Statistics and Applications {ISSN 2454 -7395 (online)}
Volume 20, No. 1, 2022 (New Series), pp 113-139

Efficient Use of Non-Sensitive Auxiliary Variable under Scrambled Randomized Response Mechanism for Estimating Sensitive Population Mean in Successive Sampling

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Received: 01 July 2020; Revised: 28 February 2021; Accepted: 02 March 2021

Abstract

This paper addresses the problem of estimation of population mean of a sensitive variable under investigation using scrambled randomized response mechanism in presence of non-sensitive auxiliary variable at current move in two occasion successive sampling. The proposed estimator is studied under scrambled randomized response models. The detail properties of the suggested estimators have been provided. To measure the scrambled model effect the envisaged estimators are compared with direct estimators. Optimum replacement policy has been elaborated. Numerical study is carried out to demonstrate the applicability of the propounded estimators and hence appropriate recommendations are given.

Key words: Scrambled response mechanism; Successive sampling; Non-sensitive auxiliary variable; Bias; Mean squared error.

AMS Classification: 62D05

1. Introduction

Surveys covering human population associated with sensitive issues, for instance, drug addiction, induced abortion, HIV infection status, excessive gambling, incidence of domestic violence, illegitimacy of offspring, drinking and driving, social security frauds, tax evasion, substance abuse, alcoholism, illegal income and AIDS *etc*. need to be addressed in social, psychological, socioeconomic and biometric research. In such cases employing the derived method of interview, people do not respond truthfully on even refuse to respond owing to social stigma and/ or fear. Under such circumstances, to gather valid and reliable data, protect respondent confidentiality and avoid unacceptable rate of non-response, randomized response procedures pioneered by Warner (1965) may be employed. Later Horvitz *et al.* (1967), Greenberg *et al.* (1969), Chaudhari and Mukerjee (1988), Kuk (1990), Mangat and Singh (1990), Christofides (2003), Mangat (1994), Arnab (2011) and Chaudhari and Christofides (2013) introduced various other devices for obtaining information on sensitive questions.

Pollock and Bek (1976) and Eichhorn and Hayre (1983) have made initial efforts to take sword of scrambled response technique ahead. Later various authors including Singh and Joardar (1996), Bar-Lev *et al.* (2004), Saha (2007), Diana and Perri (2012), Gjestvang and Singh (2009), Odumade and Singh (2009), Singh and Mathur (2005), Singh and Kim (2007),

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Tarray and Singh (2015), Arcos *et al.* (2015), Singh and Gorey (2017), and many more have discussed the problem of estimating the population mean of a sensitive variable under scrambled randomized response model.

It is to be mentioned that the above work done for single time survey associated with sensitive character analysis; instead, these issues need to be tracked constantly over time so that reflection of actual scenario in society associated with sensitive issues as well as changed level of sensitivity of issues with respect to time may be understood in a better way [see Priyanka and Trisandhya (2018)]. Interesting aspect of the scrambled response mechanism is that it can be used to protect the anonymity of individuals who have provided sensitive information. In such situations, the use of successive sampling scheme can be attractive alternative to improve the estimators of level at a point in time or to measure the change between two time points. Arnab and Singh (2013, pp. 2499-2500) have given the following examples well suited to the above mentioned situations: (i) A police department may be interested to know the average number of rapes in a large city during a particular year or a change in the number of rapes over a period of couple of years; (ii) A university administration may be interested to know the total amount of a particular drug used by students in a particular year, and after educating students about the adverse effect of drug use on health and society, if there is any significant change in the total drug use on campus or not; (iii) A social organization could be interested to know the proportion of those people who truly quit the drug after attending a lecture or seminar or after taking a medication.

Jessen (1942) first introduced the successive sampling procedure on two occasions to estimate the mean on the most recent (current) occasion. Later several authors including Patterson (1950), Narain (1953), Singh (1968), Ghangurde and Rao (1969), Sen (1973), Okafor and Arnab (1987), Biradar and Singh (2001), Singh and Priyanka (2008), Singh *et al.* (2008), Singh and Vishwakarma (2007, 2009), Singh and Pal (2017) *etc*. have paid their attention toward the estimation of mean on current occasion using successive sampling. Arnab and Singh (2013), and Yu *et al.* (2015), have used randomized response technique to deal with sensitive issues on successive occasion. Singh *et al.* (2017) applied scramble response procedure using Patterson (1950) method to tackle sensitive issues on successive occasion. Assuming non-sensitive additional auxiliary information is available at both occasions, Priyanka *et al.* (2017) and Priyanka and Trisandhya (2018, 2019) have employed both randomized and scramble response procedure to cope up with the studies related to sensitive issues on successive occasions. For example, we consider a situation, where an investigator is interested in estimating the average monthly expenditure on drug usage by undergraduate students in the current year 2016 (*i.e.* at second occasion) designated as the study variable y, then the auxiliary variable x may be taken as the average monthly expenditure on drug usage by undergraduate students in the year 2015 (*i.e.* at first occasion) and the average monthly pocket money of undergraduate students from all sources in the year 2015 may be taken as a non-sensitive additional auxiliary variable z. Here non-sensitive auxiliary data are available at both occasions. Hence this led authors to propose a class of estimators for estimating sensitive population mean of a sensitive variable at current occasion in two occasions successive sampling using non-sensitive auxiliary information. To deal with sensitive issues, randomized response technique due to Gjestvang and Singh (2009) has been applied. The detail properties of the suggested class of estimators have been discussed. Numerical illustration is given in support of the present study.

2. Survey Strategies and Analysis

2.1. Sampling procedure

Let $\Omega = (\Omega_1, \Omega_2, ..., \Omega_N)$ be a finite population of size *N*, which has been sampled over two occasions to estimate the population mean of sensitive variable at current occasion. It is supposed that the units of the population are unchanged over two occasions *i.e.* the sampling frame remain the same there by meaning is that no new units are added or deleted from the population. The character under investigation is sensitive variable designated by $x(y)$ on the first (second) occasion and *z* is a non-sensitive auxiliary variable available at both occasions. At the first occasion, a sample of *n* units is drawn from the population Ω by simple random sampling without replacement (SRSWOR). However, at the second occasion considering the partial overlap case, two independent samples are selected; one is matched sample of size $m = n\lambda$ drawn as subsample from the sample of size n and another is unmatched simple random sample of size $u = (n-m) = n\mu$ selected afresh at the second occasion so that the sample size at both the occasions is same (*i.e. n*). The sensitive variable $x(y)$ on the first (second) occasion are perturbed to $g(h)$ with the aid of scrambling variable *W*. The scrambling variable *W* so considered as it may follow any distribution. The following notations are considered further

$\mu\left(=\dfrac{u}{n}\right)$: Fraction of sample drawn afresh at current occasion,

$\lambda\left(=\dfrac{m}{n}=1-\mu\right)$: Fraction of samples matched from previous occasion,

$\overline{X}, \overline{Y}, \overline{Z}, \overline{G}, \overline{H}, \overline{W}$: Population means of variables *x, y, z, g, h* and *w* respectively,

$\overline{h}_u, \overline{g}_m, \overline{h}_m, \overline{g}_n$: Sample means of the variate based on sample sizes shown in suffices,

$\overline{z}_u, \overline{z}_m, \overline{z}_n$: Sample means of non-sensitive auxiliary variate *z* based on sample sizes shown in suffices,

$\rho_{yx}, \rho_{xz}, \rho_{yz}, \rho_{gh}, \rho_{hz}, \rho_{gz}$: Correlation coefficient between the variables depicted in suffices,

C_x, C_y, C_z : Coefficient of variation of variables depicted in suffices,

S_x^2, S_y^2, S_z^2 : Population mean square of variability *x, y, z* respectively,

$\sigma_x^2, \sigma_y^2, \sigma_z^2, \sigma_w^2$: Population variance of *x, y, z* and *w* respectively,

Note that the scrambling variable *W* such that $E(W) = \overline{W}$ and $V(W) = \sigma_w^2$.

2.2. Randomized response technique on successive occasions

For estimating the population mean (or) total of a sensitive variable Gjestvang and Singh (2009) suggested a randomized response model ($say\, M_{GS}$). In this paper Gjestvang and Singh (2009) randomized response model has been modified to be applied on successive occasions.

Let α *and* β be two known positive real numbers. Consider a deck of cards in which p is the proportion of cards bearing the statement: Multiply scrambling variable *W* with α and add to the real value of the sensitive variable $x(y)$ at first (second) move and $(1-p)$ be the proportion of cards bearing the statement: Multiply scrambling variable W with β and subtract it from the real value of the sensitive variable $x(y)$ at first (second) move. Let $p = \beta/(\alpha+\beta)$ be known. In this procedure each respondent is asked to draw one each secretly (confidentially) and report the scrambled response according at first (second) move accordingly. Using above randomization device, response given by j^{th} respondent on the first and second moves, respectively are described as

$$G_j = \begin{cases} x_j + \alpha W \text{ with probability } p \\ x_j - \beta W \text{ with probability } (1-p) \end{cases}$$ and

$$H_j = \begin{cases} y_j + \alpha W \text{ with probability } p \\ y_j - \beta W \text{ with probability } (1-p) \end{cases}$$

Therefore applying M_{GS} on two successive occasions, the sensitive variable $x(y)$ are switched to $g(h)$ and are given by

$$G = (X + \alpha W)p + (X - \beta W)(1-p)$$

and

$$H = (Y + \alpha W)p + (Y - \beta W)(1-p)$$

such that $\bar{Y} = \bar{H}$. (1)

$$\rho_{hg} = \frac{\rho_{yx}\sigma_y\sigma_x}{\sqrt{\{\sigma_y^2 + \alpha\beta(\sigma_w^2 + \bar{w}^2)\}}\sqrt{\{\sigma_x^2 + \alpha\beta(\sigma_w^2 + \bar{w}^2)\}}}, \rho_{hz} = \frac{\rho_{yz}\sigma_y}{\sqrt{\{\sigma_y^2 + \alpha\beta(\sigma_w^2 + \bar{w}^2)\}}}, \rho_{gz} = \frac{\rho_{xz}\sigma_x}{\sqrt{\{\sigma_x^2 + \alpha\beta(\sigma_w^2 + \bar{w}^2)\}}}.$$

Remark 1: Strategy is to obtain suitable estimator of population mean of coded response variable $\bar{H}$ on current occasion and substituting the same in (1) to obtain the relevant estimator for sensitive population mean $\bar{Y}$ at current occasion.

2.3. Design of the Class of Suggested Estimators

For estimating the population mean of perturbed variable H on the second (current) occasion, we have suggested two classes of estimators where one class of estimators D_u based on unmatched sample (or afresh sample) of size u on the current (second) occasion and others class of estimators based on the matched sample of size m (which is common to both the occasions).

