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A Stochastic Modeling of a Monthly Rainfall of Hillsborough County Using Frechet Distribution

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Abstract

In this paper we investigate the prediction problem for the monthly rainfall of Hillsborough County at United States of Florida by Markov chain model. We have used the monthly rainfall data from January 1915 to June 2016. Then the data is divided in 11 states and hence, 11 x 11 transition probability matrix (TPM) is prepared. The truncated Frechet distribution is used for the data in each state. To estimate the parameter of the distribution, method of moment and Bayes estimation are used. Using the estimate of the parameter in 11 states prediction method is developed based on Markov chain approach. To validate the proposed method, we have simulated the monthly rainfall for the same period of the original data and the results are compared. The simulated results come out almost similar to the original data. To predict monthly rainfall for future 5000 and 10,000 months a simulation study is also carried out and the results are shown.

Key words: Markov chain; Truncated Frechet model; Rainfall; Bayes estimation; Simulation.

1. Introduction

There will be a high impact of advancement in human necessity on natural events such as rainfall, temperature, precipitation, wind flow et cetera. Since decades there was a very complex pattern observed in climate change which was difficult to predict the parameters by the meteorologists or the hydrologists. There is still an intense scope of research is available in hydrology and meteorology. The hydrological data mainly consists of water and its application such as precipitation, rainfall, humidity level and water storage level of the dam. A rainfall is the one of the natural sources for getting water for drinking, agriculture and industrial use purpose.

The analyses of hydrological and meteorological data have a great importance amongst the scientists and researchers. Researchers must ensure the collection of hydrological data should be efficient and effective which meet the requirements (Stewart, 2015). A data from hydrological networks is used by public and private sectors for variety of applications like designing, operating and maintaining the multipurpose water management systems (USGS, 2006). Three essential elements of life are fresh water, food and house. The data related to rainfall, precipitation, temperature, humidity, wind speed is essential for the planning of any hydrological event. Analysis of rainfall data found useful in cropping pattern, providing drinking water and construction of roads, dams, bridges and culverts. Such analysis will provide useful information to farmers, water resources planner and engineers to assess the availability and requirement of storage of water.

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There are multiple research studies have been done on rainfall data and its analysis. The analysis of dry and wet spells received a special attention of many scientists, which is another aspect of the rainfall analysis. Singh and Ranade (2009) analyzed wet and dry spells and their extremes across India. Harsha (2017) describes the analysis of rainfall data in Mangalore. The classical procedure is being used for the analysis of rainfall data. To test the random fluctuations and the presence of climate changes in the yearly rainfall data run test and Kolmogorov-Smirnov two sample tests are used. G. Di Baldassarre *et al.* (2006) have used the generalized extreme value distribution to analyze rainfall extremes of northern central Italy based on L-moments and investigate its statistical properties. Nyatuame *et al.* (2014) have performed the statistical analysis for the monthly and yearly rainfall data of Volta region, Ghana using Latin squared design and analysis of variance. For trend analysis of rainfall data the linear regression model is used. Arvind *et al.* (2017) has performed statistical analysis for a rain gauge station in Trichy district. They studied various statistical distributions to analyze the rainfall data.

Not significant work has been done for the statistical analysis using stochastic process Markov chain modeling under various types of distribution, which motivate us to consider this kind of research.

We have used monthly rainfall data of Hillsborough County (latitude 27°54'36.00" N and longitude -82°20'60.00" W) at United States of Florida is considered for the period of January 1915 to June 2016. The data is taken from the pertinent website: https://www.swfwmd.state.fl.us/resources/data-maps/rainfall-summary-data-region. A separate spread sheet is available for the monthly rainfall data. Then the monthly rainfall data of Hillsborough County was concatenated for the period of January 1915 to June 2016 from that web page.

In hydrological research studies multiple statistical approaches have been applied for the estimation. The objective of this study is to develop a statistical model based on Markov chain to estimate and predict the month wise rainfall of the mentioned time period. The span of the rainfall data used is 0.00 to 19.06 mm. To consider the analysis based on the Markov chain we have bifurcated the data into some small numbers of intervals which we called the states of the Markov chain. 11 states are prepared from the data and which are shown in Table A.1.

