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Estimation of the Finite Population Mean in Stratified Random Sampling under Non-response

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Abstract

In this paper we have considered the problem of estimating the population mean using auxiliary information in presence of non-response. This situation is examined under two cases; Case I: when non-response occurs both on study variable and auxiliary variable and population mean of the auxiliary variable is known; Case II: when non-response occurs only on study variable, complete information on auxiliary variable and population mean of auxiliary variable is known. Mathematical properties of proposed estimators such as bias, mean square error and minimum mean square error are separately obtained for both the cases of all the proposed estimators up to the first order of approximation. The proposed estimators have been compared theoretically with the Hansen and Hurwitz (1946) estimator and some other existing estimators. The conditions for which proposed estimators are most efficient are obtained. Moreover, numerical illustrations shows that the proposed estimators perform better than existing estimators in terms of mean square error.

Key words: Non-response; Stratified random sampling; Auxiliary information; Mean square error; Bias; Efficiency.

1. Introduction

In sample surveys, survey statisticians are expected to gather information on each unit of the selected sample in order to provide a precise estimate of the population mean. In many circumstances, information on part of the sample units cannot be collected in the first attempt due to natural interference. The failure of some of the sample units causes the errors to be classified as non-response. Hansen and Hurwitz (1946) introduced a method of sub-sampling to deal with the non-respondents and employed it in a more expensive way using the second trial. They considered two attempts (i) mail questionnaire, and (ii) personal interview to obtain the information from the sample and tried to give the appropriate inference about the population parameter. Later, various authors such as Cochran (1977), Rao (1986), Khare and Srivastava (1997), Singh and Kumar (2008, 2009), Singh and Vishwakarma (2019), Kumar *et al*. (2022) discussed the problem of estimating the population mean of the study variable using information on an auxiliary variable in the presence of non-response following the Hansen and Hurwitz (1946) technique under the simple random sampling without replacement (*SRSWOR*) scheme. When the population units are homogeneous, the *SRSWOR* sampling strategy is usually utilized. However, in practice heterogeneous populations are also commonly encountered. In such cases, stratified random sampling is used. With this in mind,

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Chaudhary *et al*. (2009) investigated non-response in stratified random sampling, assuming that non-response happens only on the study variable. Sanaullah *et al*. (2015), Saleem *et al*. (2018), Onyeka *et al*. (2019), Shabbir *et al*. (2019) and Wani *et al*. (2021) have studied the problem of non-response in stratified single and two-phase sampling where non-response occurs on both the study and auxiliary variable, as well as on the study variable only. In this article, we attempted to propose estimators for estimating the population mean of the study variable Y using information on the auxiliary variable X in the presence of non-response for two cases. Case I occurs when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known, whereas Case II occurs when there is non-response on only the study variable Y, and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known. The mathematical properties of proposed estimators, such as bias, mean square error, and minimum mean square error, were examined using large sample approximation. The proposed estimators have been shown to outperform all other estimators tested in the literature. Numerical illustrations have also been done in support of current investigation.

2. Sampling strategy

Consider a finite heterogeneous population of N units organised into L homogenous subgroups termed as strata, with the h^{th} stratum containing N_h units, where h = 1, 2, 3, ..., L and N consists of two mutually exclusive groups, *viz.* response and non-response group. The responding and non-responding units in the h^{th} stratum, respectively, are N_{1h} and N_{2h}. We select a sample of size n_h from N_h units in the stratum by using *SRSWOR* and assume that n_{1h} units respond and n_{2h} units do not respond. We select a sub-sample of size $r_h = (n_{2h} / k_h ; k_h > 1)$ from n_{2h} non responding units in the h^{th} stratum. Following is the Hansen and Hurwitz (1946) estimator, $\bar{y}_{st}^* = \sum_{h=1}^{L} W_h \bar{y}_h^*$ and $\bar{x}_{st}^* = \sum_{h=1}^{L} W_h \bar{x}_h^*$ be the stratified sample means of y and x respectively in the h^{th} stratum under non-response, where $\bar{y}_h^* = \frac{n_{1h}\bar{y}_{n1h} + n_{2h}\bar{y}_{r2h}}{n_h}$, $\bar{x}_h^* = \frac{n_{1h}\bar{x}_{n1h} + n_{2h}\bar{x}_{r2h}}{n_h}$, and $(\bar{y}_{n1h}, \bar{x}_{n1h})$ and $(\bar{y}_{r2h}, \bar{x}_{r2h})$ are the sample means based on n_{1h} units and r_{2h} units, respectively.

The MSE of $\bar{y}_{st}^*$ and $\bar{x}_{st}^*$ are respectively given by

$$MSE(\bar{y}_{st}^*) = \sum_{h=1}^{L} W_h^2 \left\{ \left(\frac{1}{n_h} - \frac{1}{N_h} \right) S_{hy}^2 + \frac{(k_h - 1)}{n_h} W_{2h} S_{hy(2)}^2 \right\} \tag{1}$$

$$MSE(\bar{x}_{st}^*) = \sum_{h=1}^{L} W_h^2 \left\{ \left(\frac{1}{n_h} - \frac{1}{N_h} \right) S_{hx}^2 + \frac{(k_h - 1)}{n_h} W_{2h} S_{hx(2)}^2 \right\}$$

where S_{hy}^2 and $S_{hy(2)}^2$ are the population mean squares of entire group and non-response group respectively in the h^{th} stratum for the study variable.

3. Useful notations

Following are some notations used for the theoretical development of present investigation:

$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: Population mean of study variable for h^{th} stratum.

$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: Population mean of auxiliary variable for h^{th} stratum.

$\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean of the study variable.

$\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of the auxiliary variable.

$S_{hy}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2$: Population variance of study variable for h^{th} stratum.

$S_{hx}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)^2$: Population variance of auxiliary variable for h^{th} stratum.

$S_{hy(1)}^2 = \frac{1}{N_{1h}-1}\sum_{i=1}^{N_{1h}}(y_{hi} - \bar{Y}_{1h})^2$: Population variance of response group of the study variable for h^{th} stratum

$S_{hx(1)}^2 = \frac{1}{N_{1h}-1}\sum_{i=1}^{N_{1h}}(x_{hi} - \bar{X}_{1h})^2$: Population variance of response group of the auxiliary variable for h^{th} stratum

$S_{hy(2)}^2 = \frac{1}{N_{2h}-1}\sum_{i=1}^{N_{2h}}(y_{hi} - \bar{Y}_{2h})^2$: Population variance of non-response group of the study variable for h^{th} stratum

$S_{hx(2)}^2 = \frac{1}{N_{2h}-1}\sum_{i=1}^{N_{2h}}(y_{hi} - \bar{X}_{2h})^2$: Population variance of non-response group of the auxiliary variable for h^{th} stratum

$\rho_{hxy} = \frac{S_{hxy}}{S_{hy}S_{hx}}$: Correlation coefficient between the auxiliary and study variables in h^{th} stratum.

$\rho_{hxy(2)} = \frac{S_{hxy(2)}}{S_{hy(2)}S_{hx(2)}}$: Correlation coefficient between the auxiliary and study variables of non-response group in h^{th} stratum.

$f_h = \frac{n_h}{N_h}$: The sampling fraction of h^{th} stratum

And also $\sum_{h=1}^{L} N_h = N$; $\theta_h = \frac{1}{n_h} - \frac{1}{N_h}$

To derive the expressions for the bias, mean square error and minimum mean square error of existing and proposed estimators, we consider the following relative error terms along with their expectations.

3.1. For separate estimators

Relative error terms along with their expectations for separate estimators

$$\xi_{0h}^* = \frac{\bar{y}_h^* - \bar{Y}_h}{\bar{Y}_h}, \qquad \xi_{1h}^* = \frac{\bar{x}_h^* - \bar{X}_h}{\bar{X}_h}, \qquad \xi_{1h} = \frac{\bar{x}_h - \bar{X}_h}{\bar{X}_h},$$

such that $E(\xi_{0h}^*) = E(\xi_{1h}^*) = E(\xi_{1h}) = 0$ and under *SRSWOR*, we have

$$E(\xi_{0h}^{*2}) = \frac{1}{\bar{Y}_h^2}\left[\theta_h S_{hy}^2 + \frac{W_{2h}(k_h - 1)}{n_h} S_{hy(2)}^2\right] = A_h$$

$$E(\xi_{1h}^{*2}) = \frac{1}{\bar{X}_h^2}\left[\theta_h S_{hx}^2 + \frac{W_{2h}(k_h - 1)}{n_h} S_{hx(2)}^2\right] = B_h$$

$$E(\xi_{0h}^* \xi_{1h}^*) = \frac{1}{\bar{Y}_h \bar{X}_h}\left[\theta_h S_{hxy} + \frac{w_{2h}(k_h - 1)}{n_h} S_{hxy(2)}\right] = C_h$$

$$E(\xi_{1h}^2) = \frac{1}{\bar{X}_h^2}\theta_h S_{hx}^2 = D_h \quad , \quad E(\xi_{0h}^* \xi_{1h}) = \frac{1}{\bar{Y}_h \bar{X}_h}\theta_h S_{hxy} = E_h$$