2.3.1. Class of estimators based on unmatched sample on the second occasion using information on $(\bar{z}_u, \bar{Z})$ of non-sensitive auxiliary variable z

The usual ratio and product-type estimators can be ramified to estimate the population mean of coded response variable. The following estimators based on sample of size u drawn afresh at current occasion for estimating the population mean of switched variable H on current (second) move can be considered

$$d_1 = \bar{h}_u(\bar{Z}/\bar{z}_u), \qquad d_2 = \bar{h}_u(\bar{z}_u/\bar{Z}),$$

$$d_3 = \bar{h}_u \exp\left(\frac{\bar{Z} - \bar{z}_u}{\bar{Z} + \bar{z}_u}\right), \qquad d_4 = \bar{h}_u \exp\left(\frac{\bar{z}_u - \bar{Z}}{\bar{z}_u + \bar{Z}}\right),$$

$$d_5 = \bar{h}_u\left[\alpha\left(\frac{\bar{Z}}{\bar{z}_u}\right) + (1-\alpha)\left(\frac{\bar{z}_u}{\bar{Z}}\right)\right],$$

$$d_6 = \bar{h}_u\left[\alpha_1 \exp\left(\frac{\bar{Z}-\bar{z}_u}{\bar{Z}+\bar{z}_u}\right) + (1-\alpha_1)\exp\left(\frac{\bar{z}_u-\bar{Z}}{\bar{z}_u+\bar{Z}}\right)\right],$$

$$d_7 = \bar{h}_u\left(\bar{z}_u/\bar{Z}\right)^{\theta_1}, \qquad d_8 = \bar{h}_u \exp\left\{\frac{\delta_1\left(\bar{Z}-\bar{z}_u\right)}{\bar{Z}+\bar{z}_u}\right\},$$

$$d_9 = \bar{h}_u \frac{\bar{Z}}{\left\{\alpha_2\bar{Z} + (1-\alpha_2)\bar{z}_u\right\}}, \qquad d_{10} = \bar{h}_u\left(\bar{Z}/\bar{z}_u\right)^{1/2},$$

$$d_{11} = \bar{h}_u \exp\left\{\frac{\alpha_3\left(\bar{z}_u-\bar{Z}\right)}{\bar{Z}}\right\}, \qquad d_{12} = \bar{h}_u\left[2-\left(\frac{\bar{z}_u}{\bar{Z}}\right)^{\theta_2}\right],$$

$$d_{13} = \bar{h}_u\left[2-\exp\left\{\frac{\theta_3\left(\bar{z}_u-\bar{Z}\right)}{\bar{z}_u+\bar{Z}}\right\}\right], \qquad d_{14} = \bar{h}_u\left[2-\left(\frac{\bar{z}_u}{\bar{Z}}\right)^{\theta_2} .\exp\left\{\frac{\theta_3\left(\bar{z}_u-\bar{Z}\right)}{\bar{z}_u+\bar{Z}}\right\}\right],$$

$$d_{15} = \bar{h}_u \exp\left\{\frac{\alpha\left(\bar{Z}-\bar{z}_u\right)}{\bar{Z}+(a_1-1)\bar{z}_u}\right\}, \qquad d_{16} = \bar{h}_u + k\left(\bar{z}_u-\bar{Z}\right),$$

$$d_{17} = \bar{h}_u + \beta_{hz}\left(\bar{z}_u-\bar{Z}\right), \qquad d_{18} = \left\{\bar{h}_u + k\left(\bar{z}_u-\bar{Z}\right)\right\}\exp\left\{\frac{\delta_1\left(\bar{Z}-\bar{z}_u\right)}{\bar{Z}+\bar{z}_u}\right\},$$

$$d_{19} = \left\{\bar{h}_u + k\left(\bar{z}_u-\bar{Z}\right)\right\}\left(\frac{\bar{Z}}{\bar{z}_u}\right)^{\theta_1},$$

etc., where $\alpha, \alpha_1, \alpha_2, \alpha_3, k, \theta_1, \theta_2, \theta_3, \delta, \delta_1$ are suitably chosen constants to be determined such that mean squared errors (*MSEs*) of $d_i\,(i = 5 \, to \, 9)$, $d_j\,(j = 11 \, to \, 16)$, $d_l\,(l = 18, 19, 20)$are (maybe) minimized, $a(\neq 0)$ and *b* are real numbers or the values of the parameters C_z, S_z, coefficient of skewness $\beta_1(z)$, coefficient of kurtosis $\beta_2(z)$ and $\Delta = (\beta_2(z) - \beta_1(z) - 1)$ *etc*. associated with additional non-sensitive auxiliary variable z, for instance, see Upadhyaya and Singh (1999).

We propose a class of estimators of population mean of sensitive characteristic based on unmatched sample of size *u*, by following Srivastava (1980). When the population mean $\bar{Z}$ of the auxiliary variable *z* is known, we define a class of estimators for population mean of sensitive characteristic as

$$D_{u1} = D\left(\bar{h}_u, \bar{z}_u\right), \tag{2}$$

where $D\left(\bar{h}_u, \bar{z}_u\right)$ is a function of $\bar{h}_u$ and $\bar{z}_u$ such that $D\left(\bar{H}, \bar{Z}\right) = \bar{H} \Rightarrow D_1\left(\bar{H}, \bar{Z}\right) = 1$; $D_1\left(\bar{H}, \bar{Z}\right)$ being the first order partial derivative of the function $D\left(\bar{h}_u, \bar{z}_u\right)$ at the point and the function $D\left(\bar{h}_u, \bar{z}_u\right)$ also satisfies the following conditions

(i) The point $\left(\bar{h}_u, \bar{z}_u\right)$ assume values in a bounded, closed convex subset, T, of the two dimensional real space containing the point $\left(\bar{H}, \bar{Z}\right)$.

(ii) The function $D\left(\bar{h}_u, \bar{z}_u\right)$ is continuous and bounded in T.

(iii) The first and second order partial derivatives of $D\left(\bar{h}_u, \bar{z}_u\right)$ exist and are continuous and bounded in T.

Assuming that the population size is sufficiently large so that finite population correction (fpc) term can be ignored, the bias and *MSE* of D_{u1} to the first degree of approximation (fda) are respectively given by

$$B(D_{u1}) = \frac{1}{2u}\left[S_z^2 D_{22}\left(\overline{H},\overline{Z}\right) + 2S_h S_z \rho_{hz} D_{12}\left(\overline{H},\overline{Z}\right)\right], \tag{3}$$

$$MSE(D_{u1}) = \frac{1}{u}\left[S_h^2 + S_z^2 D_2^2\left(\overline{H},\overline{Z}\right) + 2S_h S_z \rho_{hz} D_2\left(\overline{H},\overline{Z}\right)\right], \tag{4}$$

where $D_2\left(\overline{H},\overline{Z}\right)$, $D_{12}\left(\overline{H},\overline{Z}\right)$ and $D_{22}\left(\overline{H},\overline{Z}\right)$ are the first and second order partial derivatives of the function $D\left(\overline{h}_u, \overline{z}_u\right)$ at the point $\left(\overline{H},\overline{Z}\right)$.

Any parametric function $D\left(\overline{h}_u, \overline{z}_u\right)$ satisfying conditions (i)-(iii) can be an acceptable estimator of population mean of a sensitive variable at current move. The class of such estimators is very large.

It can be easily identified that the estimators $d_i\,(i = 1\,to\,20)$ are members of the suggested class of estimators D_{u1} given by (2). Thus the biases and mean squared errors of the estimators d_1 to d_{20} can be easily obtained from (3) and (4) just by putting the values of ($D_{12}\left(\overline{H},\overline{Z}\right)$, $D_{22}\left(\overline{H},\overline{Z}\right)$) and $D_2\left(\overline{H},\overline{Z}\right)$ in (3) and (4) respectively.
The $MSE(D_{u1})$ at (4) is minimized for

$$D_2\left(\overline{H},\overline{Z}\right) = -\rho_{hz}\, S_h / S_z = -\beta_{hz}, \tag{5}$$

where $\beta_{hz} = S_{hz}/S_z^2$ is the population regression coefficient of *h* on *z,* and

$$S_{hz} = \frac{1}{N-1}\sum_{i=1}^{N}\left(h_i - \overline{H}\right)\left(z_i - \overline{Z}\right).$$

Thus, the resulting minimum *MSE* of D_{u1} is given by

$$MSE_{\min}(D_{u1}) = \left(S_h^2/u\right)\left(1-\rho_{hz}^2\right). \tag{6}$$

Thus, we established the following theorem.

Theorem 1: Up to terms of order u^{-1},

$$MSE_{\min}(D_{u1}) \geq \left(S_h^2/u\right)\left(1-\rho_{hz}^2\right)$$

with equality holding if $D_2\left(\overline{H},\overline{Z}\right) = -\beta_{hz}$.

2.3.2. Class of estimators based on unmatched sample of size *u* utilizing information on $\left(\overline{z}_u, \overline{Z}, S_z^2\right)$ of non-sensitive auxiliary variable *z*

Das and Tripathi (1978) and Srivastava and Jhajji (1980, 1981) have advocated that in many survey situations of practical importance, information on population variance (σ_z^2) / mean square S_z^2 is also known along with population mean $\overline{Z}$. Thus, utilizing the knowledge on $\left(\bar{z}_u, \bar{Z}, S_z^2\right)$ we define a class of estimators for population mean $\bar{H}$ of coded response variable at current (second) occasion (move) in two occasion successive sampling as

$$D_{u2} = F\left(\bar{h}_m, \bar{z}_u^*, s_{zu}^{*2}\right), \tag{7}$$

where $\bar{z}_u^* = \bar{z}_u / \bar{Z}$, $s_{zu}^{*2} = s_{zu}^2 / S_z^2$ and $F(.)$ is a function of $\left(\bar{h}_m, \bar{z}_u^*, s_{zu}^{*2}\right)$such that $F\left(\bar{H},1,1\right) = \bar{H}$ $\Rightarrow F_1\left(\bar{H},1,1\right) = 1, F_1\left(\bar{H},1,1\right)$ being the first order partial derivative of the function $F(.)$ at the point $\left(\bar{H},1,1\right)$.

The function $F(.)$ at (7) also satisfies certain regularity conditions like those given in Srivastava and Jhajji (1980, 1981).

The Bias and *MSE* of D_{u2} to the fda, ignoring fpc term, are respectively given by

$$B(D_{u2}) = \frac{1}{2u}\left[C_z^2 F_{22}(S) + (\lambda_{004} - 1)F_{33}(S) + 2\rho_{hz} S_h C_z F_{12}(S) + 2\lambda_{012} S_h F_{13}(S) + 2\lambda_{003} C_z F_{23}(S)\right], \tag{8}$$

$$\begin{aligned} MSE(D_{u2}) = \frac{1}{u}\big[&S_h^2 + C_z^2 F_2^2(S) + (\lambda_{004} - 1)F_3^2(S) + 2\rho_{hz} S_h C_z F_2(S) \\ &+ 2\lambda_{012} S_h F_3(S) + 2\lambda_{003} C_z F_2(S) F_3(S)\big] \end{aligned}, \tag{9}$$

where $\lambda_{rst} = \dfrac{\mu_{rst}}{\mu_{200}^{r/2}\ \mu_{020}^{s/2}\ \mu_{002}^{t/2}}$, $\mu_{rst} = \dfrac{1}{N}\sum_{i=1}^{N}\left(g_i - \bar{G}\right)^r \left(h_i - \bar{H}\right)^s \left(z_i - \bar{Z}\right)^t$, (r,s,t) being non-negative integers, $\left(F_2(S), F_3(S)\right)$ and $\{F_{12}(S), F_{13}(S), F_{22}(S), F_{23}(S), F_{33}(S)\}$ are the first and second order partial derivatives of the function $F(.)$ at the point $S = \left(\bar{H},1,1\right)$.

Differentiating (9) partially with respect to ($F_2(S)$ and $F_3(S)$) and equating them to zero we have

$$\begin{bmatrix} C_z^2 & C_z\lambda_{003} \\ C_z\lambda_{003} & (\lambda_{004} - 1) \end{bmatrix} \begin{bmatrix} F_2(S) \\ F_3(S) \end{bmatrix} = \begin{bmatrix} -\rho_{hz} S_h C_z \\ -\lambda_{012} S_h \end{bmatrix}. \tag{10}$$

After simplification of (10) we get the optimum values of $F_2(S)$ and $F_3(S)$ respectively as

$$F_2(S) = \frac{S_h\left[\lambda_{003}\lambda_{012} - (\lambda_{004} - 1)\rho_{hz}\right]}{C_z\left(\lambda_{004} - \lambda_{003}^2 - 1\right)} = F_{20}(S), say \tag{11}$$

$$F_3(S) = \frac{S_h\left[\lambda_{003}\rho_{hz} - \lambda_{012}\right]}{\left(\lambda_{004} - \lambda_{003}^2 - 1\right)} = F_{30}(S), say. \tag{12}$$

Thus, the resulting minimum *MSE* of D_{u2} is given by

$$MSE_{\min}(D_{u2}) = \frac{S_h^2}{u}\left[1 - \rho_{hz}^2 - \frac{\left(\lambda_{012} - \lambda_{003}\rho_{hz}\right)^2}{\left(\lambda_{004} - \lambda_{003}^2 - 1\right)}\right]. \tag{13}$$

Now we state the following theorem.

