X_{ij} and Y_{ij}, $i = 1, 2, \ldots, r$; $j = 1, 2, \ldots, m_i$, be the random variables representing damages due to the j^{th} damage shock to failing and surviving components of the i^{th} system. The $X_{ij}\prime s$ and $Y_{ij}\prime s$ are assumed to be independent exponential random variables with parameter θ $(\theta > 0)$. Let t_{ij} be the time epoch at which the i^{th} system experienced the j^{th} shock $(j = 1, 2, \ldots, m_i;\ i = 1, 2, \ldots, r)$. The inter-arrival times $(t_{i,j} - t_{i,j-1})$ are exponential random variables with parameter $p\lambda$.

For 'r_1' systems, first failure (out of two components) has occurred due to damage exceeding the threshold, the joint pdf of the *r.v's* $m_i,\ t_{i1},\ t_{i2},\ldots,t_{im_i},\ X_{i1},\ldots,X_{im_i-1},Y_{i1},\ldots,Y_{im_i}$ is

$$\prod_{i=1}^{r_1}(p\lambda)^{m_i}e^{-p\lambda t_{m_i}}\ \theta^{m_i-1}e^{-\theta\sum_{j=1}^{m_i-1}x_{ij}}\ e^{-u\theta}\ \theta^{m_i}\ e^{-\theta\sum_{j=1}^{m_i}y_{ij}} \tag{4}$$

It is assumed that the amount of damage due to a shock at which component has failed (due to damage exceeding its threshold) is not observable but is known to exceeds its threshold.

For 'r_2' systems, the first failure (out of 2 components) has occurred due to fatal shock and the joint pdf of *r.v's* $m_i,\ t_{i1},\ldots,t_{im_i},\ \ X_{i1},\ldots,X_{im_i-1},Y_{i1},\ldots,\ Y_{im_i-1}$ is given by

$$\prod_{i=1}^{r_2}(p\lambda)^{m_i-1}e^{-p\lambda t_{m_i-1}}\theta^{m_i-1}e^{-\theta\sum_{j=1}^{m_i-1}x_{ij}}.(1-p)\lambda\ e^{-(1-p)\lambda(t_{m_i}-t_{m_i-1})}\theta^{m_i-1}e^{-\theta\sum_{j=1}^{m_i-1}y_{ij}} \tag{5}$$

In this case, as the system failure has occurred due to experiencing a fatal shock at m_i^{th} shock, the damages due to fatal shock for both surviving and failing components are not observed. Combining (4) and (5), the joint pdf L_1 of all random variables of the experiment is given by

$$L_1=p^{m-r_2}\lambda^m e^{-p\lambda t..}\ \theta^{2m-r_1-2r_2}\ e^{-\theta\left(x..+y..+r_1u\right)}\left(1-p\right)^{r_2}e^{-\lambda t'} \tag{6}$$

where,

$t..\ =\ \sum_{i=1}^{r_1}t_{m_i}+2\sum_{i=1}^{r_2}t_{m_i-1}-\sum_{i=1}^{r_2}t_{m_i}$

$t'=\sum_{i=1}^{r-r_1}\left(t_{m_i}-t_{m_i-1}\right)$

$x..=\sum_{i=1}^{r}\sum_{j=1}^{ri-1}x_{ij}$

$y..=\sum_{i=1}^{r_1}\sum_{j=1}^{m_i}y_{ij}+\sum_{i=1}^{r_2}\sum_{j=1}^{m_i-1}y_{ij}$

$m=\sum_{i=1}^{r}m_i$

3. MLE's of parameters

Treating L_1 as function of parameters, the MLE's of parameters are obtained as

$$\hat{p}=\frac{\left[m-\left(r-r_1\right)\right]t'}{mt'+\left(r-r_1\right)t..} \tag{7}$$

$$\hat{\lambda}=\frac{mt'+\left(r-r_1\right)t..}{t'(t..+t')} \tag{8}$$

$$\hat{\theta}=\frac{2m-2r+r_1}{x..+y..+r_1u} \tag{9}$$

Using invariance property of MLE's, the MLE $\widehat{S_1}(t)\ of\ S_1(t)$ is obtained by substituting $\widehat{p}$, $\widehat{\lambda}$ and $\widehat{\theta}$ for $p,\ \lambda\ \ and\ \theta$ respectively in expression (3).