3.2. For combined estimators

Relative error terms along with their expectations for combined estimators

$$\xi_{0st}^* = \frac{\bar{y}_{st}^* - \bar{Y}}{\bar{Y}}, \qquad \xi_{1st}^* = \frac{\bar{x}_{st}^* - \bar{X}}{\bar{X}}, \qquad \xi_{1st} = \frac{\bar{x}_{st} - \bar{X}}{\bar{X}},$$

such that $E(\xi_{0st}^*) = E(\xi_{1st}^*) = E(\xi_{1st}) = 0$, and under *SRSWOR*, we have

$$E(\xi_{0st}^{*2}) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L} W_h^2 \left[\theta_h S_{hy}^2 + \frac{W_{2h}(k_h - 1)}{n_h} S_{hy(2)}^2\right] = A$$

$$E(\xi_{1st}^{*2}) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2 \left[\theta_h S_{hx}^2 + \frac{W_{2h}(k_h - 1)}{n_h} S_{hx(2)}^2\right] = B$$

$$E(\xi_{0st}^* \xi_{1st}^*) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h^2 \left[\theta_h S_{hxy} + \frac{w_{2h}(k_h - 1)}{n_h} S_{hxy(2)}\right] = C$$

$$E(\xi_{1st}^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2\, \theta_h S_{hx}^2 = D \quad , \quad E(\xi_{0st}^* \xi_{1st}) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h^2\, \theta_h S_{hxy} = E$$

4. Existing estimators in the literature

This section gives a brief introduction of some well-known estimators/ classes of estimators from the literature.

For the simple random sampling method, we can mention some important studies in literature when there is a complete information on the study and auxiliary variable for homogenous populations. For estimating the population mean, Cochran (1977) proposed the classical ratio type estimator as

$$t_R = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{2}$$

where $\bar{X}$ refers population mean of the auxiliary variable, $\bar{y}$ and $\bar{x}$ represents the sample means of the study and auxiliary variable respectively

Bhul and Tuteja (1991) are the first to suggest an estimator using the exponential function to estimate the population mean and is given by

$$t_{EX} = \bar{y}exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{3}$$

Motivated by Bahl and Tuteja (1991), Ozel Kadilar (2016) proposed an exponential type estimator as

$$t_O = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\delta_1} exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{4}$$

Motivated by Bahl and Tuteja (1991), Upadhyaya *et al*. (2011) proposed a ratio type exponential estimator and product type exponential estimator and is given as

$$t_{UP1} = \bar{y}exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+(\delta_2-1)\bar{x}}\right] \tag{5}$$

$$t_{UP2} = \bar{y}exp\left[\frac{\bar{x}-\bar{X}}{\bar{X}+(\delta_3-1)\bar{x}}\right] \tag{6}$$

Motivated by Bahl and Tuteja (1991), Vishwakarma *et al*. (2016) proposed exponential type estimator and is given as

$$t_V = \delta_4\bar{y} + (1-\delta_4)\bar{y}exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \tag{7}$$

The mean square error of t_R , t_{EX} , t_{OK} t_{UP1} , t_{UP2} and t_v are given as

$$MSE(t_R) = \theta\bar{Y}^2\left(C_y^2 + C_x^2 - 2C_{xy}\right) \tag{8}$$

$$MSE(t_{EX}) = \theta\bar{Y}^2\left(C_y^2 + \frac{C_x^2}{4} - C_{xy}\right) \tag{9}$$

$$MSE_{min}(t_O) = MSE_{min}(t_{UP1}) = MSE_{min}(t_{UP2}) = MSE_{min}(t_v) = \theta\bar{Y}^2C_y^2\left(1-\rho_{xy}^2\right) \tag{10}$$

where $\theta = \frac{1}{n} - \frac{1}{N}$, $C_y = \frac{S_y}{\bar{Y}}$ and $C_x = \frac{S_x}{\bar{X}}$

Because of the various reasons, the required correct information cannot be obtained completely at all times which is named as a case of non-response. In order to solve this problem, a method is considered and a new technique of sub-sampling the non-respondents is introduced by Hansen and Hurwitz (1946). In various real-life situations, the population under study is heterogeneous, and in that case, we adopt stratified random sampling to obtain precise estimators for the population parameter(s) of the study variable. Considering this fact an attempt was made in this paper to develop some improved estimators in presence of non-response using stratified random sampling. So, the Hansen and Hurwitz (1946) estimator in stratified random sampling is given as

$$\bar{y}^*_{st} = \sum_{h=1}^{L} W_h \bar{y}^*_h \tag{11}$$

The mean square error of $\bar{y}^*_{st}$ is given as

$$MSE(\bar{y}^*) = \sum_{h=1}^{L} W_h^2 A_h = \bar{Y}^2 A \tag{12}$$

The usual separate ratio estimator when non-response occurs both on study variable and auxiliary variable and the population mean of the auxiliary variable is known is given by

$$\bar{y}^*_{SR} = \sum_{h=1}^{L} W_h \frac{\bar{y}^*_h}{\bar{x}^*_h} \bar{X}_h \tag{13}$$

The mean square error of $\bar{y}^*_{SR}$ is given as

$$MSE(\bar{y}^*_{SR}) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 [A_h + B_h - 2C_h] \tag{14}$$

The separate ratio estimator when non-response occurs only on study variable, complete information on auxiliary variable and the population mean of the auxiliary variable is known is given by

$$\bar{y}'_{SR} = \sum_{h=1}^{L} W_h \frac{\bar{y}^*_h}{\bar{x}_h} \bar{X}_h \tag{15}$$

The mean square error of $\bar{y}'_{SR}$ is given as

$$MSE(\bar{y}'_{SR}) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 [A_h + D_h - 2E_h] \tag{16}$$

The usual combined ratio estimator when non-response occurs both on study variable and auxiliary variable and the population mean of the auxiliary variable is known is given by

$$\bar{y}^*_{CR} = \frac{\bar{y}^*_{st}}{\bar{x}^*_{st}} \bar{X} \tag{17}$$

The mean square error $\bar{y}^*_{CR}$ is given as

$$MSE(\bar{y}^*_{CR}) = \bar{Y}^2 [A + B - 2C] \tag{18}$$

The combined ratio estimator when non-response occurs only on study variable, complete information on auxiliary variable and the population mean of the auxiliary variable is known is given by

$$\bar{y}'_{CR} = \frac{\bar{y}^*_{st}}{\bar{x}_{st}}\bar{X} \tag{19}$$

The mean square error of $\bar{y}'_{CR}$ is given as

$$MSE(\bar{y}'_{CR}) = \bar{Y}^2\,[A + D - 2E\,] \tag{20}$$

The following are the stratified modified estimators in presence of non-response developed by Onyeka *et al*. (2019) using known values of coefficient of correlation, kurtosis, and coefficient of variation when non- response occurs both on the study variable and auxiliary variable and the population mean of the auxiliary variable is known.

$$\bar{y}^{*(i)}_{ok} = \sum_{h=1}^{L} W_h \bar{y}^*_h exp\left[\frac{\alpha_h(\bar{X}_h - \bar{x}^*_h)}{\alpha_h(\bar{X}_h - \bar{x}^*_h) + 2\beta_h}\right] \tag{21}$$

The mean square error of $\bar{y}^{*(i)}_{ok}$ is given as

$$MSE\left(\bar{y}^{*(i)}_{ok}\right) = \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\varphi_{hi}^2 B_h - \varphi_{hi}C_h\right] \tag{22}$$

When non-response occurs only on the study variable, complete information on the auxiliary variable and the population mean of the auxiliary variable is known the Onyeka *et al*. (2019) estimators are

$$\bar{y}'^{(i)}_{ok} = \sum_{h=1}^{L} W_h \bar{y}^*_h exp\left[\frac{\alpha_h(\bar{X}_h - \bar{x}_h)}{\alpha_h(\bar{X}_h - \bar{x}_h) + 2\beta_h}\right] \tag{23}$$

The mean square error of $\bar{y}'^{(i)}_{ok}$ is given as

$$MSE\left(\bar{y}'^{(i)}_{ok}\right) = \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\varphi_{hi}^2 D_h - \varphi_{hi}E_h\right] \tag{24}$$

where

$$\varphi_{hi} = \frac{\alpha_h\bar{X}_h}{\alpha_h\bar{X}_h + \beta_h}$$

$$\varphi_{h1} = 1, \quad \varphi_{h2} = \frac{\bar{X}_h}{\bar{X}_h + C_{(x)h}}, \quad \varphi_{h3} = \frac{\bar{X}_h}{\bar{X}_h + \beta_{2(x)h}}, \quad \varphi_{h4} = \frac{C_{(x)h}\bar{X}_h}{C_{(x)h}\bar{X}_h + \rho_{yxh}}$$

$$\varphi_{h5} = \frac{\beta_{2(x)h}\bar{X}_h}{\beta_{2(x)h}\bar{X}_h + C_{(x)h}}, \quad \varphi_{h6} = \frac{\bar{X}_h}{\bar{X}_h + \rho_{yxh}}, \quad \varphi_{h7} = \frac{\rho_{yxh}\bar{X}_h}{\rho_{yxh}\bar{X}_h + \beta_{2(x)h}}, \quad \varphi_{h8} = 0$$

5. Proposed estimators

In this section we propose exponential estimators for estimating the population mean in presence of non-response under stratified random sampling motivated from Upadhyaya *et al*. (2011), Vishwakarma *et al*. (2016) and Ozel Kadilar (2016). The situation of non-response is examined under two cases; Case I: when non-response occurs both on study variable and auxiliary variable and population mean of the auxiliary variable is known; Case II: when non-response occurs only on study variable, complete information on auxiliary variable and population mean of auxiliary variable is known.