In Section 3, a transition probability matrix for a Markov chain model is prepared. The truncated Frechet distribution is considered for the rainfall of each states and the estimate of the parameter of the distribution is obtained using the method of moments in Section 4. In Section 5 we have used a Bayesian approach to estimate the parameter of the distribution. A simulation study is considered in Section 6. A detailed algorithm is prepared for estimation and prediction of present and future rainfall data. Discussion about the estimated results is provided in Section 7. The conclusion is presented in the Section 8.

2. Model Creation

The rainfall of the Hillsborough lies between 0.00 mm to 19.06 mm from the period of January 1915 to June 2016 is taken. For the Markov chain model the determination of states is the first aspect. The states should non-overlapping subsets of entire data. Based on the range of our data we have constructed 11 subsets such that each subset possesses sufficient numbers of observations. Looking at the data we have considered the subsets having different length. These subsets we considered as states of our Markov chain model, which are displayed in Table A.1.

3. Markov Chain Model

A discrete parameter Markov process is known as a Markov chain. Here time space is considered as discrete. The Markov chain models are much valuable mechanism in stochastic process, which also indicates that when present value is known then the historical and future values are independent. Sericola (2013) mentioned that the present state of the procedure is known then the best future prediction can be made using very less parameters of Markov chain model.

Mahanta *et al.* (2019) applied Markov chain model for the daily temperature data of Dhaka and Chittagong stations of Bangladesh. The Markov chain model have been used as a process to search its reliability and obtain failure free operational process for long term period can be established specifically for sugar mills by Sharma and Vishwakarma (2014). Zakaria *et al.* (2019) have used the Markov chain model based on the initial state as well as transition from one state to another state for the forecasting pattern of the air pollution index of Miri, Sarawak.

Jain (1986) have also implemented the Markov model for the seasonal variation in patients who are suffering from asthma. Zhou *et al.* (2018) proposed a Markov chain model which provides prediction of daily bike production and attraction of stations with better predictive accuracy based on the daily data collected from Zhongshan city. Al-Anzi and AbuZeina (2016) have provided the hidden Markov Models (HMM) can be used for the natural language processing (NLP) applications. Patel and Patel (2020a, 2020b, 2021) have considered a first order Markov chain model for the prediction of daily high temperature and daily low temperature.

In this study, the 101 years of monthly rainfall of the Hillsborough County is being considered in millimeter (mm). A data of 1218 (=N) observations is taken for the creation of Markov chain model.

Let Z_t, $t = 1, 2, ..., N$ be the rainfall for the month t, and the states are $U_1, U_2, U_3 ... U_{11}$.

If $P[Z_{t+1} = U_j \mid Z_1 = U_1, ..., Z_{t-1} = U_{t-1}, Z_t = U_i] = P[Z_{t+1} = U_j \mid Z_t = U_i]$, then such model is called first order Markov chain model with 11 states. Here, $P[Z_{t+1} = U_j \mid Z_t = U_i]$ is independent of time t. This transition probability is denoted by pij, $i,j = 1, 2, ... ,11$, which denotes the probability that the monthly rainfall is on any month will belong to state U_j, given that it was in the state U_i a month before. Thus, 11×11 TPM, $M = [m_{ij}]$ is prepared.

The transition frequency from state U_i to U_j denotes the total number of months having rainfall in state U_j from the rainfall of earlier month in state U_i. Such transition frequencies are calculated for each state and hence, transition frequency matrix is prepared which is shown in Table A.2.

Using the transition frequency matrix a transitional probability matrix (TPM) is obtained, dividing by row total of each row to its cell values. Then the value of $(i, j)^{th}$ cell is called transition probability of j^{th} state from i^{th} state. The TPM is given in Table A.3. The cumulative TPM is provided in Table A.4.

4. Truncated Frechet Distribution for Monthly Rainfall

Very limited research work has been done about the analysis of the hydrological data using Markov chain approach along with statistical distribution. Various types of statistical distributions like exponential distribution, Weibull distribution, Gamma distribution, extreme value distribution, Frechet distribution are used to analyze the data related to meteorological data like temperature, as well as hydrological data like rainfall, wind flow, water storage capacity and precipitation. Patel and Patel (2020 a, 2020 b, 2021) have considered the truncated exponential distribution and generalized exponential distribution for the analysis of the data related to daily low and high temperature of the Ahmedabad, Gujarat, India.