In order to obtain asymptotic distribution of $\widehat{p}$, $\widehat{\lambda}$ and $\widehat{\theta}$, Fisher information matrix $I(p,\ \lambda,\ \theta)$ is obtained as

$$I(p,\ \lambda,\ \theta) = \begin{bmatrix} \frac{2m-2r+r_1}{\theta^2} & 0 & 0 \\ 0 & \frac{m-(r-r_1)}{p^2} + \frac{r-r_1}{(1-p^2)} & E(t..) \\ 0 & E(t..) & \frac{m}{\lambda^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2m-2r+r_1}{\theta^2} & 0 & 0 \\ 0 & b & E(t..) \\ 0 & E(t..) & \frac{m}{\lambda^2} \end{bmatrix} \tag{10}$$

where,

$$b = \frac{m-(r-r_1)}{p^2} + \frac{r-r_1}{(1-p^2)}, \qquad E(t..) = \sum_{i=1}^{r_1} \frac{m_i}{p\lambda} + 2\sum_{i}^{r-r_1} \frac{m_i-1}{p\lambda} - \sum_{i=1}^{r-r_1} \left[\frac{m_i-1}{p\lambda} + \frac{1}{(1-p)\lambda}\right]$$

Using multivariate central limit theorem and asymptotic properties of MLE (under regularity conditions), we have

$$\left((p-\widehat{p}), \left(\lambda - \widehat{\lambda}\right), (\theta - \widehat{\theta})\right) \to N_3(0,\ I^{-1})$$

and I^{-1} is given by

$$I^{-1} = \begin{bmatrix} \frac{\theta^2}{2m-2r+r_1} & 0 & 0 \\ 0 & \frac{m}{mb-\lambda^2(E(t..))^2} & -\frac{\lambda^2 E(t..)}{mb-\lambda^2[E(t..)]^2} \\ 0 & -\frac{\lambda^2[E(t..)]^2}{mb-\lambda^2[E(t..)]^2} & \frac{b\lambda^2}{mb-\lambda^2[E(t..)]^2} \end{bmatrix} \tag{11}$$

4. Simulation study

Validation of the model and computation of estimators is made through Monte-Carlo simulation. The random variables of the model are generated for different values of r, t and parameter combinations as below:

For the i^{th} series system of two components, the *r.v's* $t_{i1},\ t_{i2}, \ldots, t_{im_i}, X_{i1}, \ldots, X_{im_i-1}$, $Y_{i1}, Y_{i2}, \ldots, Y_{im_i-1}, Y_{im_i}$ in the case of system failure due to damage shock are generated as follows:

Step 1: Let $u = u_0$ and a random number w_i is generated from $U(0,1)$. If $0 < w_i < p(= p_0)$, then the system failure is considered as failure due to damage shock.

Step 2: Initialize $m_i = 0$, for $\theta = \theta_0$ the r.v. X_{i1}, Y_{i1} following exponential distribution with parameter θ_0 are generated. With this m_i is incremented by 1. X_{i1} *and* Y_{i1} are compared with u_0. If both $X_{i1} < u_0$ and $Y_{i1} < u_0$, the process of exponential r.v's generation with parameter θ_0 and comparing with u_0 is repeated with incrementation of m_i by 1 with every repeatation. The iteration at which either of X_{ij} *or* Y_{ij} exceeds u_0, m_i is noted.

Step 3: m_i number of inter-arrival times having exponential distribution with parameter $p_0\lambda_0$ are generated. Addition of these inter-arrival times results in t_{im_i}.