Theorem 2: Up to the fda,

$$MSE_{\min}(D_{u2}) \geq \frac{S_h^2}{u}\left[1-\rho_{hz}^2-\frac{(\lambda_{012}-\lambda_{003}\rho_{hz})^2}{(\lambda_{004}-\lambda_{003}^2-1)}\right]$$

with equality holding if $F_2(S)=F_{20}(S)$, $F_3(S)=F_{30}(S)$.

The class of estimators D_{u2} is very large. The following estimators are given below

$$D_{u2(1)} = \bar{h}_u \exp\left\{\frac{\beta(1-\bar{z}_u^*)}{1+(a_1-1)\bar{z}_u^*}\right\}\exp\left\{\frac{\gamma(1-s_{zu}^{*2})}{1+(a_2-1)s_{zu}^{*2}}\right\}, \quad D_{u2(2)} = \bar{h}_u \frac{\{1+\alpha(\bar{z}_u^*-1)\}}{\{1+\beta(s_{zu}^{*2}-1)\}},$$

$$D_{u2(3)} = \bar{h}_u\left[1+\alpha(\bar{z}_u^*-1)+\beta(s_{zu}^{*2}-1)\right]^{-1}, \qquad D_{u2(4)} = \bar{h}_u\left[1+\alpha(\bar{z}_u^*-1)\right]\left(\frac{s_{zu}^2}{S_z^2}\right)^{\beta},$$

etc. are members of the proposed class of estimators D_{u2}, where $(\alpha,\beta,\gamma,a_1,a_2)$ are suitably chosen constants. The bias and *MSE* of the estimators $D_{u2(j)}; j=1\,to\,4$ can be easily obtained from (8) and (9) just by putting the suitable values of $F_2(S)$, $F_3(S)$, $F_{12}(S)$, $F_{13}(S)$, $F_{22}(S)$, $F_{23}(S)$ and $F_{33}(S)$.

It is to be mentioned that the estimator like $D_{u2(1)}$ has been suggested by Priyanka and Trisandhya (2019). The bias and *MSE* of $D_{u2(1)}$ can be easily obtained by putting

$$\left[F_{12}(S)=-\frac{\beta\bar{H}}{a_1},\ F_{13}(S)=-\frac{\gamma\bar{H}}{a_2},\ F_{22}(S)=\bar{H}\left(\frac{2\beta}{a_1}-\frac{2\beta}{a_1^2}+\frac{\beta^2}{a_1^2}\right),\ F_{23}(S)=-\frac{\beta\gamma\bar{H}}{a_1a_2},\right.$$

$$\left.F_{33}(S)=\bar{H}\left(\frac{2\gamma}{a_2}-\frac{2\gamma}{a_2^2}+\frac{\gamma^2}{a_2^2}\right)\right] \text{ and } \left[F_2(S)=-\frac{\beta\bar{H}}{a_1}, F_3(S)=-\frac{\gamma\bar{H}}{a_2}\right] \text{ in (8) and (9) respectively.}$$

From (6) and (13) we have

$$MSE_{\min}(D_{u1})-MSE_{\min}(D_{u2}) = \frac{S_h^2}{u}\cdot\frac{(\lambda_{012}-\lambda_{003}\rho_{hz})^2}{(\lambda_{004}-\lambda_{003}^2-1)} \geq 0 \tag{14}$$

Thus, the class of estimators D_{u2} is more efficient than D_{u1} provided $\lambda_{012} \neq \lambda_{003}\rho_{hz}$. For this situation $\lambda_{012} = \lambda_{003}\rho_{hz}$, both the classes of estimators D_{u1} and D_{u2} are equally efficient. We should also add here that if the variables (*h, z*) have bivariate normal distribution, then there is no advantage of using the estimator D_{u2}. In such case, it is worth advisable to pick up the estimators belonging to the class of estimators D_{u1}.

2.3.3. Class of estimators based on matched sample at current (second) occasion using information on $(\bar{g}_m, \bar{g}_n, \bar{z}_m, \bar{z}_n, \bar{Z})$

In successive sampling it is tradition to use information gathered on first occasion as auxiliary information in addition to additional non-sensitive auxiliary variable for improving the precision of estimates at current move. The estimator based on matched sample of size *m* for population mean $\bar{H}$ at current move due to Priyanka and Trisandhya (2019) are

$$t_{m1} = \bar{h}_m \exp\left\{\frac{\theta(1-\phi)}{1+(b_1-1)\phi}\right\}\exp\left\{\frac{\delta(1-\xi)}{1+(b_2-1)\xi}\right\},$$
$$t_{m2} = \bar{h}_m \exp\left\{\frac{\theta(1-\phi)}{1+(b_1-1)\phi}\right\}\exp\left\{\frac{\delta(1-\xi)}{1+(b_2-1)\xi}\right\}\exp\left\{\frac{\eta(1-v)}{1+(b_3-1)v}\right\},$$

where $\phi = g_m/g_n$,$\xi = \bar{z}_n/\bar{Z}$, $v = s_{zn}^2/S_z^2$ *and* $(\theta, \delta, \eta, b_i; i = 1,2,3)$ are suitably chosen constants. It is to be noted that the estimators t_{mi} ,$i = 1,2$ are not utilizing the information on matched sample for additional non-sensitive auxiliary variable *z* (i.e. information on ($\bar{z}_m$,s_{zm}^2)) while information on ($\bar{z}_m$,s_{zm}^2) associated with additional non-sensitive auxiliary variable *z* can be made available easily. This led authors to propose classes of estimators utilizing information on ($\bar{z}_m$,s_{zm}^2) along with $(\bar{g}_m, \bar{g}_n, , \bar{z}_n\ s_{zn}^2, \bar{Z}, S_z^2)$ of non-sensitive auxiliary variable *z*.

We propose a class of estimators of population mean $\bar{H}$ at current (second) occasion, by following Srivastava (1971, 1980) as

$$J_{m1} = J\left(\bar{h}_m, \phi, \psi, \xi\right), \tag{15}$$

where *J*(.) is a function of $(\bar{h}_m, \phi, \psi, \xi)$with $\phi = g_m/g_n$ $\psi = \bar{z}_m/\bar{z}_n$,$\xi = \bar{z}_n/\bar{Z}$ such that

$$J(Q) = J(\bar{H}, 1, 1, 1) = \bar{H} \quad \Rightarrow J_1(Q) = 1; \tag{16}$$

$J_1(Q)$ being the first order partial derivative of the function $J(.)$ at the point $Q = (\bar{H}, 1, 1, 1)$ and satisfies certain regularity conditions similar to these given in Srivastava (1971,1980). A large number of estimators may be identified as member of the class J_{m1} at (15). The following are some examples

$$J_{m1(1)} = \bar{h}_m \phi^{\alpha_1}, \psi^{\alpha_2}\xi^{\alpha_3},$$
$$J_{m1(2)} = \bar{h}_m \exp\left\{\frac{\alpha_1(1-\phi)}{(1+\phi)}\right\}\exp\left\{\frac{\alpha_2(1-\psi)}{(1+\psi)}\right\}\exp\left\{\frac{\alpha_3(1-\xi)}{(1+\xi)}\right\},$$
$$J_{m1(3)} = \left\{\bar{h}_m + \beta_{hg}(1-\phi)\right\}, \psi^{\alpha_2}\xi^{\alpha_3}, \quad J_{m1(4)} = \left\{\bar{h}_m + \beta_{hg}(1-\phi) + \beta_{hz}(1-\psi)\right\}\xi^{\alpha_3},$$
$$J_{m1(5)} = \bar{h}_m\left[1 + \alpha_1(1-\phi) + \alpha_2(1-\psi) + \alpha_3(1-\xi)\right],$$

etc., where α_i's ,η_i's, $(i = 1,2,3)$ are suitably chosen constants.

To the fda, ignoring fpc term, the bias and *MSE* of the class of estimators J_{m1} are respectively given by

$$B(J_{m1}) = \frac{1}{2}\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left\{C_g^2 J_{22}(Q) + C_z^2 J_{33}(Q) + 2\rho_{hg}S_h C_g J_{12}(Q) + 2\rho_{hz}S_h C_z J_{13}(Q) + 2\rho_{gz}C_g C_z J_{23}(Q)\right\}\right.$$
$$\left. + \frac{1}{n}\left\{C_z^2 J_{44}(Q) + 2\rho_{hz}S_h C_z J_{14}(Q)\right\}\right], \tag{17}$$

$$MSE(J_{m1}) = \left[\left(\frac{1}{m} - \frac{1}{n}\right)\left\{S_h^2 + C_g^2 J_2^2(Q) + C_z^2 J_3^2(Q) + 2\rho_{hg} S_h C_g J_2(Q) + 2\rho_{hz} S_h C_z J_3(Q) + 2\rho_{gz} C_g C_z J_2(Q) J_3(Q)\right\} + \frac{1}{n}\left\{S_h^2 + C_z^2 J_4^2(Q) + 2\rho_{hz} S_h C_z J_4(Q)\right\}\right], \tag{18}$$

where and $J_{jl}(Q)$ $((j,l) = 1,2,3,4)$; are first and second order partial derivatives of the function $J(.)$ at the point Q.

The $MSE(J_{m1})$ at (18) is minimized for

$$\left.J_2(Q) = \frac{S_h(\rho_{hz}\rho_{gz} - \rho_{hg})}{C_g(1-\rho_{gz}^2)} = J_{20}(Q)\ ,\ J_3(Q) = \frac{S_h(\rho_{hg}\rho_{gz} - \rho_{hz})}{C_z(1-\rho_{gz}^2)} = J_{30}(Q)\ ,\ J_4(Q) = -\frac{S_h\rho_{hz}}{C_z} = J_{40}(Q)\ ,\right\}(say) \tag{19}$$

Thus, the resulting minimum *MSE* of J_{m1} is given by

$$MSE_{\min}(J_{m1}) = S_h^2\left[\frac{1}{m}\left(1 - R_{h.gz}^2\right) + \frac{1}{n}\left(R_{h.gz}^2 - \rho_{hz}^2\right)\right], \tag{20}$$

where $$R_{h.gz}^2 = \frac{\left(\rho_{hg}^2 + \rho_{hz}^2 - 2\rho_{hz}\rho_{gz}\rho_{hg}\right)}{\left(1-\rho_{gz}^2\right)}.$$

Now, we state the following theorem:

Theorem 3: Up to the fda,

$$MSE_{\min}(J_{m1}) \geq S_h^2\left[\frac{1}{m}\left(1 - R_{h.gz}^2\right) + \frac{1}{n}\left(R_{h.gz}^2 - \rho_{hz}^2\right)\right]$$
$$= S_h^2\left[\left(\frac{1}{m} - \frac{1}{n}\right)\left(1 - R_{h.gz}^2\right) + \frac{1}{n}\left(1 - \rho_{hz}^2\right)\right]$$

with equality holding if $J_i(Q) = J_{i0}(Q)$ $i = 2,34$.

2.3.3.1. Class of estimators based on matched sample of size *m* at current occasion using information on (ϕ, ξ)

If the information on (h_m, ϕ, ξ) is used to estimate the population mean $\overline{H}$ at current move, then following the procedure adopted by Srivastava (1971), we define a class of estimators as

$$J_{m2} = J^*\left(\overline{h}_m, \phi, \xi\right), \tag{21}$$

where $J(.)$ is a function of $\left(\overline{h}_m, \phi,\ \xi\right)$ such that $J^*\left(\overline{H}, 1, 1,\ \right) = \overline{H} \Rightarrow J_1^*(H, 1, 1) = 1$. $J_1^*\left(\overline{H}, 1, 1,\ \right)$ being the first and second order partial derivatives of the function about the point $\left(\overline{H}, 1, 1\right)$; and satisfies certain regularity conditions similar to these given in Srivastava (1971).