In this paper we have considered Truncated Frechet distribution to analyze the monthly rainfall data the Hillsborough County. Frechet distribution is named after a French mathematician Maurice Rene Frechet, who developed it in 1920 as a maximum value distribution. Frechet distribution is a special case of generalized extreme value distribution which is also named as extreme value type II distribution. This distribution is also referred as inverse Weibull distribution. Kotz and Nadarajah (2000) describe this distribution and discussed its various application in different fields such as rainfall, wind speeds, track race records, natural calamities and so on. Ramos *et al.* (2017) have presented the parameter estimation for the Frechet distribution in the presence of cure fraction.

Recently Ramos *et al.* (2020) have considered various methods of classical and Bayesian estimation of the parameters of the Frechet distribution. They have described the application of this distribution for five real data sets related to the minimum flow of water on Piracicaba river in Brazil.

The probability density function (pdf) of Frechet distribution:

$$g\,(x,\propto) = \frac{\propto}{\sigma}\left(\frac{x}{\sigma}\right)^{-1-\propto} e^{-\left(\frac{x}{\sigma}\right)^{-\propto}};\ \propto > 0;\ \sigma > 0;\ x > 0. \tag{1}$$

We have used truncated Frechet distribution to analyze the monthly rainfall data considering σ=1 in equation (1).

$$g\,(x,\propto) = \propto x^{-1-\propto} e^{-x^{-\propto}};\ \propto > 0;\ \sigma > 0;\ x > 0. \tag{2}$$

From equation (1) the pdf of truncated Frechet distribution whose range lies between a and b is obtained by:

$$g\,(x,\propto) = \frac{f\,(x,\ \propto)}{F(b,\ \propto) - F\,(a,\ \propto)},\ 0 < a < x < b,\ x > 0;\ \propto > 0. \tag{3}$$

where $F\,(x,\propto) = e^{-x^{-\propto}}$ can be represented as and the equation (3) can be re-written as

$$g\,(x\,/a < x < b) = \frac{\propto x^{-1-\propto} e^{-x^{-\propto}}}{e^{-b^{-\propto}} - e^{-a^{-\propto}}},\ a < x < b,\ x > 0. \tag{4}$$

The cumulative distribution function for Frechet distribution is represented as follows:

$$G\,(x\,/a < x < b) = \frac{e^{-x^{-\propto}} - G\,(a)}{G\,(b) - G\,(a)},\ a < x < b \tag{5}$$

Mean of the truncated Frechet distribution is given by

$$E(X) = \frac{\Gamma(1-\frac{1}{\propto})}{e^{-b^{-\propto}} - e^{-a^{-\propto}}} \tag{6}$$

The value of $\propto_j$ is estimated by using the method of moment by equating the observed mean with the mean of the truncated Frechet distribution of the j^{th} state, for j = 1, 2, ... , 11. The moment estimates of the parameters of 11 states are shown in Table A.5. For fitting of the truncated Frechet distribution in each state, the chi-square test of goodness of fit is performed and found that the p-values for each state appeared as > 0.05. The graph of state wise observed and expected frequencies is given below. Based on the Figure B.1 we also confirm that the Frechet distribution works well for the monthly rainfall data of each state.

5. Bayes Estimation

The Bayesian method has been applied to assess the parameters of a hydrological model. The Bayesian method also provides an estimate of uncertainty of model parameters by using prior probability distribution of the parameters. Rainfall data contains significant uncertainty, the Bayesian method has been used by several researchers to consolidate rainfall uncertainty in model calibration (Sun *et al.* (2017)). Engeland and Gottschalk (2002) have used Bayesian approach for estimation of parameters in a regional hydrological model for NOPEX area in southern Sweden. Badjana *et al.* (2017) have used Bayesian approach to investigate the long term trend in annual rainfall, annual rainfall duration and annual maximum rainfall for seven stations at Kara river basin, West Africa. The trend analysis was performed by fitting the Log normal, Normal and Generalised extreme value distribution to the annual rainfall data.