Step 4: If $w_i > p_0$, then system failure is considered as failure due to fatal shock. Step 2 and 3 are repeated with the difference that $(m_i - 1)$ interarrival times having exponential distribution with parameter $(p_0\lambda_0)$ are generated, adding all these, t_{i,m_i-1} is obtained. One inter-arrival time $(t_{i,m_i} - t_{i,m_i-1})$ is generated having $exp((1-p_0)\,\lambda_0)$ distribution.

Steps 1 to 4 are repeated for $r = 25, 30, 40, 50$.

The statistics $x..,\ y..,\ r_1,\ m,\ t..,\ t'$ are computed, using which MLE $\widehat{S}_1(t)\ of\ S_1(t)$ are obtained at given mission time. Also using the considered set of parameter combinations $S_1(t)$ is also obtained for same mission times. The discrepancy between theoretical $S_1(t)$ and estimated $\widehat{S}_1(t)$ is studied through bias for three sets of parameter combinations and are presented in Table 1, Table 2 and Table 3.

Table 1: Survival probabilities and bias for $\mathbf{p = 0.7,\ \lambda = 0.3,\ u = 0.95,\ \theta = 0.8}$

		Absolute Bias			
t	$S_1(t)$	r=25	r=30	r=40	r=50
0.5	0.886703	0.024142	0.009437	0.008541	0.002917
0.75	0.834963	0.034296	0.013352	0.01208	0.004119
1	0.786242	0.043307	0.016791	0.015188	0.005171
1.25	0.740364	0.051269	0.019796	0.017901	0.006085
1.5	0.697163	0.058268	0.022405	0.020256	0.006874
1.75	0.656483	0.064384	0.024654	0.022283	0.00755
2	0.618176	0.06969	0.026574	0.024013	0.008124

Table 2: Survival probabilities and bias for $\mathbf{p = 0.4,\ \lambda = 0.4,\ u = 1.75,\ \theta = 0.7}$

		Absolute Bias			
t	$S_1(t)$	r=25	r=30	r=40	r=50
0.5	0.849827	0.047447	0.042041	0.005374	0.003764
0.75	0.783422	0.066517	0.058848	0.007443	0.005211
1	0.722206	0.082895	0.073223	0.009163	0.006412
1.25	0.665773	0.096855	0.085421	0.010575	0.007397
1.5	0.61375	0.108646	0.095668	0.011717	0.008192
1.75	0.565792	0.118495	0.104174	0.012622	0.00882
2	0.521581	0.126606	0.111126	0.013319	0.009303
2	0.52709	0.091367	0.047135	0.04284	0.007813

Table 3: Survival probabilities and bias for $\mathbf{p = 0.65,\ \lambda = 0.5,\ u = 1.2,\ \theta = 1.1}$

		Absolute Bias			
t	$S_1(t)$	r=25	r=30	r=40	r=50
0.5	0.849827	0.047447	0.042041	0.005374	0.003764
0.75	0.783422	0.066517	0.058848	0.007443	0.005211
1	0.722206	0.082895	0.073223	0.009163	0.006412
1.25	0.665773	0.096855	0.085421	0.010575	0.007397
1.5	0.61375	0.108646	0.095668	0.011717	0.008192
1.75	0.565792	0.118495	0.104174	0.012622	0.00882
2	0.521581	0.126606	0.111126	0.013319	0.009303

5. A random threshold case

Assuming that the thresholds of two components of the series system are independent *r.v's* having exponential distribution with parameter σ, $\sigma > 0$; and with other modelling features same as in Section 1, the reliability of the system at mission time 't' is given by

$$S_2(t) = e^{-\lambda t\left[1-p\left(\frac{\theta}{\theta+\sigma}\right)\right]^2} \tag{12}$$