To the fda, ignoring fpc term, the bias and *MSE* of the class of estimators J_{m2} are respectively given by

$$B(J_{m2})=\frac{1}{2}\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left\{C_g^2 J_{22}^*(Q^*)+2\rho_{hg}S_h C_g J_{12}^*(Q^*)\right\}+\frac{1}{n}\left\{C_z^2 J_{33}^*(Q^*)+2\rho_{hz}S_h C_z J_{13}^*(Q^*)\right\}\right], \tag{22}$$

$$MSE(J_{m2})=\left[\begin{array}{l}\left(\frac{1}{m}-\frac{1}{n}\right)\left\{S_h^2+C_g^2 J_{22}^{*2}(Q^*)+2\rho_{hg}S_h C_g J_2^*(Q^*)\right\}\\ +\frac{1}{n}\left\{S_h^2+C_z^2 J_3^{*2}(Q^*)+2\rho_{hz}S_h C_z J_3^{*2}(Q^*)\right\}\end{array}\right] \tag{23}$$

where $J_i^*(Q^*)\,(i=2,\,3,4)$ and $J_{jl}^*(Q^*)\,((j,l)=1,2\;3,4)$; are first and second order partial derivatives of the function $J(.)$ at the point $Q^*=(\overline{H},1,1)$.

The $MSE(J_{m2})$ at (23) is minimized when

$$J_2^*(Q^*)=-\frac{S_h\rho_{hg}}{C_g}=J_{20}^*(Q^*)\;,say \tag{24}$$

$$J_3^*(Q^*)=-\frac{S_h\rho_{hz}}{C_z}=J_{30}^*(Q^*)\;,say \tag{25}$$

Thus, the resulting minimum *MSE* of J_{m2} is given by

$$MSE_{\min}(J_{m2})=S_h^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left(1-\rho_{hg}^2\right)+\frac{1}{n}\left(1-\rho_{hz}^2\right)\right]. \tag{26}$$

Now, we state the following theorem.

Theorem 4: To the fda,

$$MSE_{\min}(J_{m2})\geq S_h^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left(1-\rho_{hg}^2\right)+\frac{1}{n}\left(1-\rho_{hz}^2\right)\right]$$

with equality holding if $J_2^*(Q^*)=J_{20}^*(Q^*)$ and $J_3^*(Q^*)=J_{30}^*(Q^*)$.

The class of estimators J_{m2} is very large. In addition to Priyanka and Trisandhya (2019) estimator t_{m1}, the following estimators

$$J_{m2(1)}=\bar{h}_m\,\phi^{\alpha_1}\xi^{\alpha_2},\qquad J_{m2(2)}=\bar{h}_m\exp\left\{\frac{\alpha_1(1-\phi)}{(1+\phi)}\right\}\exp\left\{\frac{\alpha_2(1-\xi)}{(1+\xi)}\right\},$$

$$J_{m2(3)}=\bar{h}_m\frac{\{1+\alpha_1(\phi-1)\}}{\{1+\alpha_2(\xi-1)\}},\qquad J_{m2(4)}=\bar{h}_m\phi^{\alpha_1}\exp\left\{\frac{\alpha_2(1-\xi)}{(1+\xi)}\right\},$$

etc. are the members of the proposed class of estimators J_{m2}, where (α_1,α_2) are suitably chosen constants. The bias and *MSE* of the estimators $t_{m1}, J_{m2(j)}; j=1\,to\,4$ can be obtained easily just by putting the values of derivatives $\left(J_{12}^*(Q^*), J_{13}^*(Q^*), J_{22}^*(Q^*), J_{33}^*(Q^*), J_{23}^*(Q^*)\right)$ and $\left(J_2^*(Q^*),\;J_3^*(Q^*)\right)$ in (22) and (23) respectively.

We also note that the proposed class of estimators J_{m2} is a member of class of estimators J_{m1} at (15).

From (20) and (16) we have

$$MSE_{\min}(J_{m2})-MSE_{\min}(J_{m1})=S_h^2\left(\frac{1}{m}-\frac{1}{n}\right)\rho_{hg.z}^2\left(1-\rho_{hg}^2\right)\geq 0 \tag{27}$$

where $\rho_{hg.z}=\dfrac{\left(\rho_{hg}-\rho_{hz}\rho_{gz}\right)}{\sqrt{\left(1-\rho_{hz}^2\right)\left(1-\rho_{gz}^2\right)}}$ is the partial correlation coefficient between h and g.

It follows from (27) that the proposed class of estimator J_{m1} is more efficient than the estimator J_{m2}.

2.3.4. Class of estimators based on matched sample at current move using information on $\left(\bar{h}_m,\bar{g}_m,\bar{z}_m,s_{zm}^2,\bar{g}_n,\bar{z}_n\, s_{zn}^2,\bar{Z},S_z^2\right)$

It is to be noted that the estimator t_{m2} due to Priyanka and Trisandhya (2019) utilizing information on $\left(\bar{Z},S_z^2\right)$ based on matched sample of size m can be further generalized as

$$t_{m3}=\bar{h}_m\exp\left\{\frac{\theta_1(1-\phi)}{1+(b_1-1)\phi}\right\}\exp\left\{\frac{\theta_2(1-\psi)}{1+(b_2-1)\psi}\right\}\exp\left\{\frac{\theta_3(1-\xi)}{1+(b_3-1)\xi}\right\}\exp\left\{\frac{\theta_4(1-w)}{1+(b_4-1)w}\right\}\exp\left\{\frac{\theta_5(1-v)}{1+(b_5-1)v}\right\}, \tag{28}$$

where

$\phi=g_m/g_n\,,\psi=\bar{z}_m/\bar{z}_n\;\;\xi=\bar{z}_n/\bar{Z}\;,\;v=s_{zn}^2/\bar{Z}\;,w=s_{zm}^2/s_{zn}^2\;,v=s_{zn}^2/S_z^2$

$and\;\;\theta_{i's}\,,b_i's\,(i=1,to5)$

are suitably chosen constants.

Keeping the class of estimators (28) in view and adopting the same procedure as adopted by Srivastava and Jhajji (1981) we define a class of estimators of sensitive population mean $\bar{H}$ of coded response variable h based on the matched sample of size m at current move as

$$J_{m3}=L\left(\bar{h}_m,\phi,\psi,\xi,w,v\right), \tag{29}$$

where $L(.)$ is a function of $\left(\bar{h}_m,\phi,\psi,\xi,w,v\right)$ such that $L(B)=\bar{H}\;\Rightarrow L_1(B)=1,\;L_1(B)$ being the first order partial derivative of the function $L(.)$ at $B=\left(\bar{H},1,1,1,1,1\right)$and also satisfies certain regularity conditions similar to these given in Srivastava and Jhajji (1981).

To the fda, ignoring fpc term, the bias and MSE of the class of estimators J_{m3} are respectively given by

$$\begin{aligned}B(J_{m3})=\frac{1}{2}\Bigg[\left(\frac{1}{m}-\frac{1}{n}\right)&\Big\{C_g^2L_{22}(B)+C_z^2L_{33}(B)+(\lambda_{004}-1)L_{55}(B)+2\rho_{hg}S_hC_gL_{12}(B)\\&+2\rho_{hz}S_hC_zL_{13}(B)+2\lambda_{012}S_hL_{15}(B)+2\rho_{gz}C_gC_zL_{23}(B)+2\lambda_{102}C_gL_{25}(B)\\&+2\lambda_{003}C_zL_{35}(B)\Big\}+\frac{1}{n}\Big\{C_z^2L_{44}(B)+(\lambda_{004}-1)L_{66}(B)+2\rho_{hz}S_hC_zL_{14}(B)\\&+2\lambda_{012}S_hL_{16}(B)+2\lambda_{003}C_zL_{46}(B)\Big\}\Bigg]\end{aligned}, \tag{30}$$

$$MSE(J_{m3}) = \frac{1}{2}\Bigg[\left(\frac{1}{m}-\frac{1}{n}\right)\Big\{S_h^2 + C_g^2 L_2^2(B) + C_z^2 L_3^2(B) + (\lambda_{004}-1)L_5^2(B) + 2\rho_{hg} S_h C_g L_2(B)$$
$$+ 2\rho_{hz} S_h C_z L_3(B) + 2\lambda_{012} S_h L_5(B) + 2\rho_{gz} C_g C_z L_2(B) L_3(B) + 2\lambda_{102} C_g L_2(B) L_5(B)$$
$$+ 2\lambda_{003} C_z L_3(B) L_5(B)\Big\} + \frac{1}{n}\Big\{S_h^2 + C_z^2 L_4^2(B) + (\lambda_{004}-1)L_6^2(B) + 2\rho_{hz} S_h C_z L_4(B)$$
$$+ 2\lambda_{012} S_h L_6(B) + 2\lambda_{003} C_z L_4(B) L_6(B)\Big\}\Bigg] \tag{31}$$

where $L_i(B)$ $(i = 2\ 3,4,5,6)$ and $L_{ij}(B)$ $((i,j) = 1,2\ 3,4,5)$; are the first and second order partial derivatives of the function $L(\bar{h}_m, \phi, \psi, \xi, w, v)$ at the point B.

The $MSE(J_{m3})$ at (31) is minimized for

$$L_2(B) = \frac{S_h \delta_1^*}{C_g \delta^*} = L_{20}(B)\ , say, \tag{32}$$

$$L_3(B) = \frac{S_h \delta_2^*}{C_z \delta^*} = L_{30}(B)\ , say, \tag{33}$$

$$L_4(B) = \frac{S_h[\lambda_{003}\lambda_{012} - (\lambda_{004}-1)\rho_{hz}]}{C_z(\lambda_{004} - \lambda_{003}^2 - 1)} = L_{40}(B)\ , say\ , \tag{34}$$

$$L_5(B) = \frac{S_h \delta_3^*}{\delta^*} = L_{50}(B)\ , say, \tag{35}$$

$$L_6(B) = \frac{S_h[\lambda_{003}\rho_{hz} - \lambda_{012}]}{(\lambda_{004} - \lambda_{003}^2 - 1)} = L_{60}(B), say\ , \tag{36}$$

where
$$\delta^* = [(\lambda_{004} - \lambda_{003}^2 - 1) - \rho_{gz}\{\rho_{gz}(\lambda_{004}-1) - \lambda_{102}\lambda_{003}\} + \lambda_{102}(\rho_{gz}\lambda_{003} - \lambda_{102})]$$
$$\delta_1^* = [\rho_{hg}(\lambda_{004} - \lambda_{003}^2 - 1) - \rho_{gz}\{\rho_{hz}(\lambda_{004}-1) - \lambda_{012}\lambda_{003}\} + \lambda_{102}(\rho_{hz}\lambda_{003} - \lambda_{012})]$$
$$\delta_2^* = [\{\rho_{hz}(\lambda_{004}-1) - \lambda_{012}\lambda_{003}\} - \rho_{hg}\{\rho_{gz}(\lambda_{004}-1) - \lambda_{102}\lambda_{003}\} + \lambda_{102}\{\lambda_{012}\rho_{gz} - \lambda_{102}\rho_{hz}\}]$$
$$\delta_3^* = [(\lambda_{012} - \rho_{hg}\lambda_{003}) - \rho_{gz}(\rho_{gz}\lambda_{012} - \lambda_{102}\rho_{hz}) + \rho_{hg}(\rho_{gz}\lambda_{003} - \lambda_{102})].$$

Substitution of (32) to (36) in (31) yields the minimum MSE of J_{m3} as

$$MSE_{\min}(J_{m3}) = S_h^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left\{1 - R_{h.gz}^2 - \frac{\rho_1^2}{Q_1(1-\rho_{gz}^2)}\right\} + \frac{1}{n}\left\{1 - \rho_{hz}^2 - \frac{(\lambda_{003}\rho_{hz} - \lambda_{012})^2}{(\lambda_{004} - \lambda_{003}^2 - 1)}\right\}\right], \tag{37}$$
$$\rho_1 = [\rho_{hg}(\lambda_{102} - \rho_{gz}\lambda_{003}) + \rho_{hz}(\lambda_{003} - \rho_{gz}\lambda_{102}) + \lambda_{012}(1-\rho_{gz}^2)],$$
$$Q_1 = [(\lambda_{004}-1)(1-\rho_{gz}^2) - \lambda_{102}(\lambda_{102} - \rho_{gz}\lambda_{003}) - \lambda_{003}(\lambda_{003} - \rho_{gz}\lambda_{102})].$$

Now, we state the following theorem.