The similar type of research work around Bayesian analysis and statistical modeling can be found in, for example, Fortin *et al.* (1997), P.H.A.J.M Van Gelder (1996) and Noortwijk *et al.* (1998). Morita (1993) has applied the Bayesian estimates as the symptomatic tool for the clinical practice. Various priors of the Bayes estimators based on the power law distribution, of the double Gamma-Exponential distribution has the minimum posterior standard error as well as minimum Akaike's Information Criteria (AIC) and Bayesian Information Criteria (BIC) by Sultan *et al.* (2014).

Verma *et al.* (2019) has proved that Bayesian technique is quite helpful if any prior data information is available, which reduces the variability for making the effective clinically meaningful decisions.

In this section Bayes estimates of the parameters of truncated Frechet distribution under squared error loss function are derived for 11 states. The prior distribution for the j^{th} state is considered as exponential distribution with mean θ_j having pdf

$$\pi_j(\propto_j) = \frac{1}{\theta_j} e^{\frac{-\propto_j}{\theta_j}}\ ;\ \alpha_j > 0,\ \theta_j > 0,\ j = 1,2, \dots ,12. \tag{7}$$

That is $\propto_j \sim Exp\ (mean\ \theta_j),\ j = 1,2, \dots ,12.$ (8)

The likelihood function based on the observations $x_{1j}, x_{2j}, \dots x_{n_j j}$ of the j^{th} state is given by

$$L(\underline{x} / \propto_j) = \prod_{i=1}^{n_j} \frac{\propto_j x_{ij}{}^{1-\propto_j} \; e^{-x_{ij}{}^{-\propto_j}}}{e^{-b^{-\propto_j}} - e^{-a^{-\propto_j}}} \tag{9}$$

Using likelihood function and prior distribution, the posterior distribution of θ_j for j^{th} state is obtained as:

$h(\propto_j / \underline{x}) \sim L(\underline{x} / \propto_j)\,\pi(\propto_j)$

$$= \frac{\propto_j{}^{n_j} \prod_{i=1}^{n_j} x_{ij}{}^{-\propto_j} e^{-\sum_{i=1}^{n_j} x_{ij}{}^{-\propto_j}}}{(e^{-b^{-\propto_j}} - e^{-a^{-\propto_j}})^{n_j} \prod_{i=1}^{n_j} x_{ij}} \frac{1}{\theta_j} e^{\frac{-\propto_j}{\theta_j}} \tag{10}$$

$$\sim \frac{\frac{1}{\theta_j} \propto_j{}^{n_j} e^{-(\sum_{i=1}^{n_j} log x_{ij} + \frac{1}{\theta})\propto_j} e^{-\sum_{i=1}^{n_j} x_i{}^{-\propto_j}}}{(e^{-b^{-\propto_j}} - e^{-a^{-\propto_j}})^{n_j}}, \; \theta_j > 0 \tag{11}$$

Under squared error loss function the Bayes estimator of θ_j is nothing but mean of its posterior distribution.

That is, $\widehat{\propto_J}_{Bayes} = E_h(\propto_j / \underline{x}), j = 1,2, \dots ,11.$ (12)

$$\widehat{\propto_J}_{Bayes} = \int_0^\infty \frac{\frac{1}{\theta_j} \propto_j{}^{n_j+1} e^{-(\sum_{i=1}^{j} log x_{ij} + \frac{1}{\theta})\propto} e^{-\sum_{i=1}^{n_j} x_i{}^{-\propto_j}}}{k(e^{-b^{-\propto_j}} - e^{-a^{-\propto_j}})^{n_j}} d\alpha_j \tag{13}$$

where

$$k = \int_0^\infty \frac{\frac{1}{\theta_j} \propto_j{}^{n_j} e^{-(\sum_{i=1}^{n_j} log x_{ij} + \frac{1}{\theta})\propto_j} e^{-\sum_{i=1}^{n_j} x_i{}^{-\propto_j}}}{(e^{-b^{-\propto_j}} - e^{-a^{-\propto_j}})^{n_j}} d\alpha_j$$

is a function of $\underline{x}$, independent of $\propto_j$.