Proposed Estimator 1: Based on Upadhyaya *et al*. (2011), we propose a ratio type exponential estimator for estimating the population mean in stratified random sampling in presence of non-response for case I and case II.

Case I: When non-response occurs both on the study variable and auxiliary variable and population mean of the auxiliary variable is known. The proposed estimator t_1^* is given as

$$t_1^* = \bar{y}_{st}^* exp\left[\frac{\bar{X} - \bar{x}_{st}^*}{\bar{X} + (a-1)\bar{x}_{st}^*}\right] \tag{25}$$

Now, we express the equation (25) in terms of ξ_{0st}^* and ξ_{1st}^* up to first order of approximation and is given as

$$t_1^* = \bar{Y}(1+\xi_{0st}^*)\left[\frac{\bar{X} - \bar{X}(1+\xi_{1st}^*)}{\bar{X} + (a-1)\bar{X}(1+\xi_{1st}^*)}\right]$$

$$t_1^* = \bar{Y}(1+\xi_{0st}^*)\left[\frac{-\xi_{1st}^*}{a}\left\{1+\frac{(a-1)}{a}\xi_{1st}^*\right\}^{-1}\right]$$

$$t_1^* - \bar{Y} = \bar{Y}\left[\xi_{0st}^* - \frac{\xi_{1st}^*}{a} + \frac{\xi_{1st}^{*2}}{a^2}\left(a-\frac{1}{2}\right) - \frac{\xi_{0st}^*\xi_{1st}^*}{a}\right] \tag{26}$$

Taking expectation in equation (26), the bias of t_1^* to first order of approximation is given as

$$E(t_1^* - \bar{Y}) = \bar{Y}\left[E(\xi_{0st}^*) - \frac{E(\xi_{1st}^*)}{a} + \frac{E(\xi_{1st}^{*2})}{a^2}\left(a-\frac{1}{2}\right) - \frac{E(\xi_{0st}^*\xi_{1st}^*)}{a}\right]$$

$$E(t_1^* - \bar{Y}) = \bar{Y}\left[\frac{E(\xi_{1st}^{*2})}{a^2}\left(a-\frac{1}{2}\right) - \frac{E(\xi_{0st}^*\xi_{1st}^*)}{a}\right]$$

$$Bias(t_1^*) = \frac{\bar{Y}}{a}\left[\frac{(2a-1)}{2a}B - C\right] \tag{27}$$

Squaring up to first order of approximation and then taking expectation in equation (26), The *MSE* of t_1^* is given as

$$E(t_1^* - \bar{Y})^2 = \bar{Y}^2 E\left[\xi_{0st}^* - \frac{\xi_{1st}^*}{a} + \frac{\xi_{1st}^{*2}}{a^2}\left(a-\frac{1}{2}\right) - \frac{\xi_{0st}^*\xi_{1st}^*}{a}\right]^2$$

$$E(t_1^* - \bar{Y})^2 = \bar{Y}\left[E(\xi_{0st}^{*2}) + \frac{E(\xi_{1st}^{*2})}{a^2} - 2\frac{1}{a}E(\xi_{0st}^*\xi_{1st}^*)\right]$$

$$MSE(t_1^*) = \bar{Y}^2\left[A + \frac{1}{a^2}B - 2\frac{1}{a}C\right] \tag{28}$$

For obtaining the optimal values of a , differentiating equation (28) w.r.t a and equating to zero we have

$$\frac{\partial MSE(t_1^*)}{\partial\, a} = 0$$

$$a_{opt} = \frac{B}{C}$$

Substituting the optimal value of a in equation (28), we have MSE as

$$MSE(t_1^*) = \bar{Y}^2\left[A + \frac{1}{\left(\frac{B}{C}\right)^2}B - 2\frac{1}{\left(\frac{B}{C}\right)}C\right] \tag{29}$$

Simplifying equation (29), we have the minimum mean square error of the proposed estimator t_1^*

$$MSE(t_1^*)_{min} = \bar{Y}^2\left[A - \frac{C^2}{B}\right] \tag{30}$$

Case II: When non-response occurs only on study variable, complete information on auxiliary variable and population mean of the auxiliary variable is known. The proposed estimator t_1' is given as

$$t_1' = \bar{y}_{st}^* exp\left[\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + (a' - 1)\bar{x}_{st}}\right] \tag{31}$$

Now, we express the equation (31) in terms of ξ_{0st}^* and ξ_{1st} up to first order of approximation and is given as

$$t_1' = \bar{Y}(1 + \xi_{0st}^*)\left[\frac{\bar{X} - \bar{X}(1 + \xi_{1st})}{\bar{X} + (a' - 1)\bar{X}(1 + \xi_{1st})}\right]$$

$$t_1' = \bar{Y}(1 + \xi_{0st}^*)\left[\frac{\xi_{1st}}{a'}\left\{1 + \frac{(a' - 1)}{a'}^{-1}\xi_{1st}\right\}\right]$$

$$t_1' - \bar{Y} = \bar{Y}\left[\xi_{0st}^* - \frac{\xi_{1st}}{a'} + \frac{\xi_{1st}^2}{a'^2}\left(a' - \frac{1}{2}\right) - \frac{\xi_{0st}^*\xi_{1st}}{a'}\right] \tag{32}$$

Taking expectation in equation (32), the bias of t_1' to first order of approximation to get bias and is given as

$$E(t_1' - \bar{Y}) = \bar{Y}\left[E(\xi_{0st}^*) - \frac{E(\xi_{1st})}{a'} + \frac{E(\xi_{1st}^2)}{a'^2}\left(a' - \frac{1}{2}\right) - \frac{E(\xi_{0st}^*\xi_{1st})}{a'}\right]$$

$$E(t_1' - \bar{Y}) = \bar{Y}\left[\frac{E(\xi_{1st}^2)}{a'^2}\left(a' - \frac{1}{2}\right) - \frac{E(\xi_{0st}^*\xi_{1st})}{a'}\right]$$

$$Bias(t_1') = \frac{\bar{Y}}{a'}\left[\frac{(2a'-1)}{2a'}D - E\right] \quad (33)$$

Squaring up to first order of approximation and then taking expectation in equation (32), The *MSE* of t_1' is given as

$$E(t_1' - \bar{Y})^2 = \bar{Y}^2 E\left[\xi_{0st}^* - \frac{\xi_{1st}}{a'} + \frac{\xi_{1st}^2}{a'^2}\left(a' - \frac{1}{2}\right) - \frac{\xi_{0st}^*\xi_{1st}}{a'}\right]^2$$

$$E(t_1' - \bar{Y})^2 = \bar{Y}\left[E(\xi_{0st}^{*2}) + \frac{E(\xi_{1st}^2)}{a'^2} - 2\frac{1}{a'}E(\xi_{0st}^*\xi_{1st})\right]$$

$$MSE(t_1') = \bar{Y}^2\left[A + \frac{1}{a'^2}D - 2\frac{1}{a'}E\right] \quad (34)$$

For obtaining the optimal values of a' , differentiating equation (34) w.r.t a' and equating to zero we have

$$\frac{\partial MSE(t_1')}{\partial a'} = 0$$

$$a'_{opt} = \frac{D}{E}$$

Substituting the optimal value of a' in equation (34), we have *MSE* as

$$MSE(t_1') = \bar{Y}^2\left[A + \frac{1}{\left(\frac{D}{E}\right)^2}D - 2\frac{1}{\left(\frac{D}{E}\right)}E\right] \quad (35)$$

Simplifying equation (35), we have the minimum mean square error of the proposed estimator t_1'

$$MSE(t_1')_{min} = \bar{Y}^2\left[A - \frac{E^2}{D}\right] \quad (36)$$

Proposed Estimator 2: Based on Upadhyaya *et al*. (2011), we propose a product type exponential estimator for estimating the population mean in stratified random sampling in presence of non–response for case I and case II.