Theorem 5: To the fda,

$$MSE_{\min}(J_{m3}) \geq S_h^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left\{1 - R_{h.gz}^2 - \frac{\rho_1^2}{Q_1(1-\rho_{gz}^2)}\right\} + \frac{1}{n}\left\{1 - \rho_{hz}^2 - \frac{(\lambda_{003}\rho_{hz} - \lambda_{012})^2}{(\lambda_{004} - \lambda_{003}^2 - 1)}\right\}\right],$$

with equality holding if $L_j(B)=L_{j0}(B)$, $j=2,3,4,5,6$.

The class of estimators J_{m3} at (29) is very large. The following estimators:

$$J_{m3(1)}=\bar{h}_m\,\phi^{\alpha_1}\ \psi^{\alpha_2}\xi^{\alpha_3}w^{\alpha_4}v^{\alpha_5},$$
$$J_{m3(2)}=\bar{h}_m\left[1+\alpha_1(\phi-1)+\alpha_2(\psi-1)+\alpha_3(\xi-1)+\alpha_4(w-1)+\alpha_5(v-1)\right]^{-1},$$
$$J_{m3(3)}=\bar{h}_m\left[2-\phi^{\alpha_1},\psi^{\alpha_2}\xi^{\alpha_3}w^{\alpha_4}v^{\alpha_5}\right],$$

etc. are the members of the suggested class of estimators J_{m3}, where $\alpha_i{}'s\,(i=1 to 5)$ are suitably chosen constants. The bias and *MSE* of the estimators, $J_{m3(j)}; j=1 to 3$ and t_{m3} at (28) can be obtained easily from (30) and (31) just by putting the values of derivatives.

Keeping the form of Priyanka and Trisandhya (2019) the estimator t_{m2} and motivated by Srivastava and Jhajji (1981) we define a subclass of estimators J_{m3} of the class of estimators J_{m2} for the population mean $\bar{H}$ of the coded response at current move as

$$J_{m4}=L^*\left(\bar{h}_m,\phi,\xi,v\right), \tag{38}$$

where $L^*(.)$is a function of $\left(\bar{h}_m,\phi,\xi,v\right)$ such that $L^*(B^*)=\bar{H}$, $\Rightarrow L_1^*(B^*)=1$, $L_1^*(B^*)$ being the first order partial derivative of the function $L^*(.)$ at the point $B^*=\left(\bar{H},1,1,1\right)$ and also satisfies
certain regularity conditions similar to these given in Srivastava and Jhajji (1981).

To the fda, ignoring fpc term, the bias and *MSE* of the class of estimators J_{m4} are respectively given by

$$\begin{aligned}B(J_{m4})=\frac{1}{2}\Big[\left(\frac{1}{m}-\frac{1}{n}\right)&\left\{C_g^2L_{22}^*(B^*)+2\rho_{hg}S_hC_gL_{12}^*(B^*)\right\}+\frac{1}{n}\left\{C_z^2L_{33}^*(B^*)+(\lambda_{004}-1)L_{44}^*(B^*)\right.\\&\left.+2\lambda_{003}C_zL_{34}^*(B^*)+2\rho_{hz}S_hC_zL_{13}^*(B^*)+2\lambda_{012}S_hL_{14}^*(B^*)\right\}\Big],\end{aligned} \tag{39}$$

$$\begin{aligned}MSE(J_{m4})=\Big[\left(\frac{1}{m}-\frac{1}{n}\right)&\left\{S_h^2+C_g^2L_2^{*2}(B^*)+2\rho_{hg}S_hC_gL_2^*(B^*)\right\}+\frac{1}{n}\left\{S_h^2+C_z^2L_3^{*2}(B^*)\right.\\&\left.+(\lambda_{004}-1)L_4^{*2}(B^*)+2\lambda_{003}C_zL_3^*(B^*)L_4^*(B^*)+2\rho_{hz}S_hC_zL_3^*(B^*)+2\lambda_{012}S_hL_4^*(B^*)\right\}\Big],\end{aligned} \tag{40}$$

where $L_2^*(B^*), L_3^*(B^*), L_4^*(B^*)$ *and* $L_{ij}^*(B^*),((i,j)=1,2,3,4)$ are the first and second order partial derivatives of the function $L^*\left(\bar{h}_m,\phi,\xi,v\right)$ at the point $B^*=\left(\bar{H},1,1,1\right)$.

The $MSE(J_{m4})$ at (40) is minimized for

$$\left.\begin{aligned} L_2^*\left(B^*\right) &= -\rho_{hg}\frac{S_h}{C_g} = L_{20}^*\left(B^*\right), say \\ L_3^*\left(B^*\right) &= \frac{S_h\left[(\lambda_{004}-1)\rho_{hz}-\lambda_{003}\lambda_{012}\right]}{C_z\left(\lambda_{004}-\lambda_{003}^2-1\right)} = L_{30}^*\left(B^*\right), say \\ L_4^*\left(B^*\right) &= \frac{S_h\left[\lambda_{012}-\lambda_{003}\rho_{hz}\right]}{\left(\lambda_{004}-\lambda_{003}^2-1\right)} = L_{40}^*\left(B^*\right), say \end{aligned}\right\}. \tag{41}$$

Thus, the resulting minimum MSE of J_{m4} is given by

$$MSE_{\min}\left(J_{m4}\right) = S_h^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left(1-\rho_{gz}^2\right)+\frac{1}{n}\left\{1-\rho_{hz}^2-\frac{\left(\lambda_{003}\rho_{hz}-\lambda_{012}\right)^2}{\left(\lambda_{004}-\lambda_{003}^2-1\right)}\right\}\right]. \tag{42}$$

Thus, we arrived at the following theorem.

Theorem 6: To the fda,

$$MSE_{\min}\left(J_{m4}\right) \geq S_h^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left(1-\rho_{gz}^2\right)+\frac{1}{n}\left\{1-\rho_{hz}^2-\frac{\left(\lambda_{003}\rho_{hz}-\lambda_{012}\right)^2}{\left(\lambda_{004}-\lambda_{003}^2-1\right)}\right\}\right],$$

with equality holding if $L_j^*\left(B^*\right)=L_{j0}^*\left(B^*\right)$, $j=2,3,4$; where $L_{j0}^*\left(B^*\right)$ is given by (41), $j=2,3,4$.

The class of estimators J_{m3} is very large. The following estimators

$$\begin{aligned} J_{m4(1)} &= \bar{h}_m\,\phi^{\alpha_1}\xi^{\alpha_2}v^{\alpha_3}, \\ J_{m4(2)} &= \bar{h}_m\left[1+\alpha_1(\phi-1)+\alpha_2(\xi-1)+\alpha_3(v-1)\right]^{-1}, \\ J_{m4(3)} &= \bar{h}_m\left[2-\phi^{\alpha_1}\xi^{\alpha_2}v^{\alpha_3}\right], \\ J_{m4(4)} &= \left[\bar{h}_m+\alpha_1(1-\phi)+\alpha_3(1-\xi)+\alpha_3(1-v)\right], \end{aligned}$$

etc. are the members of the suggested class of estimators J_{m4}, where α_i's $(i=1 to 3)$ are suitably chosen constants. The bias and *MSE* of the estimators can easily be obtained from (29) and (40) just by putting the values of derivatives.

From (37) and (42) we have

$$MSE_{\min}\left(J_{m4}\right)-MSE_{\min}\left(J_{m3}\right)=\left(\frac{1}{m}-\frac{1}{n}\right)\frac{S_h^2}{\left(1-\rho_{hg}^2\right)}\left[\left(\rho_{hg}\rho_{gz-}\rho_{hz}\right)^2+\frac{\rho_1^2}{Q_1}\right]\geq 0. \tag{43}$$

It follows from (43) that the proposed class of estimator J_{m3} is more efficient than the estimator J_{m4}, and hence better than the Priyanka and Trisandhya (2019) -type estimator t_{m2}.

3. Combined Classes of Estimators

Taking the convex linear combination of class of estimators D_{ui} $(i=1,2)$ and J_{mj} $(j=1 to 4)$based sample of size *u* and *m* respectively, the final estimator for the population mean $\overline{H}$ of coded response at current occasion in two occasion successive sampling is defined by

$$T_{ij} = \Phi_{ij} D_{ui} + \left(1 - \Phi_{ij}\right) J_{mj}, \tag{44}$$

where $D_{u1}, D_{u2}, J_{m1}, J_{m2}, J_{m3}$, and J_{m4} are respectively defined in (2), (7), (15), (21), (29) and (38) and $\Phi_{ij} \in [0, 1]$ is a scalar quantity to be chosen suitably.

Theorem 7: Bias of the combined class of estimators T_{ij} to the fda, ignoring fpc term, is obtained as

$$B\left(T_{ij}\right) = \Phi_{ij} B\left(D_{ui}\right) + \left(1 - \Phi_{ij}\right) B\left(J_{mj}\right), \tag{45}$$

where $i = 1,2$ and $j = 1\,to\,4$.

Proof is simple so omitted.

Putting the values of $B\left(D_{u1}\right), B\left(D_{u2}\right), B\left(j_{m1}\right), B\left(j_{m2}\right), B\left(j_{m3}\right)$, and $B\left(j_{m4}\right)$ as respectively defined in (3), (8), (17), (22), (30) and (39) in the above equation, we get the expression for the bias of the class of estimators T_{ij} in (45).

Theorem 8: The mean squared error of the class of estimators T_{ij} is given by

$$MSE\left(T_{ij}\right) = \Phi_{ij}^2 MSE_{\min}\left(D_{ui}\right) + \left(1 - \Phi_{ij}\right)^2 MSE_{\min}\left(J_{mj}\right), \tag{46}$$

where $\Phi_{ij}'s$ $\{i = 1,2 \; and \; j = 1\,to\,4\}$ are constants to be determined such that mean squared errors of $T_{ij}'s$ are minimum.

Proof: The mean squared error of the class of estimators T_{ij} is derived as

$$\begin{aligned} MSE\left(T_{ij}\right) &= E\left(T_{ij} - \overline{H}\right)^2 = E\left[\Phi_{ij}\left(D_{ui} - \overline{H}\right) + \left(1 - \Phi_{ij}\right)\left(J_{mj} - \overline{H}\right)\right]^2, \\ &= \Phi_{ij}^2 E\left(D_{ui} - \overline{H}\right)^2 + \left(1 - \Phi_{ij}\right)^2 E\left(J_{mj} - \overline{H}\right)^2 + 2\Phi_{ij}\left(1 - \Phi_{ij}\right) E\left\{\left(D_{ui} - \overline{H}\right)\left(J_{mj} - \overline{H}\right)\right\}, \\ &= \left[\Phi_{ij}^2 MSE\left(D_{ui}\right) + \left(1 - \Phi_{ij}\right)^2 MSE\left(J_{mj}\right) + 2\Phi_{ij}\left(1 - \Phi_{ij}\right) Cov\left(D_{ui}, J_{mj}\right)\right]. \end{aligned} \tag{47}$$

The minimum *MSE's* of D_{ui} $(i = 1,2)$ and J_{mj} $(j = 1\,to\,4)$ are given by (6), (13), (20), (26), (37) and (42) respectively and as the two sets of estimators $\{D_{u1}, D_{u2}\}$ and $\{J_{m1}, J_{m2}, J_{m3}, J_{m4}\}$ are based on two non-overlapping samples of sizes *u* and *m*, respectively, therefore $Cov\left(D_{ui}, J_{mj}\right) = 0$. Thus inserting $MSE_{\min}\left(D_{ui}\right)$ and $MSE_{\min}\left(J_{mj}\right)$ in place of $MSE\left(D_{ui}\right)$ and $MSE\left(J_{mj}\right)$ respectively and $Cov\left(D_{ui}, J_{mj}\right) = 0$ in (47), we get the *MSE* of T_{ij} as in (46).