In order to assess $S_2(t)$, considering the life testing experiment of 'r' systems with life distribution $(1 - S_2(t))$ and following on the lines of Section 2, the joint distribution of the random variables $m_i, t_{i1},\ t_{i2}, \ldots, t_{im_i}, X_{i1}, \ldots, X_{im_i-1}, Y_{i1}, Y_{i2}, \ldots, Y_{im_i-1}, Y_{im_i},\ u_{i,1},\ u_{i,2}$ for all 'r' systems is given by

$$L_2 = p^{m-r_2}\lambda^m e^{-p\lambda t_{..}}\theta^{2m-r_1-2r_2}e^{-\theta(x_{..}+y_{..})}(1-p)^{r_2}e^{-\lambda t'}\left(\frac{\sigma}{\sigma+\theta}\right)^{r_1}\sigma^{2r}e^{-\sigma u} \tag{13}$$

where, $t_{..},\ t',\ x_{..},\ y_{..},\ m$ are as defined in (6) with $u_. = \sum_{i=1}^{r_1} u_{i,1} + \sum_{i=1}^{r_2} u_{i,2}$ Using L_2, the MLE's of $p,\ \lambda,\ \theta,\ \sigma$ are obtained as

$$\hat{p} = \frac{[m-(r-r_1)]\,t'}{mt' + (r-r_1)\,t_{..}} \tag{14}$$

$$\hat{\lambda} = \frac{mt' + (r-r_1)\,t_{..}}{t'(t_{..}+t')} \tag{15}$$

and $\hat{\sigma}$ and $\hat{\theta}$ are obtained numerically using Newton-Raphson method by solving the equations given below

$$(\sigma+\theta)\,(x_{..}+y_{..}) + r_1 = 0 \tag{16}$$

$$\frac{1}{\sigma}\left(2r_1 + 2r_2 + \frac{r_1\theta}{\sigma+\theta}\right) - u_. = 0 \tag{17}$$

Using invariance property of MLE, MLE $\widehat{S}_2(t)\ of\ S_2(t)$ is obtained as

$$\widehat{S}_2(t) = e^{-\hat{\lambda}t\left[1-\hat{p}\left(\frac{\hat{\theta}}{\hat{\theta}+\hat{\sigma}}\right)\right]^2} \tag{18}$$

$\widehat{S}_2(t)$ is computed using Monte-Carlo simulation procedure. For the i^{th} system, for generation of random variables m_i,t_{i1},t_{i2},...,t_{im_i},X_{i1},...,X_{im_i-1},Y_{i1},Y_{i2},...,Y_{im_i} and computation of $\widehat{S}_2(t)$, Section 4 is referred. The random thresholds U_{i1}, U_{i2} are generated from exponential distribution with parameter $\sigma = \sigma_0$ and results are presented in Table 4, Table 5 and Table 6.

From above tables, it is evident that for both the sets of parameter combinations under the two cases of fixed and random thresholds of components, bias of estimators decreases as the number of systems on test (r) increases.

Table 4: Survival probabilities and bias for $\mathbf{p = 0.7,\ \lambda = 0.3,\ \sigma = 0.7,\ \theta = 0.8}$

		Absolute Bias			
t	$S_2(t)$	$r = 25$	$r = 30$	$r = 40$	$r = 50$
0.5	0.886802	0.072797	0.069117	0.06289	0.061482
0.75	0.835103	0.104912	0.099509	0.090392	0.088335
1	0.786418	0.134412	0.127362	0.115497	0.112826
1.25	0.740571	0.161466	0.152842	0.138364	0.135112
1.5	0.697397	0.186231	0.176103	0.159144	0.155342
1.75	0.65674	0.208854	0.19729	0.177978	0.173656
2	0.618454	0.229475	0.216541	0.194998	0.190186

Table 5: Survival probabilities and bias for $\mathbf{p = 0.4,\ \lambda = 0.4,\ \sigma = 0.3,\ \theta = 0.7}$