Case 1: When non-response occurs both on the study variable and auxiliary variable and population mean of the auxiliary variable is known. The proposed estimator t_2^* is given as

$$t_2^* = \bar{y}_{st}^* exp\left[\frac{\bar{x}_{st}^* - \bar{X}}{\bar{X} + (b-1)\bar{x}_{st}^*}\right] \quad (37)$$

Now, we express the equation (37) in terms of ξ_{0st}^* and ξ_{1st}^* up to first order of approximation and is given as

$$t_2^* = \bar{Y}(1+\xi_{0st}^*)\left[\frac{\bar{X}(1+\xi_{1st}^*)-\bar{X}}{\bar{X}+(b-1)\bar{X}(1+\xi_{1st}^*)}\right]$$

$$t_2^* = \bar{Y}(1+\xi_{0st}^*)\left[\frac{\xi_{1st}^*}{b}\left\{1+\frac{(b-1)}{b}^{-1}\xi_{1st}^*\right\}\right]$$

$$t_2^* - \bar{Y} = \bar{Y}\left[\xi_{0st}^* + \frac{\xi_{1st}^*}{b} - \frac{\xi_{1st}^{*2}}{b^2}\left(b-\frac{3}{2}\right) + \frac{\xi_{0st}^*\xi_{1st}^*}{b}\right] \tag{38}$$

Taking expectation in equation (38), the bias of t_2^* to first order of approximation is given as

$$E(t_2^* - \bar{Y}) = \bar{Y}\left[E(\xi_{0st}^*) + \frac{E(\xi_{1st}^*)}{b} - \frac{E(\xi_{1st}^{*2})}{b^2}\left(b-\frac{3}{2}\right) + \frac{E(\xi_{0st}^*\xi_{1st}^*)}{b}\right]$$

$$E(t_2^* - \bar{Y}) = \bar{Y}\left[\frac{E(\xi_{0st}^*\xi_{1st}^*)}{b} - \frac{E(\xi_{1st}^{*2})}{b^2}\left(b-\frac{3}{2}\right)\right]$$

$$Bias(t_2^*) = \frac{\bar{Y}}{b}\left[C - \frac{(2b-3)}{2b}B\right] \tag{39}$$

Squaring up to first order of approximation and then taking expectation in equation (38), The *MSE* of t_2^* is given as

$$E(t_2^* - \bar{Y})^2 = \bar{Y}^2 E\left[\xi_{0st}^* + \frac{\xi_{1st}^*}{b} - \frac{\xi_{1st}^{*2}}{b^2}\left(b-\frac{3}{2}\right) + \frac{\xi_{0st}^*\xi_{1st}^*}{b}\right]^2$$

$$E(t_2^* - \bar{Y})^2 = \bar{Y}^2\left[E(\xi_{0st}^{*2}) + \frac{E(\xi_{1st}^{*2})}{b^2} + 2\frac{1}{b}E(\xi_{0st}^*\xi_{1st}^*)\right]$$

$$MSE(t_2^*) = \bar{Y}^2\left[A + \frac{1}{b^2}B + 2\frac{1}{b}C\right] \tag{40}$$

For obtaining the optimal values of b , differentiating equation (40) w.r.t b and equating to zero we have

$$\frac{\partial MSE(t_2^*)}{\partial\, b} = 0$$

$$b_{opt} = -\frac{B}{C}$$

Substituting the optimal value of b in equation (40), we have *MSE* as

$$MSE(t_2^*) = \bar{Y}^2\left[A + \frac{1}{\left(-\frac{B}{C}\right)^2}B + 2\frac{1}{\left(-\frac{B}{C}\right)}C\right] \tag{41}$$

Simplifying equation (41), we have the minimum mean square error of the proposed estimator t_2^*

$$MSE(t_2^*)_{min} = \bar{Y}^2\left[A - \frac{C^2}{B}\right] \tag{42}$$

Case II: When non-response occurs only on study variable, complete information on auxiliary variable and population mean of the auxiliary variable is known. The proposed estimator t_2' is given as

$$t_2' = \bar{y}_{st}^* exp\left[\frac{\bar{x}_{st} - \bar{X}}{\bar{X} + (b' - 1)\bar{x}_{st}}\right] \tag{43}$$

Now, we express the equation (43) in terms of ξ_{0st}^* and ξ_{1st} up to first order of approximation and is given as

$$t_2' = \bar{Y}(1 + \xi_{0st}^*)\left[\frac{\bar{X}(1 + \xi_{1st}) - \bar{X}}{\bar{X} + (b' - 1)\bar{X}(1 + \xi_{1st})}\right]$$

$$t_2' = \bar{Y}(1 + \xi_{0st}^*)\left[\frac{\xi_{1st}}{b'}\left\{1 + \frac{(b-1)}{b'}^{-1}\xi_{1st}\right\}\right]$$

$$t_2' - \bar{Y} = \bar{Y}\left[\xi_{0st}^* + \frac{\xi_{1st}}{b'} - \frac{\xi_{1st}^2}{b'^2}\left(b' - \frac{3}{2}\right) + \frac{\xi_{0st}^*\xi_{1st}}{b'}\right] \tag{44}$$

Taking expectation in equation (44), the bias of t_2' to first order of approximation is given as

$$E(t_2' - \bar{Y}) = \bar{Y}\left[E(\xi_{0st}^*) + \frac{E(\xi_{1st})}{b'} - \frac{E(\xi_{1st}^2)}{b'^2}\left(b' - \frac{3}{2}\right) + \frac{E(\xi_{0st}^*\xi_{1st})}{b'}\right]$$

$$E(t_2' - \bar{Y}) = \bar{Y}\left[\frac{E(\xi_{0st}^*\xi_{1st})}{b'} - \frac{E(\xi_{1st}^2)}{b'^2}\left(b' - \frac{3}{2}\right)\right]$$

$$Bias(t_2') = \frac{\bar{Y}}{b'}\left[E - \frac{(2b' - 3)}{2b'}D\right] \tag{45}$$

Squaring and then taking expectation in equation (44), The *MSE* of t_2' is given as

$$E(t_2' - \bar{Y})^2 = \bar{Y}^2 E\left[\xi_{0st}^* + \frac{\xi_{1st}}{b'} - \frac{\xi_{1st}^2}{b'^2}\left(b' - \frac{3}{2}\right) + \frac{\xi_{0st}^*\xi_{1st}}{b'}\right]^2$$

$$E(t_2' - \bar{Y})^2 = \bar{Y}^2\left[E(\xi_{0st}^{*2}) + \frac{E(\xi_{1st}^2)}{b'^2} + 2\frac{1}{b'}E(\xi_{0st}^*\xi_{1st})\right]$$

$$MSE(t_2') = \bar{Y}^2\left[A + \frac{1}{b'^2}D + 2\frac{1}{b'}E\right] \tag{46}$$

For obtaining the optimal values of b' , differentiating equation (46) w.r.t b' and equating to zero we have

$$\frac{\partial MSE(t_2')}{\partial\, b'}=0$$

$$b'_{opt}=-\frac{D}{E}$$

Substituting the optimal value of b' in equation (46), we have *MSE* as

$$MSE(t_2')=\bar{Y}^2\left[A+\frac{1}{\left(-\frac{D}{E}\right)^2}D+2\frac{1}{\left(-\frac{D}{E}\right)}E\right] \quad (47)$$

Simplifying equation (47), we have the minimum mean square error of the proposed estimator t_2'

$$MSE(t_2')_{min}=\bar{Y}^2\left[A-\frac{E^2}{D}\right] \quad (48)$$

Proposed Estimator 3: Based on Vishwakarma *et al*. (2016), we propose a stratified exponential estimator in presence of non-response for case I and case II.