Here Bayes estimate cannot be simplified and obtained in a closed form. So, we use the important sampling method, proposed by Kundu *et al.* (2009).
We rewrite the posterior distribution of $\propto_j$ as

$$h(\widehat{\propto_J}_{Bayes} / \underline{x}) = Gamma\,(n_j + 2, \sum_{i=1}^{n_j} log\, x_{ji} + \frac{1}{\theta_j})\omega(\alpha_j) \tag{14}$$

where $\omega(\alpha_j) = \frac{e^{-\sum_{i=1}^{j} x_i{}^{-\propto}}}{\theta\,(e^{-b^{-\propto}} - e^{-a^{-\propto}})^{n_j}}$ (15)

Using important sampling the Bayes estimates of the $\propto_j$ can be obtained by following algorithm:

Step 1: Generate $\propto_j$ from Gamma ($n_j + 2, \sum_{i=1}^{n_j} log\, x_{ji} + \frac{1}{\beta_j}$) distribution.
Step 2: Repeat the above steps S=1000 times to generate ($\propto_{j1}, \propto_{j2}, \ldots \propto_{jS}$).
Step 3: Compute the S values of $\omega(\alpha_j)$ using the values of $\propto_j$ in Step 2.
Step 4: The Bayes estimate of parameter $\propto_j$ is given by

$$\widehat{\propto_J}_{Bayes} = \frac{\sum_{i=1}^{S} \propto_{ji} * \omega(\alpha_{ji})}{\sum_{i=1}^{S} \omega(\alpha_{ji})}$$

The values of the Bayes estimates of the parameters obtained for all the states are given in Table A.6.

6. Simulation and Prediction

In this section we check the performance of the proposed methods of prediction. We consider a simulation to check whether the simulated results are approximately accurate to the original data or not. To estimate the monthly rainfall, the moment estimates and Bayes estimates of the parameters of the 11 states are used.

6.1. Simulation algorithm

The simulations algorithm steps are mentioned below:

1. Let us consider the initial state as the state observed for the first value of the rainfall data. say j (j = 1, 2, 3, … 11). Generate the uniform random number from uniform distribution U (0, 1), say rnx.
2. To decide the next state, say l, the random value (rnx) is compared with the cumulative transition probabilities of the state j, till the random value (rnx) outstrip the cumulative transition probability of the state.
3. Let us consider the relevant values of a parameter for l-th state from Table A.6.
4. Insert the value of a parameter in the cumulative distribution function of truncated Frechet distribution.

$$F(x \,/\, a_j < x < b_j) = \frac{\propto_j x^{-1-\propto_j} e^{-x^{-\propto_j}}}{e^{-b^{-\propto_j}} - e^{-a^{-\propto_j}}},\ \mathrm{j} = 1,2, \ldots, 11. \tag{16}$$

Here (a_j, b_j) are the lower and upper limits of the j^{th} state respectively. Replace $F(x \,/\, a_j < x < b_j)$ by the random number between 0 to 1 in Equation (16).

5. Solving the Equation (16) we get the estimate of rainfall for next month.
6. Continue the step 1 to step 6 by considering initial state j=l till we have 1218 estimated rainfall values.

In similar manner, prediction for future monthly rainfall is being done using the above steps considering the initial state j as the state of the last rainfall value of the data. The simulation is continuing for next 5000 and 10,000 months. The estimation is carried out for the monthly rainfall of the Hillsborough County of the same period from January 1915 to June 2016 under the proposed methods. The average rainfall obtained from both the methods reflect almost close to each other.

The simulated results obtained from moment estimates and Bayes estimates, the descriptive statistics (minimum, maximum, average and standard deviation) are presented in Table A.7. A comparison of state wise frequencies obtained through the proposed methods is made with frequencies obtained based on the actual data. The results are shown in the Table A.8 and Table A.9. The prediction is being carried out for the future months. Using 101 years of monthly rainfall data of Hillsborough County the next 5000 and 10,000 months of rainfall can be predicted through method of moments and Bayes estimation.

A prediction is being made for number of months and percentage for the rainfall, higher than 0.50mm, 2.00mm, 4.00mm, 7.00mm, 9.00mm and 11.00mm as well as the numbers of months having rainfall below 0.50mm, 2.00mm, 4.00mm, 7.00mm, 9.00mm and 11.00mm of Hillsborough County (refer Table A.10 and Table A.11).

7. Results and Discussion

The state wise frequency obtain from the method of moments (MOM) and Bayes estimation are almost near to the actual data. Prediction made by method of moments is almost similar to the results obtained under the Bayes estimation. The prediction done under the proposed methods are near to actual value which reveals that the prediction based on truncated Frechet distribution under the Markov model is appropriate.