3.1. Minimum *MSE* of the suggested combined class of estimators T_{ij}

Differentiating (46) with respect to Φ_{ij} and equating them to zero, we get the optimum value of Φ_{ij} as

$$\Phi_{ij(opt)} = \frac{MSE_{\min}\left(J_{mj}\right)}{MSE_{\min}\left(D_{ui}\right)+MSE_{\min}\left(J_{mj}\right)},\ \left(i=1,2\ ;\ j=1\,to\,4\right). \tag{48}$$

Inserting the value of $\Phi_{ij(opt)}$ from equation (48) in (46), we get the minimum *MSE* of classes of estimators T_{ij} as

$$MSE_{\min}\left(T_{ij}\right)=\frac{MSE_{\min}\left(D_{ui}\right)*MSE_{\min}\left(J_{mj}\right)}{MSE_{\min}\left(D_{ui}\right)+MSE_{\min}\left(J_{mj}\right)},\left(i=1,2\ ;\ j=1\,to\,4\right). \tag{49}$$

Putting the value of $MSE_{\min}\left(D_{ui}\right)$ and $MSE_{\min}\left(J_{mj}\right)$ from (6), (13), (20), (26), (37) and (42) respectively in (49), the simplified values of $MSE_{\min}\left(T_{ij}\right)$ are obtained as

$$MSE_{\min}\left(T_{11}\right)=\frac{S_h^2}{n}\frac{\left(1-\rho_{hg}^2\right)\left(1-\mu_{11}\rho_{hg.z}^2\right)}{\left(1-\mu_{11}^2\rho_{hg.z}^2\right)}, \tag{50}$$

$$MSE_{\min}\left(T_{12}\right)=\frac{S_h^2}{n}\frac{\left(1-\rho_{hg}^2\right)\left[\left(1-\rho_{hg}^2\right)-\mu_{12}\left(\rho_{hg}^2-\rho_{hz}^2\right)\right]}{\left[\left(1-\rho_{hg}^2\right)-\mu_{12}^2\left(\rho_{hg}^2-\rho_{hz}^2\right)\right]}, \tag{51}$$

$$MSE_{\min}\left(T_{13}\right)=\frac{S_h^2}{n}\frac{A_{11}\left(B_{13}-\mu_{13}C_{13}\right)}{\left[A_{13}-\mu_{13}\left(A_{11}-B_{13}\right)-\mu_{13}^2C_{13}\right]}, \tag{52}$$

$$MSE_{\min}\left(T_{14}\right)=\frac{S_h^2}{n}\frac{A_{11}\left(B_{13}+\mu_{14}C_{14}\right)}{\left[A_{11}-\mu_{14}C_{13}+\mu_{14}^2C_{14}\right]}, \tag{53}$$

$$MSE_{\min}\left(T_{21}\right)=\frac{S_h^2}{n}\frac{A_{11}B_{13}\left(1-\mu_{21}\rho_{hg.z}^2\right)}{\left[B_{13}+\mu_{21}C_{13}-\mu_{21}^2\rho_{hg.z}^2A_{11}\right]}, \tag{54}$$

$$MSE_{\min}\left(T_{22}\right)=\frac{S_h^2}{n}\frac{\left(A_{11}-C_{13}\right)\left[A_{11}-\mu_{22}\left(\rho_{hg}^2-\rho_{hz}^2\right)\right]}{\left[A_{11}-C_{13}+\mu_{22}C_{13}-\mu_{22}^2\left(\rho_{hg}^2-\rho_{hz}^2\right)\right]}, \tag{55}$$

$$MSE_{\min}\left(T_{23}\right)=\frac{S_h^2}{n}\frac{\left(A_{11}-C_{13}\right)\left[A_{11}-\mu_{23}C_{23}\right]}{\left[\left(A_{11}-C_{13}\right)-\mu_{22}^2C_{23}\right]}, \tag{56}$$

$$MSE_{\min}\left(T_{24}\right)=\frac{S_h^2}{n}\frac{\left(A_{11}-C_{13}\right)\left[\left(A_{11}-C_{13}\right)-\mu_{24}\left(\rho_{hg}^2-\rho_{hz}^2-C_{13}\right)\right]}{\left[\left(A_{11}-C_{13}\right)-\mu_{24}^2\left(\rho_{hg}^2-\rho_{hz}^2-C_{13}\right)\right]}, \tag{57}$$

where $A_{11}=\left(1-\rho_{hg}^2\right)$, $\rho_{hg.z}=\dfrac{\left(\rho_{hg}-\rho_{gz}\rho_{hz}\right)}{\sqrt{\left(1-\rho_{hg}^2\right)}\sqrt{\left(1-\rho_{gz}^2\right)}}$, $A_{13}=\left\{1-R_{h.gz}^2-\dfrac{\rho_1^2}{Q_1\left(1-\rho_{gz}^2\right)}\right\}$,

$$B_{13}=\left\{1-\rho_{hz}^2-\frac{\left(\lambda_{003}\rho_{hz}-\lambda_{012}\right)^2}{\left(\lambda_{004}-\lambda_{003}^2-1\right)}\right\}=\left(A_{11}-C_{13}\right),$$

$$C_{13}=\left(A_{11}-B_{13}\right)=\frac{\left(\lambda_{003}\rho_{hz}-\lambda_{012}\right)^2}{\left(\lambda_{004}-\lambda_{003}^2-1\right)},$$

$$C_{14}=\left[\frac{(\lambda_{003}\rho_{hz}-\lambda_{012})^2}{(\lambda_{004}-\lambda_{003}^2-1)}+d_{11}\right],\ d_{11}=(\rho_{hz}^2-\rho_{hg}^2),$$

$$C_{23}=\left\{\rho_{h.g.z}^2(1-\rho_{hz}^2)+\frac{\rho_1^2}{Q_1(1-\rho_{gz}^2)}-C_{13}\right\}.$$

3.2. Optimum rotation rate

It is observed from (50) to (57) that $MSE_{\min}(T_{ij}),\ (i=1,2;\ j=1,2,3,4)$ is the function of μ_{ij} which is rotation rate or the fraction of sample to be drawn afresh at current occasion. As less the sample need to be selected afresh, less is the total cost of the survey so to estimate population mean with maximum precision and minimum cost $MSE_{\min}(T_{ij})$ at (50)-(57) have been minimized with respect to μ_{ij}. The optimum values μ_{ij} have been derived as

$$\hat{\mu}_{11}=\frac{1}{\left[1+\sqrt{(1-\rho_{h.g.z}^2)}\right]}, \tag{58}$$

$$\hat{\mu}_{12}=\frac{\sqrt{(1-\rho_{h.z}^2)}}{\left[\sqrt{(1-\rho_{h.z}^2)}+\sqrt{(1-\rho_{h.g}^2)}\right]}, \tag{59}$$

$$\hat{\mu}_{13}=\min\left[\frac{B_{13}-\sqrt{(B_{13}-C_{13}C_{23})}}{C_{13}},\frac{B_{13}+\sqrt{(B_{13}-C_{13}C_{23})}}{C_{13}}\right]\in(0,1), \tag{60}$$

$$\hat{\mu}_{14}=\min\left[\frac{-B_{13}-\sqrt{(B_{13}(B_{13}-C_{13})-A_{11}C_{14})}}{C_{14}},\frac{-B_{13}+\sqrt{(B_{13}(B_{13}-C_{13})-A_{11}C_{14})}}{C_{14}}\right]\in(0,1), \tag{61}$$

$$\hat{\mu}_{21}=\min\left[\frac{A_{11}-\sqrt{A_{11}B_{13}(1-\rho_{h.g.z}^2)}}{\rho_{h.g.z}^2},\frac{A_{11}+\sqrt{A_{11}B_{13}(1-\rho_{h.g.z}^2)}}{\rho_{h.g.z}^2}\right]\in(0,1), \tag{62}$$

$$\hat{\mu}_{22}=\min\left[\frac{A_{11}-\sqrt{(A_{11}C_{13}-d_{11}B_{13})}}{d_{11}},\frac{A_{11}+\sqrt{(A_{11}C_{13}-d_{11}B_{13})}}{d_{11}}\right]\in(0,1), \tag{63}$$

$$\hat{\mu}_{23}=\min\left[\frac{A_{11}-\sqrt{(A_{11}^2-B_{13}C_{23})}}{C_{23}},\frac{A_{11}+\sqrt{(A_{11}^2-B_{13}C_{23})}}{C_{23}}\right]\in(0,1), \tag{64}$$

$$\hat{\mu}_{24}=\min\left[\frac{B_{13}-\sqrt{B_{13}(A_{11}-d_{11})}}{(d_{11}-C_{13})},\frac{B_{13}+\sqrt{B_{13}(A_{11}-d_{11})}}{(d_{11}-C_{13})}\right]\in(0,1), \tag{65}$$

Inserting $\hat{\mu}_{11},\hat{\mu}_{12}$ admissible value of $\hat{\mu}_{13},\hat{\mu}_{14},\hat{\mu}_{21},\hat{\mu}_{22},\hat{\mu}_{23}$ *and* $\hat{\mu}_{24}$ from (58)-(65) respectively in (50) - (57) we get the optimum values of $MSE_{\min}(T_{ij}),\ (i=1,2;\ j=1,2,3,4)$ as

$$MSE_{\min}(T_{11})_{opt}=\frac{S_h^2\ (1-\rho_{hz}^2)\left(1+\sqrt{(1-\rho_{hg.z}^2)}\right)}{2n}, \tag{66}$$

$$MSE_{\min}(T_{12})_{opt} = \frac{S_h^2\sqrt{(1-\rho_{hz}^2)}\left[\sqrt{(1-\rho_{hz}^2)}+\sqrt{(1-\rho_{hg}^2)}\right]}{2n}, \tag{67}$$

$$MSE_{\min}(T_{13})_{opt} = \frac{S_h^2}{n}\frac{A_{11}(B_{13}-\hat{\mu}_{13}C_{13})}{\left[A_{13}-\hat{\mu}_{13}(A_{11}-B_{13})-\hat{\mu}_{13}^2C_{13}\right]}, \tag{68}$$

$$MSE_{\min}(T_{14})_{opt} = \frac{S_h^2}{n}\frac{A_{11}(B_{13}+\hat{\mu}_{14}C_{14})}{\left[A_{11}-\hat{\mu}_{14}C_{13}+\hat{\mu}_{14}^2C_{14}\right]}, \tag{69}$$

$$MSE_{\min}(T_{21})_{opt} = \frac{S_h^2}{n}\frac{A_{11}B_{13}(1-\hat{\mu}_{21}\rho_{hg.z}^2)}{\left[B_{13}+\hat{\mu}_{21}C_{13}-\hat{\mu}_{21}^2\rho_{hg.z}^2A_{11}\right]}, \tag{70}$$

$$MSE_{\min}(T_{22})_{opt} = \frac{S_h^2}{n}\frac{(A_{11}-C_{13})[A_{11}-\hat{\mu}_{22}d_{11}]}{\left[(A_{11}-C_{13})+\hat{\mu}_{22}C_{13}-\hat{\mu}_{22}^2d_{11}\right]}, \tag{71}$$

$$MSE_{\min}(T_{23})_{opt} = \frac{S_h^2}{n}\frac{(A_{11}-C_{13})[A_{11}-\hat{\mu}_{23}C_{23}]}{\left[(A_{11}-C_{13})-\hat{\mu}_{22}^2C_{23}\right]}, \tag{72}$$

$$MSE_{\min}(T_{24})_{opt} = \frac{S_h^2}{n}\frac{(A_{11}-C_{13})[(A_{11}-C_{13})-\hat{\mu}_{24}(d_{11}-C_{13})]}{\left[(A_{11}-C_{13})-\hat{\mu}_{24}^2(d_{11}-C_{13})\right]}. \tag{73}$$

4. Performances of the Suggested Classes of Estimators

For examining the relevance and utility of the information used on non-sensitive auxiliary variable with the proposed classes of estimators, we have considered a class of estimators where no additional non-sensitive auxiliary information is used, known as modified general class of successive sampling estimators.