Case 1: When non-response occurs both on the study variable and auxiliary variable and population mean of the auxiliary variable is known. The proposed estimator t_3^* is given as

$$t_3^*=c\bar{y}_{st}^*+(1-c)\bar{y}_{st}^*exp\left[\frac{\bar{X}-\bar{x}_{st}^*}{\bar{X}+\bar{x}_{st}^*}\right] \quad (49)$$

Now, we express the equation (49) in terms of ξ_{0st}^* and ξ_{1st}^* up to first order of approximation and is given as

$$t_3^*=\bar{Y}(c+c\xi_{0st}^*)+\bar{Y}(1+\xi_{0st}^*-c-c\xi_{0st}^*)exp\left[\frac{-\xi_{1st}^*}{2}\left(1+\frac{\xi_{1st}^*}{2}\right)^{-1}\right]$$

$$t_3^*=\bar{Y}\left(1+\xi_{0st}^*-\frac{\xi_{1st}^*}{2}+\frac{c\xi_{1st}^*}{2}+\frac{3\xi_{1st}^{*2}}{8}-\frac{3c\xi_{1st}^{*2}}{8}-\frac{\xi_{0st}^*\xi_{1st}^*}{2}+\frac{c\xi_{0st}^*\xi_{1st}^*}{2}\right)$$

$$t_3^*-\bar{Y}=\bar{Y}\left(\xi_{0st}^*+\left(\frac{c}{2}-\frac{1}{2}\right)\xi_{1st}^*+\left(\frac{3}{8}-\frac{3c}{8}\right)\xi_{1st}^{*2}+\left(\frac{c}{2}-\frac{1}{2}\right)\xi_{0st}^*\xi_{1st}^*\right) \quad (50)$$

Taking expectation in equation (50), the bias of t_3^* to first order of approximation is given as

$$E(t_3^*-\bar{Y})=\bar{Y}\left(E(\xi_{0st}^*)+\left(\frac{c}{2}-\frac{1}{2}\right)E(\xi_{1st}^*)+\left(\frac{3}{8}-\frac{3c}{8}\right)E(\xi_{1st}^{*2})+\left(\frac{c}{2}-\frac{1}{2}\right)E(\xi_{0st}^*\xi_{1st}^*)\right)$$

$$E(t_3^*-\bar{Y})=\bar{Y}\left(\left(\frac{3}{8}-\frac{3c}{8}\right)E(\xi_{1st}^{*2})+\left(\frac{c}{2}-\frac{1}{2}\right)E(\xi_{0st}^*\xi_{1st}^*)\right)$$

$$Bias(t_3^*)=\bar{Y}\left[\left(\frac{3}{8}-\frac{3c}{8}\right)B+\left(\frac{c}{2}-\frac{1}{2}\right)C\right] \quad (51)$$

Squaring up to first order of approximation and then taking expectation in equation (50), The *MSE* of t_3^* is given as

$$E(t_3^* - \bar{Y})^2 = \bar{Y}^2 E\left(\xi_{0st}^* + \left(\frac{c}{2} - \frac{1}{2}\right)\xi_{1st}^* + \left(\frac{3}{8} - \frac{3c}{8}\right)\xi_{1st}^{*2} + \left(\frac{c}{2} - \frac{1}{2}\right)\xi_{0st}^*\xi_{1st}^*\right)^2$$

$$E(t_3^* - \bar{Y})^2 = \bar{Y}^2\left(E(\xi_{0st}^*) + \left(\frac{c}{2} - \frac{1}{2}\right)^2 E(\xi_{1st}^{*2}) + (c-1)E(\xi_{0st}^*\xi_{1st}^*)\right)^2$$

$$MSE(t_3^*) = \bar{Y}^2\left[A + \left(\frac{c^2}{4} - \frac{c}{2} + \frac{1}{4}\right)B + (c-1)C\right] \tag{52}$$

For obtaining the optimal values of c, differentiating equation (52) w.r.t c and equating to zero we have

$$\frac{\partial MSE(t_3^*)}{\partial\, c} = 0$$

$$c_{opt} = \frac{B - 2C}{B}$$

Substituting the optimal value of c in equation (52), we have *MSE* as

$$MSE(t_3^*) = \bar{Y}^2\left[A + \left(\frac{\left(\frac{B-2C}{B}\right)^2}{4} - \frac{\left(\frac{B-2C}{B}\right)}{2} + \frac{1}{4}\right)B + \left(\left(\frac{B-2C}{B}\right) - 1\right)C\right] \tag{53}$$

Simplifying equation (53), we have the minimum mean square error of the proposed estimator t_3^*

$$MSE_{min}(t_3^*) = \bar{Y}^2\left[A - \frac{C^2}{B}\right] \tag{54}$$

Case II: When non-response occurs only on study variable, complete information on auxiliary variable and population mean of the auxiliary variable is known. The proposed estimator t_3' is given as

$$t_3' = c'\bar{y}_{st}^* + (1-c')\bar{y}_{st}^*\left[\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right] \tag{55}$$

Now, we express the equation (55) in terms of ξ_{0st}^* and ξ_{1st} up to first order of approximation and is given as

$$t_3' = \bar{Y}(c' + c'\xi_{0st}^*) + \bar{Y}(1 + \xi_{0st}^* - c' - c'\xi_{0st}^*)exp\left[\frac{-\xi_{1st}}{2}\left(1 + \frac{\xi_{1st}}{2}\right)^{-1}\right]$$

$$t_3' = \bar{Y}\left(1 + \xi_{0st}^* - \frac{\xi_{1st}}{2} + \frac{c'\xi_{1st}}{2} + \frac{3\xi_{1st}^2}{8} - \frac{3c'\xi_{1st}^2}{8} - \frac{\xi_{0st}^*\xi_{1st}}{2} + \frac{c'\xi_{0st}^*\xi_{1st}}{2}\right)$$

$$t'_3 - \bar{Y} = \bar{Y}\left(\xi^*_{0st} + \left(\frac{c'}{2} - \frac{1}{2}\right)\xi_{1st} + \left(\frac{3}{8} - \frac{3c'}{8}\right)\xi^2_{1st} + \left(\frac{c'}{2} - \frac{1}{2}\right)\xi^*_{0st}\xi_{1st}\right) \tag{56}$$

Taking expectation in equation (56), the bias of t'_3 to first order of approximation is given as

$$E(t'_3 - \bar{Y}) = \bar{Y}\left(E(\xi^*_{0st}) + \left(\frac{c'}{2} - \frac{1}{2}\right)E(\xi_{1st}) + \left(\frac{3}{8} - \frac{3c'}{8}\right)E(\xi^2_{1st}) + \left(\frac{c'}{2} - \frac{1}{2}\right)E(\xi^*_{0st}\xi_{1st})\right)$$

$$E(t'_3 - \bar{Y}) = \bar{Y}\left(\left(\frac{3}{8} - \frac{3c'}{8}\right)E(\xi^2_{1st}) + \left(\frac{c'}{2} - \frac{1}{2}\right)E(\xi^*_{0st}\xi_{1st})\right)$$

$$Bias(t'_3) = \bar{Y}\left[\left(\frac{3}{8} - \frac{3c'}{8}\right)D + \left(\frac{c'}{2} - \frac{1}{2}\right)E\right] \tag{57}$$

Squaring up to first order of approximation and then taking expectation in equation (56), The *MSE* of t'_3 is given as

$$E(t'_3 - \bar{Y})^2 = \bar{Y}^2 E\left(\xi^*_{0st} + \left(\frac{c}{2} - \frac{1}{2}\right)\xi_{1st} + \left(\frac{3}{8} - \frac{3c}{8}\right)\xi^2_{1st} + \left(\frac{c}{2} - \frac{1}{2}\right)\xi^*_{0st}\xi_{1st}\right)^2$$

$$E(t'_3 - \bar{Y})^2 = \bar{Y}^2\left(E(\xi^*_{0st}) + \left(\frac{c}{2} - \frac{1}{2}\right)^2 E(\xi^2_{1st}) + (c-1)E(\xi^*_{0st}\xi_{1st})\right)^2$$

$$MSE(t'_3) = \bar{Y}^2\left[A + \left(\frac{c'^2}{4} - \frac{c'}{2} + \frac{1}{4}\right)D + (c'-1)E\right] \tag{58}$$

For obtaining the optimal values of c' , differentiating equation (58) w.r.t c' and equating to zero we have

$$\frac{\partial MSE(t'_3)}{\partial\, c'} = 0$$

$$c'_{opt} = \frac{D - 2E}{D}$$

Substituting the optimal value of c' in equation (58), we have *MSE* as

$$MSE(t'_3) = \bar{Y}^2\left[A + \left(\frac{\left(\frac{D-2E}{D}\right)^2}{4} - \frac{\left(\frac{D-2E}{D}\right)}{2} + \frac{1}{4}\right)D + \left(\left(\frac{D-2E}{D}\right) - 1\right)E\right] \tag{59}$$

Simplifying equation (59), we have the minimum mean square error of the proposed estimator t'_3

$$MSE_{min}(t'_3) = \bar{Y}^2\left[A - \frac{E^2}{D}\right] \tag{60}$$

Proposed Estimator 4: Based on Ozel Kadilar (2016), we propose a stratified exponential estimator in presence of non-response for case I and case II.