Based on Table A.8, The state wise frequency and percentage results achieved thorough both these methods are completely identical to each other. The outcome obtained through method of moments and Bayes estimation for most of the states are very much similar in frequency and percentage values of the observed data.

The Table A.10 and Table A.11 exhibits that there are approximately 95% chances of having < 11.00mm rainfall during the next 5,000 and 10,000 months. In a similar way there are only around 5% probability of having > 11.00mm rainfall during the next 5,000 and 10,000 days.

8. Conclusion

In this paper we have analysed monthly rainfall data of Hillsborough County at United states of Florida. The overall data is divided into 11 states. We have applied the truncated Frechet distribution for the monthly rainfall data for each state. Two types of approaches have been jointly used *viz*: 1. Markov chain model and 2. Distribution theory approach. To estimate the parameters of the distribution we have used method of moments and method of Bayes estimation. In case of the Bayes estimation important sampling is used to estimate the parameters. Simulation study is considered to judge the performance of the proposed methods and for prediction of future monthly rainfall. The models work good for estimation and prediction of the monthly rainfall. This work may be useful to water resource management to take precautionary steps in advance on the basis of future predictions.

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Appendix A

Table A.1: States for rainfall of Hillsborough

States	Rainfall
1	0.00--0.50
2	0.51--1.00
3	1.01--1.50
4	1.51--2.00
5	2.01--3.00
6	3.01--4.00
7	4.01--5.50
8	5.51--7.00
9	7.01--9.00
10	9.01--11.00
11	11.01--19.06

Table A.2: Transition frequency matrix for rainfall of Hillsborough

	Transition frequency											Row
i \ j	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**Total**
1	10	9	13	5	14	13	6	5	4	0	0	79
2	11	9	14	6	12	14	7	9	7	2	2	93
3	8	16	12	19	14	13	10	5	3	0	1	101
4	8	13	7	15	19	12	14	7	6	2	0	103
5	15	17	14	13	30	23	19	11	9	6	5	162
6	8	12	11	13	18	16	18	19	6	7	5	133
7	7	9	10	17	16	13	17	17	21	8	8	143
8	7	4	6	4	16	13	23	21	21	13	10	138
9	2	2	9	7	12	9	12	22	32	21	7	135
10	0	0	1	1	8	8	10	10	18	18	9	83
11	3	9	4	2	3	2	6	5	8	5	6	53

Table A.3: Transition probability matrix (TPM) for rainfall of Hillsborough

	Transition probability										
$i \times j$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	0.1266	0.1139	0.1646	0.0633	0.1772	0.1646	0.0759	0.0633	0.0506	0.0000	0.0000
2	0.1183	0.0968	0.1505	0.0645	0.1290	0.1505	0.0753	0.0968	0.0753	0.0215	0.0215
3	0.0792	0.1584	0.1188	0.1881	0.1386	0.1287	0.0990	0.0495	0.0297	0.0000	0.0099
4	0.0777	0.1262	0.0680	0.1456	0.1845	0.1165	0.1359	0.0680	0.0583	0.0194	0.0000
5	0.0926	0.1049	0.0864	0.0802	0.1852	0.1420	0.1173	0.0679	0.0556	0.0370	0.0309
6	0.0602	0.0902	0.0827	0.0977	0.1353	0.1203	0.1353	0.1429	0.0451	0.0526	0.0376
7	0.0490	0.0629	0.0699	0.1189	0.1119	0.0909	0.1189	0.1189	0.1469	0.0559	0.0559
8	0.0507	0.0290	0.0435	0.0290	0.1159	0.0942	0.1667	0.1522	0.1522	0.0942	0.0725
9	0.0148	0.0148	0.0667	0.0519	0.0889	0.0667	0.0889	0.1630	0.2370	0.1556	0.0519
10	0.0000	0.0000	0.0120	0.0120	0.0964	0.0964	0.1205	0.1205	0.2169	0.2169	0.1084
11	0.0566	0.1698	0.0755	0.0377	0.0566	0.0377	0.1132	0.0943	0.1509	0.0943	0.1132