4.1. Modified general class of estimators based on matched sample of size *m*

Following the procedure adopted by Srivastava (1971,1980) we consider the following class of estimators of the population mean $\overline{H}$ of coded response variable on the current (second) occasion as

$$t_G = G(\bar{h}_m, \phi), \tag{74}$$

where $G(.)$ is a function of $(\bar{h}_m, \phi = \bar{g}_m/\bar{g}_n)$ such that

$$G(\overline{H},1) = \overline{H} \Rightarrow G_1(\overline{H},1) = 1, \tag{75}$$

$G_1(\overline{H},1)$ being the first order partial derivative of the function $G(\bar{h}_m,\phi)$ at the point $(\overline{H},1)$and satisfies certain regularity conditions similar to these given in Srivastava (1971,1980).

To the fda, ignoring fpc term, the bias and *MSE* of t_G are respectively given by

$$B(t_G) = \frac{1}{2}\left(\frac{1}{m}-\frac{1}{n}\right)\left[C_g^2G_{22}(\overline{H},1)+2\rho_{hg}S_hC_gG_{12}(\overline{H},1)\right], \tag{76}$$

$$MSE(t_G) = \left[\frac{1}{m}S_h^2+\left(\frac{1}{m}-\frac{1}{n}\right)\left\{C_g^2G_2^2(\overline{H},1)+2\rho_{hg}S_hC_gG_2(\overline{H},1)\right\}\right], \tag{77}$$

where $G_2(\overline{H},1)=\left.\dfrac{\partial G(.)}{\partial \phi}\right|_{(\overline{H},1)}$, $G_{12}(\overline{H},1)=\left.\dfrac{\partial^2 G(.)}{\partial \phi \partial \overline{h}_m}\right|_{(\overline{H},1)}$ and $G_{22}(\overline{H},1)=\left.\dfrac{\partial^2 G(.)}{\partial^2 \phi}\right|_{(\overline{H},1)}$.

The *MSE* of t_G at (77) is minimum when

$$G_2(\overline{H},1)=-\rho_{hg}(S_h/C_g). \tag{78}$$

Substitution (78) in (77) yields the minimum *MSE* of the class of estimators t_G as

$$MSE_{\min}(t_G)=S_h^2\left[\frac{1}{m}\left(1-\rho_{hg}^2\right)+\frac{1}{n}\rho_{hg}^2\right] \tag{79}$$

which is equal to the minimum *MSE* of the difference estimator

$$t_{dm}=\overline{h}_m+\beta_{hg}(\overline{g}_n-\overline{g}_m), \tag{80}$$

where β_{hg} is the population regression coefficient of h on g.

We note that the class of estimators t_G at (74) is very vast. The following estimators (including t_{dm})

$$t_{G1}=\overline{h}_m\phi^{\alpha_1},\quad t_{G2}=\overline{h}_m\left(2-\phi^{\alpha_1}\right),\quad t_{G3}=\overline{h}_m\exp\left\{\frac{\alpha_1(\overline{g}_m-\overline{g}_n)}{(\overline{g}_m+\overline{g}_n)}\right\},$$

etc. are the members of the class of estimators t_G at (74). The bias and *MSE* of the estimators t_{Gj} $(j=1 to 3)$ and t_{dm} can be easily obtained from (76) and (77) just by putting the suitable values of derivatives $G_2(\overline{H},1)$, $G_{12}(\overline{H},1)$ and $G_{22}(\overline{H},1)$.

Now we state the following theorem.

Theorem 9: Up to the first order of approximation,

$$MSE_{\min}(t_G)\geq S_h^2\left[\frac{1}{m}\left(1-\rho_{hg}^2\right)+\frac{1}{n}\rho_{hg}^2\right]$$

with equality holding if $G_2(\overline{H},1)=-\rho_{hg}(S_h/C_g)$.

4.2. Combined class of estimators

We consider the following combined classes of estimators for population mean response $\overline{H}$ of coded response variable at current (second) move at

$$T_C=\Phi\overline{h}_u+(1-\Phi)t_G, \tag{81}$$

where $\Phi\in[0,1]$ is unknown constant.

We note that the class of estimators

$$T_J=\Phi_{(1)}\overline{h}_u+\left(1-\Phi_{(1)}\right)\left[\overline{h}_m+k_1(\overline{g}_n-\overline{g}_m)\right], \tag{82}$$

due to Priyanka and Trisandhya (2019) is a member of the class of estimators T_C at (81), $\Phi_{(1)} \in [0,1]$ is unknown constant and k_1 is a suitably chosen scalar.

The *MSE* of T_C is given by

$$MSE(T_C) = \Phi^2 MSE(\bar{h}_u) + (1-\Phi)^2 MSE(t_G), \tag{83}$$

as the term $Cov(\bar{h}_u, t_G) = 0$.

Replacing $MSE(t_G)$ by its $MSE_{\min}(t_G)$ in (83) we have

$$MSE(T_C) = \Phi^2 \left[MSE(\bar{h}_u) + MSE_{\min}(t_G)\right] - 2\Phi MSE_{\min}(t_G) + MSE_{\min}(t_G), \tag{84}$$

which is minimum when

$$\Phi = \frac{MSE_{\min}(t_G)}{MSE(\bar{h}_u) + MSE_{\min}(t_G)}, \tag{85}$$

and thus the resulting minimum *MSE* of class of estimators T_C is given by

$$MSE_{\min}(T_C) = \frac{S_h^2}{n} \frac{\left(1-\mu_J \rho_{hg}^2\right)}{\left(1-\mu_J^2 \rho_{hg}^2\right)} . \tag{86}$$

Expression (86) is optimized for

$$\hat{\mu}_J = \frac{1}{\left\{1+\sqrt{\left(1-\rho_{hg}^2\right)}\right\}} . \tag{87}$$

Thus the optimum value of $MSE_{\min}(T_C)$ is

$$MSE_{\min}(T_C)_{opt} = \frac{S_h^2 \left\{1+\sqrt{\left(1-\rho_{hg}^2\right)}\right\}}{2n} . \tag{88}$$

4.3. Theoretical comparison of the estimators $T_{11} = \{\Phi_{11} D_{u1} + (1-\Phi_{11}) J_{m1}\}$ and $T_{12} = \{\Phi_{12} D_{u1} + (1-\Phi_{12}) J_{m2}\}$ with the estimators $T_C = \{\Phi \bar{h}_u + (1-\Phi) t_G\}$, [or with the estimators $T_J = \Phi_{(1)} \bar{h}_u + (1-\Phi_{(1)}) [\bar{h}_m + k_1 (\bar{g}_n - \bar{g}_m)]$]

From (66), (67) and (68) we have

$$MSE_{\min}(T_C)_{opt} - MSE_{\min}(T_{11})_{opt} = \frac{S_h^2}{2n}\left(\rho_{hg}^2 + \frac{N_1}{D_1}\right) > 0, \tag{89}$$

$$MSE_{\min}(T_C)_{opt} - MSE_{\min}(T_{12})_{opt} = \frac{S_h^2}{2n}\left[\rho_{hz}^2 + \sqrt{\left(1-\rho_{hg}^2\right)}\left\{1-\sqrt{\left(1-\rho_{hz}^2\right)}\right\}\right] > 0, \tag{90}$$

$$MSE_{\min}(T_{12})_{opt} - MSE_{\min}(T_{11})_{opt} = \frac{S_h^2}{2n} \frac{\sqrt{\left(1-\rho_{hz}^2\right)}\left(\rho_{hz} - \rho_{hg}\rho_{gz}\right)}{\left(1-\rho_{gz}^2\right) D_2} > 0, \tag{91}$$

where $N_1 = \left[\rho_{hz}^2\left(1-R_{h.gz}^2\right) + \left(\rho_{hz} - \rho_{hg}\rho_{gz}\right)^2\right]$,

$D_1 = \left(1-\rho_{gz}^2\right)\left[\sqrt{\left(1-\rho_{hz}^2\right)} + \left(1-\rho_{hz}^2\right)\left\{\sqrt{\left(1-\rho_{hg.z}^2\right)}\right\}\right]$, and $D_2 = \left[\sqrt{\left(1-\rho_{hg}^2\right)} + \sqrt{\left(1-\rho_{hz}^2\right)}\sqrt{\left(1-\rho_{hg.z}^2\right)}\right]$.

From (89), (90) and (91) we have the inequality

$$MSE_{\min}(T_{11})_{opt} < MSE_{\min}(T_{12})_{opt} < MSE_{\min}(T_C)_{opt} \,. \tag{92}$$

It follows from (92) that the proposed estimator T_{11} is more efficient than the estimators T_{12} and T_C. Theoretical Comparison among the estimators $T_{13}, T_{14}, T_{21}, T_{22}, T_{23}$ and T_{24} are tedious, therefore we have not made the comparison among these estimators.

5. Estimators of Sensitive Population Mean at Current (Second) Move under Model M_G

The population mean $\overline{H}$ of the coded response variable *h* at current occasion in (1) is replaced by its estimators $T_{ij}, T_C\ (i = 1,2;\, j = 1,2,3,4)$ given in (44) and (81) respectively, to derive the estimators $\hat{\hat{Y}}_{ij}$ and $\hat{\hat{Y}}_C$ for sensitive population mean which are given below

$$\hat{\hat{Y}}_{11} = T_{11} = \{\Phi_{11}D_{u1} + (1-\Phi_{11})J_{m1}\}, \qquad \hat{\hat{Y}}_{12} = T_{12} = \{\Phi_{12}D_{u1} + (1-\Phi_{12})J_{m2}\},$$

$$\hat{\hat{Y}}_{13} = T_{13} = \{\Phi_{13}D_{u1} + (1-\Phi_{13})J_{m3}\}, \qquad \hat{\hat{Y}}_{14} = T_{14} = \{\Phi_{14}D_{u1} + (1-\Phi_{14})J_{m4}\},$$

$$\hat{\hat{Y}}_{21} = T_{21} = \{\Phi_{21}D_{u2} + (1-\Phi_{21})J_{m1}\}, \qquad \hat{\hat{Y}}_{22} = T_{22} = \{\Phi_{22}D_{u2} + (1-\Phi_{22})J_{m2}\},$$

$$\hat{\hat{Y}}_{23} = T_{23} = \{\Phi_{23}D_{u2} + (1-\Phi_{23})J_{m3}\}, \qquad \hat{\hat{Y}}_{24} = T_{24} = \{\Phi_{24}D_{u2} + (1-\Phi_{24})J_{m4}\},$$

$$\hat{\hat{Y}}_C = T_C = \{\Phi\bar{h}_u + (1-\Phi)t_G\}.$$

5.1. Numerical illustration

To have tangible idea about the performance of the suggested estimators $\hat{\hat{Y}}_{11}$ and $\hat{\hat{Y}}_{12}$ (for the sake of convenience we have considered only two estimators $\hat{\hat{Y}}_{11}$ and $\hat{\hat{Y}}_{12}$ for purpose of comparison), we have considered artificial parametric values

$$\sigma_x^2 = 6,\ \sigma_y^2 = 2,\ \sigma_z^2 = 2,\ \rho_{yx} = 0.6820,\ \rho_{yz} = 0.7520,\ \rho_{xz} = 0.650\ ,\ \overline{Y} = 5.00,\ \overline{X} = 4.50, \overline{Z} = 5.50$$

Here we suppose that $W \sim F(10,5)$ for which $\overline{W} = 1.6667, \sigma_w = 2.6874$.