Case 1: When non-response occurs both on the study variable and auxiliary variable and population mean of the auxiliary variable is known. The proposed estimator t_4^* is given as

$$t_4^* = \bar{y}_{st}^* \left(\frac{\bar{x}_{st}^*}{\bar{X}}\right)^d exp\left[\frac{\bar{X} - \bar{x}_{st}^*}{\bar{X} + \bar{x}_{st}^*}\right] \tag{61}$$

Now, we express the equation (61) in terms of ξ_{0st}^* and ξ_{1st}^* up to first order of approximation and is given as

$$t_4^* = \bar{Y}(1 + \xi_{0st}^*)(1 + \xi_{1st}^*)^d exp\left[\frac{\bar{X} - \bar{X}(1 + \xi_{1st}^*)}{\bar{X} + \bar{X}(1 + \xi_{1st}^*)}\right]$$

$$t_4^* = \bar{Y}(1 + \xi_{0st}^*)(1 + \xi_{1st}^*)^d exp\left[\frac{-\xi_{1st}^*}{2}\left(1 + \frac{\xi_{1st}^*}{2}\right)^{-1}\right]$$

$$t_4^* - \bar{Y} = \bar{Y}\left[\left(\frac{d^2}{2} - d + \frac{3}{8}\right)\xi_{1st}^{2*} + \left(d - \frac{1}{2}\right)\xi_{0st}^*\xi_{1st}^* + \xi_{0st}^* - \frac{\xi_{1st}^*}{2} + d\xi_{1st}^*\right] \tag{62}$$

Taking expectation in equation (62), the bias of t_4^* to first order of approximation to get bias and is given as

$$E(t_4^* - \bar{Y}) = \bar{Y}\begin{bmatrix}\left(\frac{d^2}{2} - d + \frac{3}{8}\right)E(\xi_{1st}^{2*}) + \left(d - \frac{1}{2}\right)E(\xi_{0st}^*\xi_{1st}^*) \\ +E(\xi_{0st}^*) - \frac{E(\xi_{1st}^*)}{2} + dE(\xi_{1st}^*)\end{bmatrix}$$

$$E(t_4^* - \bar{Y}) = \bar{Y}\left[\left(\frac{d^2}{2} - d + \frac{3}{8}\right)E(\xi_{1st}^{2*}) + \left(d - \frac{1}{2}\right)E(\xi_{0st}^*\xi_{1st}^*)\right]$$

$$Bias(t_4^*) = \bar{Y}\left[\left(\frac{d^2}{2} - d + \frac{3}{8}\right)B + \left(d - \frac{1}{2}\right)C\right] \tag{63}$$

Squaring up to first order of approximation and then taking expectation in equation (62), The *MSE* of t_4^* is given as

$$E(t_4^* - \bar{Y})^2 = \bar{Y}^2 E\left[\left(\frac{d^2}{2} - d + \frac{3}{8}\right)\xi_{1st}^{2*} + \left(d - \frac{1}{2}\right)\xi_{0st}^*\xi_{1st}^* + \xi_{0st}^* - \frac{\xi_{1st}^*}{2} + d\xi_{1st}^*\right]^2$$

$$E(t_4^* - \bar{Y})^2 = \bar{Y}^2\left[(\xi_{0st}^{*2}) + \left(d^2 - d + \frac{1}{4}\right)E(\xi_{1st}^{2*}) + (2d - 1)E(\xi_{0st}^*\xi_{1st}^*)\right]$$

$$MSE(t_4^*) = \bar{Y}^2\left[A + \left(d^2 - d + \frac{1}{4}\right)B + (2d - 1)C\right] \tag{64}$$

For obtaining the optimal values of d, differentiating equation (64) w.r.t d and equating to zero we have

$$\frac{\partial MSE(t_4^*)}{\partial\, d} = 0$$

$$d_{opt} = \frac{B - 2C}{2B}$$

Substituting the optimal value of d in equation (64), we have MSE as

$$MSE(t_4^*) = \bar{Y}^2\left[A + \left(\left(\frac{B-2C}{2B}\right)^2 - \left(\frac{B-2C}{2B}\right) + \frac{1}{4}\right)B + \left(2\left(\frac{B-2C}{2B}\right) - 1\right)C\right] \quad (65)$$

Simplifying equation (65), we get the minimum mean square error of the proposed estimator t_4^*

$$MSE_{min}(t_4^*) = \bar{Y}^2\left[A - \frac{C^2}{B}\right] \quad (66)$$

Case II: When non-response occurs only on study variable, complete information on auxiliary variable and population mean of the auxiliary variable is known. The proposed estimator t_4' is given as

$$t_4' = \bar{y}_{st}^*\left(\frac{\bar{x}_{st}}{\bar{X}}\right)^{d'}\left[\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right] \quad (67)$$

Now, we express the equation (67) in terms of ξ_{0st}^* and ξ_{1st} up to first order of approximation and is given as

$$t_4' = \bar{Y}(1 + \xi_{0st}^*)(1 + \xi_{1st})^{d'} exp\left[\frac{\bar{X} - \bar{X}(1 + \xi_{1st})}{\bar{X} + \bar{X}(1 + \xi_{1st})}\right]$$

$$t_4' = \bar{Y}(1 + \xi_{0st}^*)(1 + \xi_{1st})^{d'} exp\left[\frac{-\xi_{1st}}{2}\left(1 + \frac{\xi_{1st}}{2}\right)^{-1}\right]$$

$$t_4' - \bar{Y} = \bar{Y}\left[\left(\frac{d'^2}{2} - d' + \frac{3}{8}\right)\xi_{1st}^2 + \left(d' - \frac{1}{2}\right)\xi_{0st}^*\xi_{1st} + \xi_{0st}^* - \frac{\xi_{1st}^*}{2} + d'\xi_{1st}\right] \quad (68)$$

Taking expectation in equation (68), the bias of t_4' to first order of approximation to get bias and is given as

$$E(t_4' - \bar{Y}) = \bar{Y}\left[\begin{array}{c}\left(\frac{d'^2}{2} - d' + \frac{3}{8}\right)E(\xi_{1st}^2) + \left(d' - \frac{1}{2}\right)E(\xi_{0st}^*\xi_{1st}) \\ +E(\xi_{0st}^*) - \frac{E(\xi_{1st})}{2} + d'E(\xi_{1st})\end{array}\right]$$

$$E(t_4' - \bar{Y}) = \bar{Y}\left[\left(\frac{d'^2}{2} - d' + \frac{3}{8}\right)E(\xi_{1st}^2) + \left(d' - \frac{1}{2}\right)E(\xi_{0st}^*\xi_{1st})\right]$$

$$Bias(t_4') = \bar{Y}\left[\left(\frac{d'^2}{2} - d' + \frac{3}{8}\right)D + \left(d' - \frac{1}{2}\right)E\right] \quad (69)$$

Squaring up to first order of approximation and then taking expectation in equation (68), The MSE of t'_4 is given as

$$E(t'_4 - \bar{Y})^2 = \bar{Y}^2 E\left[\left(\frac{d'^2}{2} - d' + \frac{3}{8}\right)\xi_{1st}^2 + \left(d' - \frac{1}{2}\right)\xi_{0st}^*\xi_{1st} + \xi_{0st}^* - \frac{\xi_{1st}}{2} + d'\xi_{1st}\right]^2$$

$$E(t'_4 - \bar{Y})^2 = \bar{Y}^2\left[(\xi_{0st}^{*2}) + \left(d'^2 - d' + \frac{1}{4}\right)E(\xi_{1st}^2) + (2d' - 1)E(\xi_{0st}^*\xi_{1st})\right]$$

$$MSE(t'_4) = \bar{Y}^2\left[A + \left({d'}^2 - d' + \frac{1}{4}\right)D - (2d' - 1)E\right] \tag{70}$$

For obtaining the optimal values of d' , differentiating equation (70) w.r.t d' and equating to zero we have

$$\frac{\partial MSE(t'_{14})}{\partial\, d'} = 0$$

$$d'_{opt} = \frac{D - 2E}{2D}$$

Substituting the optimal value of d' in equation (70), we have MSE as

$$MSE(t'_4) = \bar{Y}^2\left[A + \left(\left(\frac{D - 2E}{2D}\right)^2 - \left(\frac{D - 2E}{2D}\right) + \frac{1}{4}\right)D - \left(2\left(\frac{D - 2E}{2D}\right) - 1\right)E\right] \tag{71}$$

Simplifying equation (71), we have the minimum mean square error of the proposed estimator t'_4

$$MSE_{min}(t'_4) = \bar{Y}^2\left[A - \frac{E^2}{D}\right] \tag{72}$$

Interesting Note: We have proposed four estimators for case I and four estimators for case II having same MSE respectively and are given as

$$MSE_{min}(t_i^*) = \bar{Y}^2\left[A - \frac{C^2}{B}\right]; i = 1 - 4 \tag{73}$$

$$MSE_{min}(t'_i) = \bar{Y}^2\left[A - \frac{E^2}{D}\right]; i = 1 - 4 \tag{74}$$

6. Efficiency comparison

Now we will investigate the efficiencies of $t_i^*, i = 1 - 4$ and $t'_i, i = 1 - 4$ given in equation (73) and (74) with various estimators from the literature.