Table A.4: Transition cumulative probability matrix for rainfall of Hillsborough

$i \times j$	Transition cumulative probability										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	0.1266	0.2405	0.4051	0.4684	0.6456	0.8101	0.8861	0.9494	1.0000	1.0000	1.0000
2	0.1183	0.2151	0.3656	0.4301	0.5591	0.7097	0.7849	0.8817	0.9570	0.9785	1.0000
3	0.0792	0.2376	0.3564	0.5446	0.6832	0.8119	0.9109	0.9604	0.9901	0.9901	1.0000
4	0.0777	0.2039	0.2718	0.4175	0.6019	0.7184	0.8544	0.9223	0.9806	1.0000	1.0000
5	0.0926	0.1975	0.2840	0.3642	0.5494	0.6914	0.8086	0.8765	0.9321	0.9691	1.0000
6	0.0602	0.1504	0.2331	0.3308	0.4662	0.5865	0.7218	0.8647	0.9098	0.9624	1.0000
7	0.0490	0.1119	0.1818	0.3007	0.4126	0.5035	0.6224	0.7413	0.8881	0.9441	1.0000
8	0.0507	0.0797	0.1232	0.1522	0.2681	0.3623	0.5290	0.6812	0.8333	0.9275	1.0000
9	0.0148	0.0296	0.0963	0.1481	0.2370	0.3037	0.3926	0.5556	0.7926	0.9481	1.0000
10	0.0000	0.0000	0.0120	0.0241	0.1205	0.2169	0.3373	0.4578	0.6747	0.8916	1.0000
11	0.0566	0.2264	0.3019	0.3396	0.3962	0.4340	0.5472	0.6415	0.7925	0.8868	1.0000

Table A.5: Moment estimates of $\propto_j$ for each state

State	Moment Estimates ($\hat{\propto}$)
1	1.00024008
2	1.00486095
3	1.00532035
4	1.00425636
5	1.01327133
6	1.00793604
7	1.01226263
8	1.00726517
9	1.00860501
10	1.00545760
11	1.04569409

Table A.6: Bayes estimates of $\propto_j$ for each state

State	Bayes Estimates ($\hat{\propto}$)
1	1.001338
2	1.005927
3	1.006125
4	1.002706
5	1.053751
6	1.009213
7	1.049640
8	1.008237
9	1.010347
10	1.006579
11	1.073141

Table A.7: Descriptive statistics for observed and simulated results

Statistics	Observed	Simulations using Moment estimates			Simulations using Bayes estimates		
	N=1218	N=1218	N=5000	N=10,000	N=1218	N=5000	N=10,000
Minimum rainfall (mm)	0.00	0.11	0.09	0.09	0.11	0.09	0.09
Maximum rainfall (mm)	19.06	19.05	19.06	19.06	19.05	19.06	19.06
Average rainfall (mm)	4.38	4.60	4.70	4.63	4.60	4.70	4.63
Standard deviation of rainfall (mm)	3.39	4.27	4.40	4.27	4.27	4.40	4.27

Table A.8: Observed and estimated frequencies based on method of moments and Bayes estimates

Sr. No.	State	Actual data	Method of Moments		
		N=1218	N=1218	N=5000	N=10,000
1	0.0-0.5	79 (6.49%)	81 (6.65%)	355 (7.10%)	676 (6.76%)
2	0.51-1.0	93 (7.64%)	102 (8.37%)	405 (8.10%)	824 (8.24%)
3	1.01-1.50	101 (8.29%)	94 (7.72%)	414 (8.28%)	815 (8.15%)
4	1.51-2.0	103 (8.46%)	105 (8.62%)	418 (8.36%)	840 (8.40%)
5	2.01-3.00	162 (13.30%)	168 (13.79%)	662 (13.24%)	1350 (13.50%)
6	3.01-4.00	133 (10.92%)	155 (12.73%)	563 (11.26%)	1172 (11.72%)
7	4.01-5.50	143 (11.74%)	141 (11.58%)	593 (11.86%)	1164 (11.64%)
8	5.51-7.00	138 (11.33%)	117 (9.61%)	509 (10.18%)	1041 (10.41%)
9	7.01-9.00	135 (11.08%)	123 (10.10%)	526 (10.52%)	1052 (10.52%)
10	9.01-11.00	78 (6.40%)	78 (6.40%)	302 (6.04%)	619 (6.19%)
11	11.01-19.06	53 (4.35%)	54 (4.43%)	253 (5.06%)	447 (4.47%)