The optimum values of fraction of sample to be drawn afresh at current (second) occasion (move) and percent relative efficiencies (*PRE*s) have been computed by using the following formulae

$$\hat{\mu}_{11} = \left[\frac{1}{\left[1+\sqrt{\left(1-\rho_{h.g.z}^2\right)}\right]}\right], \tag{93}$$

$$\hat{\mu}_{12} = \left[\frac{\sqrt{\left(1-\rho_{h.z}^2\right)}}{\left[\sqrt{\left(1-\rho_{h.z}^2\right)}+\sqrt{\left(1-\rho_{h.g}^2\right)}\right]}\right], \tag{94}$$

$$PRE\left(\hat{\hat{Y}}_{11}, \hat{\hat{Y}}_C\right) = \left[\frac{\left[1+\sqrt{\left(1-\rho_{h.g}^2\right)}\right]}{\left(1-\rho_{h.z}^2\right)\left[1+\sqrt{\left(1-\rho_{h.g.z}^2\right)}\right]}\right]*100, \tag{95}$$

$$PRE\left(\hat{\bar{Y}}_{12}, \hat{\bar{Y}}_{C}\right) = \frac{\left[1+\sqrt{\left(1-\rho_{h.g}^{2}\right)}\right]}{\sqrt{\left(1-\rho_{h.z}^{2}\right)}\left[\sqrt{\left(1-\rho_{h.z}^{2}\right)}+\sqrt{\left(1-\rho_{h.g}^{2}\right)}\right]} * 100, \tag{96}$$

Findings are given in Table 5.1 and 5.2.

Table 5.1: Optimum Values of $\hat{\mu}_{11}$ (in bracket) and *PRE* of $\hat{\bar{Y}}_{11}$ with respect to $\hat{\bar{Y}}_{C}$.

α / β	0.01	0.05	0.1	0.3	0.5	0.7	0.9	1.0	1.50	2.00
0.01	**207.13** (0.5201)	**206.63** (0.52)	**206.02** (0.5199)	**203.64** (0.5194)	**201.36** (0.5189)	**199.18** (0.5185)	**197.1** (0.518)	**196.09** (0.5178)	**191.35** (0.5169)	**187.06** (0.516)
0.05	**206.63** (0.52)	**204.22** (0.5195)	**201.36** (0.5189)	**191.35** (0.5169)	**183.16** (0.5152)	**176.33** (0.5139)	**170.56** (0.5127)	**167.99** (0.5122)	**157.56** (0.5101)	**149.94** (0.5086)
0.1	**206.02** (0.5199)	**201.36** (0.5189)	**196.09** (0.5178)	**179.6** (0.5145)	**167.99** (0.5122)	**159.37** (0.5105)	**152.72** (0.5092)	**149.94** (0.5086)	**139.56** (0.5065)	**132.82** (0.5052)
0.3	**203.64** (0.5194)	**191.35** (0.5169)	**179.6** (0.5145)	**152.72** (0.5092)	**139.56** (0.5065)	**131.75** (0.505)	**126.58** (0.5039)	**124.6** (0.5035)	**118.05** (0.5021)	**114.37** (0.5014)
0.5	**201.36** (0.5189)	**183.16** (0.5152)	**167.99** (0.5122)	**139.56** (0.5065)	**128.09** (0.5042)	**121.91** (0.5029)	**118.05** (0.5021)	**116.61** (0.5019)	**112.01** (0.501)	**109.48** (0.5005)
0.7	**199.18** (0.5185)	**176.33** (0.5139)	**159.37** (0.5105)	**131.75** (0.505)	**121.91** (0.5029)	**116.88** (0.5019)	**113.82** (0.5013)	**112.7** (0.5011)	**109.11** (0.5005)	**107.14** (0.5003)
0.9	**197.1** (0.518)	**170.56** (0.5127)	**152.72** (0.5092)	**126.58** (0.5039)	**118.05** (0.5021)	**113.82** (0.5013)	**111.28** (0.5008)	**110.35** (0.5007)	**107.37** (0.5003)	**105.74** (0.5001)
1	**196.09** (0.5178)	**167.99** (0.5122)	**149.94** (0.5086)	**124.6** (0.5035)	**116.61** (0.5019)	**112.7** (0.5011)	**110.35** (0.5007)	**109.48** (0.5005)	**106.73** (0.5002)	**105.23** (0.5001)
1.5	**191.35** (0.5169)	**157.56** (0.5101)	**139.56** (0.5065)	**118.05** (0.5021)	**112.01** (0.501)	**109.11** (0.5005)	**107.37** (0.5003)	**106.73** (0.5002)	**104.7** (0.5001)	**103.6** (0.5)
2	**187.06** (0.516)	**149.94** (0.5086)	**132.82** (0.5052)	**114.37** (0.5014)	**109.48** (0.5005)	**107.14** (0.5003)	**105.74** (0.5001)	**105.23** (0.5001)	**103.6** (0.5)	**102.75** (0.5)

Table 5.2: Optimum Values of $\hat{\mu}_{12}$ (in bracket) and *PRE* of $\hat{\bar{Y}}_{12}$ with respect to $\hat{\bar{Y}}_{C}$.

α / β	0.01	0.05	0.1	0.3	0.5	0.7	0.9	1.0	1.50	2.00
0.01	**188.8** (0.4741)	**188.42** (0.4742)	**187.96** (0.4743)	**186.15** (0.4748)	**184.41** (0.4752)	**182.74** (0.4757)	**181.14** (0.4761)	**180.36** (0.4763)	**176.68** (0.4773)	**173.33** (0.4781)
0.05	**188.42** (0.4742)	**186.59** (0.4747)	**184.41** (0.4752)	**176.68** (0.4773)	**170.26** (0.479)	**164.84** (0.4804)	**160.2** (0.4816)	**158.12** (0.4821)	**149.58** (0.4843)	**143.23** (0.4859)
0.1	**187.96** (0.4743)	**184.41** (0.4752)	**180.36** (0.4763)	**167.44** (0.4797)	**158.12** (0.4821)	**151.08** (0.4839)	**145.57** (0.4853)	**143.23** (0.4859)	**134.43** (0.4879)	**128.61** (0.4892)
0.3	**186.15** (0.4748)	**176.68** (0.4773)	**167.44** (0.4797)	**145.57** (0.4853)	**134.43** (0.4879)	**127.68** (0.4894)	**123.14** (0.4902)	**121.39** (0.4905)	**115.51** (0.4914)	**112.17** (0.4918)
0.5	**184.41** (0.4752)	**170.26** (0.479)	**158.12** (0.4821)	**134.43** (0.4879)	**124.48** (0.49)	**118.99** (0.4909)	**115.51** (0.4914)	**114.21** (0.4915)	**110.02** (0.4921)	**107.73** (0.4925)
0.7	**182.74** (0.4757)	**164.84** (0.4804)	**151.08** (0.4839)	**127.68** (0.4894)	**118.99** (0.4909)	**114.46** (0.4915)	**111.67** (0.4918)	**110.64** (0.492)	**107.4** (0.4926)	**105.67** (0.4934)
0.9	**181.14** (0.4761)	**160.2** (0.4816)	**145.57** (0.4853)	**123.14** (0.4902)	**115.51** (0.4914)	**111.67** (0.4918)	**109.35** (0.4922)	**108.51** (0.4923)	**105.87** (0.4933)	**104.48** (0.4942)
1	**180.36** (0.4763)	**158.12** (0.4821)	**143.23** (0.4859)	**121.39** (0.4905)	**114.21** (0.4915)	**110.64** (0.492)	**108.51** (0.4923)	**107.73** (0.4925)	**105.32** (0.4936)	**104.05** (0.4945)
1.5	**176.68** (0.4773)	149.58 (0.4843)	**134.43** (0.4879)	**115.51** (0.4914)	**110.02** (0.4921)	**107.4** (0.4926)	**105.87** (0.4933)	**105.32** (0.4936)	**103.62** (0.4949)	**102.74** (0.4959)
2	**173.33** (0.4781)	**143.23** (0.4859)	**128.61** (0.4892)	**112.17** (0.4918)	**107.73** (0.4925)	**105.67** (0.4934)	**104.48** (0.4942)	**104.05** (0.4945)	**102.74** (0.4959)	**102.08** (0.4968)

It is observed from Tables 5.1 and 5.2 that

(i) the suggested class of estimators $\hat{\hat{\bar{Y}}}_{11}$ and $\hat{\hat{\bar{Y}}}_{12}$ perform better than the class of estimators $\hat{\hat{\bar{Y}}}_{C}\left(or\ \hat{\hat{\bar{Y}}}_{J}\right)$ which does not utilize information on additional auxiliary variable '*z*' in terms of optimum fraction of sample to be drawn afresh and also in terms of percent relative efficiency.

(ii) the PRE's of the proposed estimators $\hat{\hat{\bar{Y}}}_{11}$ and $\hat{\hat{\bar{Y}}}_{12}$ decreases with increasing values of (α,β).

(iii) the larger gain in efficiency is observed by using the proposed classes of estimators $\hat{\hat{\bar{Y}}}_{11}$ and $\hat{\hat{\bar{Y}}}_{12}$ over the class of estimators $\hat{\hat{\bar{Y}}}_{C}\left(or\ \hat{\hat{\bar{Y}}}_{J}\right)$ when the value of (α,β) are small.

(iv) the gain in efficiency by using the proposed class of estimators $\hat{\hat{\bar{Y}}}_{11}$ over $\hat{\hat{\bar{Y}}}_{C}\left(or\ \hat{\hat{\bar{Y}}}_{J}\right)$ is larger as compared to the class of estimators $\hat{\hat{\bar{Y}}}_{12}$ over $\hat{\hat{\bar{Y}}}_{C}\left(or\ \hat{\hat{\bar{Y}}}_{J}\right)$.

It is to be mentioned that a practical choice of α and β, fixed by the experience of the experimenter from repeated surveys can always provide better results than the class of estimators $\hat{\hat{\bar{Y}}}_{C}\left(or\ \hat{\hat{\bar{Y}}}_{J}\right)$.

Remark 2: The procedure outlined in this paper can be also applied to the randomized response models mentioned in Priyanka and Trisandhya (2019) {see Arcos *et al.* (2015) and Odumade and Singh (2008) *etc*} to get the efficient estimators of the population mean at current (second) move using information on additional non-sensitive auxiliary variable at both the occasion in two occasion successive sampling.

6. Conclusion

This article presents some classes of estimators for estimating the population mean at current (second) occasion in two occasions successive sampling using information on an additional non-sensitive auxiliary variable in presence of randomized response model. The properties of the suggested classes are studied under randomized response models. Optimum replacement policies have been elaborated. It has been demonstrated that the proposed classes of estimators are better than the class of estimators which does not utilize non-sensitive auxiliary information. Numerical illustration is given in support of the present study. It has been shown that there is appreciable gain in efficiency by using the proposed classes of estimators over the class of estimators $\hat{\hat{\bar{Y}}}_{C}\left(or\ \hat{\hat{\bar{Y}}}_{J}\right)$. Thus the proposed study is recommended for its use in practice.

Acknowledgement

Authors are thankful to the learned referee and the Editor – in - Chief for their valuable suggestions regarding improvement of the paper.

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