6.1. Efficiency comparison for case I

Using equations (12), (14), (18), (22) and (73) we find the efficiency conditions of $t_i^*, i = 1 - 4$ as follows

1. t_i^*, $\text{i} = 1 - 4$ Perform better than $\bar{y}^*$ if:

$$MSE(t_i^*) < MSE(\bar{y}^*)$$

$$\bar{Y}^2\left[A - \frac{C^2}{B}\right] < \bar{Y}^2 A$$

$$-\frac{C^2}{B} < 0$$

which is obviously true because $C > 0$ and $B > 0$

2. $t_i^*,$ $\mathrm{i} = 1 - 4$ Perform better than $\bar{y}_{SR}^*$ if:

$$MSE(t_i^*) < MSE(\bar{y}_{SR}^*)$$

$$\bar{Y}^2\left[A - \frac{C^2}{B}\right] < \sum_{h=1}^{L} W_h^2\,[A_h + R_h^2 B_h - 2R_h C_h\,]$$

$$\bar{Y}^2\left[A - \frac{C^2}{B}\right] - \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2[A_h + B_h - 2C_h\,] < 0$$

3. $t_i^*,$ $\mathrm{i} = 1 - 4$ Perform better than $\bar{y}_{CR}^*$ if:

$$MSE(t_i^*) < MSE(\bar{y}_{CR}^*)$$

$$\bar{Y}^2\left[A - \frac{C^2}{B}\right] < \bar{Y}^2[A + B - 2C\,]$$

$$\left[2C - \frac{C^2}{B} - B\right] < 0$$

4. $t_i^*,$ $\mathrm{i} = 1 - 4$ Perform better than $\bar{y}_{ok}^{*(i)}$ if:

$$MSE(t_i^*) < MSE\left(\bar{y}_{ok}^{*(i)}\right)$$

$$\bar{Y}^2\left[A - \frac{C^2}{B}\right] < \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\varphi_{hi}^2 B_h - \varphi_{hi} C_h\,\right]$$

$$\bar{Y}^2\left[A - \frac{C^2}{B}\right] - \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\varphi_{hi}^2 B_h - \varphi_{hi} C_h\,\right] < 0$$

6.2. Efficiency comparison for case II

Using equations (12), (16), (20), (24) and (74) we find the efficiency conditions of $t_i', i = 1 - 4$ as follows.

1. $t_i',\ i = 1 - 4$ Perform better than $\bar{y}^*$ if:

$$MSE(t_i') < MSE(\bar{y}^*)$$

$$\bar{Y}^2\left[A - \frac{E^2}{D}\right] < \bar{Y}^2 A$$

$$-\frac{E^2}{D} < 0$$

which is obviously true because $E > 0$ and $D > 0$

2. $t_i',\ i = 1 - 4$ Perform better than $\bar{y}_{SR}'$ if:

$$MSE(t_i') < MSE(\bar{y}_{SR}')$$

$$\bar{Y}^2\left[A - \frac{E^2}{D}\right] < \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2[A_h + D_h - 2E_h\,]$$

$$\bar{Y}^2\left[A - \frac{E^2}{D}\right] - \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2[A_h + D_h - 2E_h\,] < 0$$

3. $t_i',\ i = 1 - 4$ Perform better than $\bar{y}_{CR}'$ if:

$$MSE(t_i') < MSE(\bar{y}_{CR}')$$

$$\bar{Y}^2\left[A - \frac{E^2}{D}\right] < \bar{Y}^2\sum_{h=1}^{L} W_h^2\,[A + D - 2E\,]$$

$$\left[2E - \frac{E^2}{D} - D\right] < 0$$

4. $t_i',\ i = 1 - 4$ Perform better than $\bar{y}_{ok}'^{(i)}$ if:

$$MSE(t_i') < MSE\left(\bar{y}_{ok}'^{(i)}\right)$$

$$\bar{Y}^2\left[A - \frac{E^2}{D}\right] < \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\varphi_{hi}^2 D_h - \varphi_{hi}E_h\,\right]$$

$$\bar{Y}^2\left[A - \frac{E^2}{D}\right] - \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\varphi_{hi}^2 D_h - \varphi_{hi}E_h\,\right] < 0$$

7. Empirical study

To illustrate the performance of the proposed estimators $t_i^*, i = 1 - 4$ and $t_i', i = 1 - 4$ over other existing estimators using the real data set given as

Data Set Source: Koyuncu and Kadilar (2009)

We consider number of teachers as study variable *(Y)* and number of classes in primary and secondary schools as auxiliary variable *(X)*, for 923 districts at six 6 regions (1) Marmara; (2) Agean; (3) Mediterranean; (4) Central Anatolia; (5) Black Sea; and (6) East and Southeast Anatolia in Turkey in 2007.

Table 1: The descriptive statistics

h	1	2	3	4	5	6
N_h	127	117	103	170	205	201
n_h	31	21	29	38	22	39
S_{hy}	883.84	644.92	1033.40	810.58	403.65	771.72
S_{hx}	555.58	365.46	612.95	458.03	260.85	397.05
$\bar{Y}_h$	703.74	413	573.17	424.66	267.03	393.84
$\bar{X}_h$	498.28	318.33	431.36	311.32	227.2	313.71
ρ_{hxy}	0.979	0.976	0.984	0.983	0.964	0.983
$\mathbf{W_{2h} = 10\%\ Non-response}$						
$S_{hy(2)}$	510.57	386.77	1872.88	1603.3	264.19	497.84
$S_{hx(2)}$	303.92	278.51	960.71	821.29	190.85	287.99
$\rho_{hxy(2)}$	0.9931	0.9871	0.9972	0.9942	0.985	0.9647
$\mathbf{W_{2h} = 20\%\ Non-response}$						
$S_{hy(2)}$	396.77	406.15	1654.40	1333.35	335.83	903.91
$S_{hx(2)}$	244.56	274.42	965.42	680.28	214.49	469.86
$\rho_{hxy(2)}$	0.9898	0.9798	0.9846	0.9940	0.9818	0.9874
$\mathbf{W_{2h} = 30\%\ Non-response}$						
$S_{hy(2)}$	500.26	356.95	1383.70	1193.47	289.41	825.24
$S_{hx(2)}$	284.44	247.63	811.21	631.28	188.30	437.90
$\rho_{hxy(2)}$	0.9739	0.9793	0.9839	0.9904	0.9799	0.9829

Table 2: The percent relative efficiency of the existing and proposed estimators with respect to Hansen and Hurwitz estimator for Case 1

$\mathbf{W_{2h} = 10\%\ Non-response}$					
Estimators	*K*=2	*K*=2.5	*K*=3	K=3.5	*K*=4
$\bar{y}^*$	100	100	100	100	100
$\bar{y}^*_{SR}$	1021.72	1022.33	1019.68	1023.33	1023.73
$\bar{y}^*_{CR}$	1031.44	1023.39	1013.42	1010.73	1005.66
$\bar{y}^{*(1)}_{ok}$	245.18	244.57	243.28	243.58	243.19
$\bar{y}^{*(2)}_{ok}$	244.13	243.52	242.24	242.54	242.14
$\bar{y}^{*(3)}_{ok}$	237.43	236.89	235.70	236.04	235.69
$\bar{y}^{*(4)}_{ok}$	244.56	243.95	242.68	242.98	242.59
$\bar{y}^{*(5)}_{ok}$	245.05	244.44	243.15	243.46	243.06
$\bar{y}^{*(6)}_{ok}$	244.39	243.78	242.50	242.80	242.41