		Absolute Bias			
t	$S_2(t)$	$r = 25$	$r = 30$	$r = 40$	$r = 50$
0.5	0.851462	0.054733	0.045812	0.040406	0.035388
0.75	0.785685	0.076962	0.064254	0.056585	0.049486
1	0.724988	0.096202	0.080112	0.070441	0.061515
1.25	0.668981	0.112745	0.093647	0.082213	0.07169
1.5	0.6173	0.126859	0.105096	0.092118	0.08021
1.75	0.569612	0.138785	0.114674	0.100354	0.087252
2	0.525608	0.148745	0.122579	0.107099	0.092979

Table 6: Survival probabilities and bias for $\mathbf{p = 0.65,\ \lambda = 0.5,\ \sigma = 0.4,\ \theta = 1.1}$

		Absolute Bias			
t	$S_2(t)$	$r = 25$	$r = 30$	$r = 40$	$r = 50$
0.5	0.849922	0.097285	0.087879	0.069802	0.029125
0.75	0.783553	0.138312	0.124615	0.098482	0.04062
1	0.722367	0.174835	0.157104	0.123524	0.050357
1.25	0.665959	0.207239	0.185722	0.14527	0.058528
1.5	0.613955	0.23588	0.210813	0.164031	0.065305
1.75	0.566013	0.261086	0.232694	0.180093	0.070845
2	0.521814	0.283157	0.251655	0.193718	0.075288

6. Comparison, results analysis and conclusion

The estimators for two models of series systems with components having fixed threshold and random threshold are compared by computing the mean square errors of $\widehat{S}_i(t),\ \ i = 1,2$ using

$$MSE_i\left(\hat{S}_i(t)\right) = \tfrac{1}{M}\textstyle\sum_{j=1}^{M}\left(S_i(t) - \hat{S}_{ij}(t)\right)^2; \quad i = 1,2 \quad \text{for} \quad m = 10000$$

The relative efficiencies of $\widehat{S}_2(t)$ as compared $\widehat{S}_1(t)$ are obtained as the ratio of MSE $(\widehat{S}_1(t))$ to MSE $(\widehat{S}_2(t))$ and are presented in Table 7.

From Table 7, it is clear that the estimators of the series system with fixed threshold are more efficient as compared to estimators of series system with random threshold. Hence,

Table 7: Relative efficiency of $\widehat{S}_2(t)$ as compared to $\widehat{S}_1(t)$

	P=0.7, λ=0.3, u=0.95, σ=0.7, θ=0.8			P=0.4, λ=0.4, u=1.75, σ=0.3, θ=0.7			P=0.65, λ=0.5, u=1.2, σ=0.4, θ=1.1		
t	$S_1(t)$	$S_2(t)$	Efficiency	$S_1(t)$	$S_2(t)$	Efficiency	$S_1(t)$	$S_2(t)$	Efficiency
0.5	0.8848	0.8868	0.0423	0.8521	0.8515	0.06055	0.84985	0.84995	0.1773
0.75	0.8323	0.8351	0.0411	0.7865	0.7857	0.0593	0.7834	0.7836	0.1732
1	0.7830	0.7864	0.0400	0.7260	0.7250	0.0581	0.7222	0.7224	0.1692
1.25	0.7365	0.7406	0.0389	0.6702	0.6690	0.0570	0.6658	0.6660	0.1652
1.5	0.6928	0.6974	0.0378	0.6186	0.6173	0.0558	0.6138	0.6140	0.1612
1.75	0.6517	0.6567	0.0368	0.5710	0.5696	0.0547	0.5658	0.5660	0.1573
2	0.6130	0.6185	0.0357	0.5271	0.5256	0.0536	0.5216	0.5218	0.1534

the study is suggestive of series system with components having fixed threshold, which results in gain in reliability of series system. This is because when the thresholds are *r.v.'s* and if one of the component's thresholds turns out to be too small, then system will be less reliable. Instead, maintaining the threshold of weakest component at certain level (optimum) would be the wise criteria to enhance system reliability.

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