Table A.9: Observed and estimated frequencies based on Bayes estimates

Sr. No.	State	Actual data	Bayes		
		N=1218	N=1218	N=5000	N=10,000
1	0.00-0.50	79 (6.49%)	81 (6.65%)	355 (7.10%)	676 (6.76%)
2	0.51-1.00	93 (7.64%)	102 (8.37%)	405 (8.10%)	824 (8.24%)
3	1.01-1.50	101 (8.29%)	94 (7.72%)	414 (8.28%)	815 (8.15%)
4	1.51-2.00	103 (8.46%)	105 (8.62%)	418 (8.36%)	840 (8.40%)
5	2.01-3.00	162 (13.30%)	168 (13.79%)	662 (13.24%)	1350 (13.50%)
6	3.01-4.00	133 (10.92%)	155 (12.73%)	563 (11.26%)	1172 (11.72%)
7	4.01-5.50	143 (11.74%)	141 (11.58%)	593 (11.86%)	1164 (11.64%)
8	5.51-7.00	138 (11.33%)	117 (9.61%)	509 (10.18%)	1041 (10.41%)
9	7.01-9.00	135 (11.08%)	123 (10.10%)	526 (10.52%)	1052 (10.52%)
10	9.01-11.00	78 (6.40%)	78 (6.40%)	302 (6.04%)	619 (6.19%)
11	11.01-19.06	53 (4.35%)	54 (4.43%)	253 (5.06%)	447 (4.47%)

Table A.10: Frequency of simulated observations with different ranges based on method of moments

Sr. No.	Rainfall (mm)	Method of Moments		
		N=1218	N=5000	N=10,000
1	<0.50	81 (6.65%)	355 (7.10%)	676 (6.76%)
2	<2.00	382 (31.36%)	1592 (31.84%)	3155 (31.55%)
3	<4.00	705 (57.88%)	2817 (56.34%)	5677 (56.77%)
4	<7.00	963 (79.06%)	3919 (78.38%)	7882 (78.82%)
5	<9.00	1086 (89.16%)	4445 (88.90%)	8934 (89.34%)
6	<11.00	1164 (95.57%)	4747 (94.94%)	9553 (95.53%)
7	>0.50	1137 (93.35%)	4645 (92.90%)	9324 (93.24%)
8	>2.00	836 (68.64%)	3408 (68.16%)	6845 (68.45%)
9	>4.00	513 (42.12%)	2183 (43.66%)	4323 (43.23%)
10	>7.00	255 (20.94%)	1081 (21.62%)	2118 (21.18%)
11	>9.00	132 (10.84%)	555 (11.10%)	1066 (10.66%)
12	>11.00	54 (4.43%)	253 (5.06%)	447 (4.47%)

Table A.11: Frequency of simulated observations with different ranges based on Bayes estimates

Sr. No.	Rainfall (mm)	Bayes estimates		
		N=1218	N=5000	N=10,000
1	<0.50	81 (6.65%)	355 (7.10%)	676 (6.76%)
2	<2.00	382 (31.36%)	1592 (31.84%)	3155 (31.55%)
3	<4.00	705 (57.88%)	2817 (56.34%)	5677 (56.77%)
4	<7.00	963 (79.06%)	3919 (78.38%)	7882 (78.82%)
5	<9.00	1086 (89.16%)	4445 (88.90%)	8934 (89.34%)
6	<11.00	1164 (95.57%)	4747 (94.94%)	9553 (95.53%)
7	>0.50	1137 (93.35%)	4645 (92.90%)	9324 (93.24%)
8	>2.00	836 (68.64%)	3408 (68.16%)	6845 (68.45%)
9	>4.00	513 (42.12%)	2183 (43.66%)	4323 (43.23%)
10	>7.00	255 (20.94%)	1081 (21.62%)	2118 (21.18%)
11	>9.00	132 (10.84%)	555 (11.10%)	1066 (10.66%)
12	>11.00	54 (4.43%)	253 (5.06%)	447 (4.47%)

Appendix B

Table B.1: State wise observed and expected frequency of monthly rainfall data of Hillsborough County