$\bar{y}_{ok}^{*(7)}$	237.27	236.74	235.55	235.89	235.55
$\bar{y}_{ok}^{*(8)}$	1021.71	1022.35	1019.68	1023.33	1023.72
t_i^*; $i = 1 - 4$	2564.04	2644.96	2717.59	2803.98	2884.16
$\mathbf{W_{2h} = 20\%}$ **Non − response**					
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_{SR}^*$	1021.71	1037.16	1040.68	1043.60	1046.05
$\bar{y}_{CR}^*$	1034.36	1043.18	1041.25	1039.66	1038.42
$\bar{y}_{ok}^{*(1)}$	243.05	245.32	222.94	244.78	244.58
$\bar{y}_{ok}^{*(2)}$	242.00	244.26	244.43	243.72	243.52
$\bar{y}_{ok}^{*(3)}$	235.28	237.51	237.24	237.02	236.84
$\bar{y}_{ok}^{*(4)}$	242.43	244.71	244.42	244.18	243.98
$\bar{y}_{ok}^{*(5)}$	242.92	245.29	244.90	244.66	244.46
$\bar{y}_{ok}^{*(6)}$	242.26	244.53	244.23	243.99	243.79
$\bar{y}_{ok}^{*(7)}$	235.13	237.35	237.09	236.87	236.69
$\bar{y}_{ok}^{*(8)}$	1021.71	1037.16	1040.68	1043.60	1046.05
t_i^*; $i = 1 - 4$	2618.32	2748.12	2839.69	2922.43	2997.38
$\mathbf{W_{2h} = 30\%}$ **Non − response**					
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_{SR}^*$	1027.85	1030.21	1032.04	1033.48	1034.66
$\bar{y}_{CR}^*$	1043.22	1040.35	1038.15	1036.42	1035.02
$\bar{y}_{ok}^{*(1)}$	245.42	245.01	244.70	244.45	244.25
$\bar{y}_{ok}^{*(2)}$	244.36	243.95	243.63	243.35	243.18
$\bar{y}_{ok}^{*(3)}$	237.49	237.09	236.78	236.54	236.34
$\bar{y}_{ok}^{*(4)}$	244.80	244.39	244.08	243.84	243.64
$\bar{y}_{ok}^{*(5)}$	245.29	244.88	244.57	244.32	244.12
$\bar{y}_{ok}^{*(6)}$	244.62	244.21	243.90	243.66	243.46
$\bar{y}_{ok}^{*(7)}$	237.33	236.93	236.63	236.39	236.19
$\bar{y}_{ok}^{*(8)}$	1027.85	1030.21	1032.04	1033.48	1034.66
t_i^*; $i = 1 - 4$	2595.87	2664.30	2722.05	2771.39	2814.02

Table 3: The percent relative efficiency of the existing and proposed estimators with respect to Hansen and Hurwitz estimator for Case II

$\mathbf{W_{2h} = 10\%}$ **Non − response**					
Estimators	*K*=2	*K*=2.5	*K*=3	*K*=3.5	*K*=4
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_{SR}'$	407.23	330.47	283.52	253.68	231.73
$\bar{y}_{CR}'$	411.50	333.06	285.29	255.00	232.77
$\bar{y}_{ok}^{\prime(1)}$	199.01	185.15	174.15	166.53	159.97

$\bar{y}_{ok}^{\prime(2)}$	198.44	184.69	173.77	166.20	159.69
$\bar{y}_{ok}^{\prime(3)}$	194.63	181.63	171.25	164.05	157.82
$\bar{y}_{ok}^{\prime(4)}$	198.66	184.87	173.92	166.33	159.80
$\bar{y}_{ok}^{\prime(5)}$	198.94	185.09	174.11	166.48	159.93
$\bar{y}_{ok}^{\prime(6)}$	198.57	184.80	173.86	166.28	159.75
$\bar{y}_{ok}^{\prime(7)}$	194.54	181.56	171.19	163.99	157.77
$\bar{y}_{ok}^{\prime(8)}$	407.23	330.47	283.52	253.68	231.73
t_i'; $i = 1 - 4$	504.55	386.38	320.64	280.77	252.63
$\mathbf{W_{2h} = 20\%\ Non-response}$					
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_{SR}'$	314.60	255.12	221.46	199.80	184.70
$\bar{y}_{CR}'$	316.89	256.47	222.37	200.48	185.23
$\bar{y}_{ok}^{\prime(1)}$	181.76	166.94	156.66	149.12	143.35
$\bar{y}_{ok}^{\prime(2)}$	181.33	166.61	156.40	148.91	143.17
$\bar{y}_{ok}^{\prime(3)}$	178.45	164.43	154.67	147.47	141.95
$\bar{y}_{ok}^{\prime(4)}$	181.50	166.74	156.50	148.99	143.24
$\bar{y}_{ok}^{\prime(5)}$	181.71	166.90	156.63	149.09	143.33
$\bar{y}_{ok}^{\prime(6)}$	181.43	166.68	156.46	148.96	143.21
$\bar{y}_{ok}^{\prime(7)}$	178.38	164.38	154.63	147.44	141.92
$\bar{y}_{ok}^{\prime(8)}$	314.60	255.12	221.46	199.80	184.70
t_i'; $i = 1 - 4$	363.59	282.6541	239.75	213.16	195.08
$\mathbf{W_{2h} = 30\%\ Non-response}$					
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_{SR}'$	269.24	220.20	193.19	176.09	164.30
$\bar{y}_{CR}'$	270.79	221.09	193.80	176.55	164.66
$\bar{y}_{ok}^{\prime(1)}$	170.80	156.24	146.65	139.85	134.78
$\bar{y}_{ok}^{\prime(2)}$	170.45	155.99	146.45	139.69	134.65
$\bar{y}_{ok}^{\prime(3)}$	168.09	154.27	145.11	138.60	133.73
$\bar{y}_{ok}^{\prime(4)}$	170.58	156.09	146.53	139.75	134.70
$\bar{y}_{ok}^{\prime(5)}$	170.75	156.21	146.62	139.83	134.77
$\bar{y}_{ok}^{\prime(6)}$	170.53	156.04	146.50	139.73	134.68
$\bar{y}_{ok}^{\prime(7)}$	168.04	154.23	145.08	138.57	133.70
$\bar{y}_{ok}^{\prime(8)}$	269.24	220.20	193.19	176.09	164.30
t_i'; $i = 1 - 4$	301.25	238.17	205.20	184.93	171.21

Table 2 presents the empirical comparison based on percent relative efficiencies (*PREs*) of the proposed class of combined exponential type of estimators t_1^* , t_2^* , t_3^* and t_4^* and it clearly shows that the proposed estimators are more efficient than the Hansen and

Hurwitz estimator as well as from the other existing estimators taken in literature when non-response occurs both on the study variable and on the auxiliary variable and the population mean of the auxiliary variable is known. The proposed estimators t_1^* , t_2^* , t_3^* and t_4^* are equally efficient. The *PREs* of the proposed estimators $t_i^*, i = 1-4$ at 10% non-response rate and at $K_h = 2$ is (2564.04). Similarly, the *PREs* of the proposed estimators $t_i^*, i = 1-4$ at 20% non-response rate and at $K_h = 2$ is (2618.32) and also, the *PREs* of the proposed estimators $t_i^*, i = 1-4$ at 30% non-response rate and at $K_h = 2$ is (2595.87). Further an increasing trend has been observed in *PREs* with increase in the value of $\mathrm{K_h}$ at 10%, 20% and 30% non-response rates.

Table 3 presents the empirical comparison based on percent relative efficiencies *PRE*s of the proposed class of combined exponential type of estimators t_1' , t_2' , t_3' and t_4' clearly shows that the proposed estimators are more efficient than the Hansen and Hurwitz estimator as well as from the other existing estimators taken in literature when non-response occurs only on the study variable, complete information on the auxiliary variable and the population mean of the auxiliary variable is known. The proposed estimators t_1' , t_2' , t_3' and t_4' are equally efficient. The *PREs* of the proposed estimators $t_i', i = 1-4$ at 10% non-response rate and at $K_h = 2$ is (504.55). Similarly, the *PREs* of the proposed estimators $t_i', i = 1-4$ at 20% non-response rate and at $K_h = 2$ is (363.59) and also, the *PREs* of the proposed estimators $t_i', i = 1-4$ at 30% non-response rate and at $K_h = 2$ is (301.25). Further a decreasing trend has been observed in *PREs* with increase in the value of K_h at 10%, 20% and 30% non-response rates.

8. Conclusion

In this paper, we have discussed the problem of estimating the population mean using auxiliary information in stratified random sampling under non-response. The situation of non-response is examined under two cases; Case I: when non-response occurs both on study variable and auxiliary variable and population mean of the auxiliary variable is known; Case II: when non-response occurs only on study variable, complete information on auxiliary variable and population mean of auxiliary variable is known. Four exponential estimators t_1^* , t_2^* , t_3^* and t_4^* have been proposed in the Case I of non-response when non-response occurs both on study variable and auxiliary variable and population mean of the auxiliary variable is known. Similarly, four exponential estimators t_1' , t_2' , t_3' and t_4' have been proposed in Case II, when non-response occurs only on study variable, complete information on auxiliary variable and population mean of auxiliary variable is known. Expression of bias and mean square error of the proposed estimators $t_i^*, i = 1-4$ and $t_i', i = 1-4$ are obtained separately for all the proposed estimators. Optimum conditions of the proposed estimators are obtained at which the mean squared error of the proposed estimators $t_i^*, i = 1-4$ and $t_i', i = 1-4$ are minimized. The proposed estimators compared with Hansen and Hurwitz (1946) estimator and some other existing estimators theoretically. We have also carried out empirical study to validate the performance of the proposed estimators $t_i^*, i = 1-4$ and $t_i', i = 1-4$ over the existing estimators. Thus, the proposed study is recommended for its use in practice.

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References

Chaudhary, M. K. and Kumar, A. (2015). Estimating the population mean in stratified sampling using two phase sampling in the presence of non-response. *World Applied Sciences Journal,* **33**, 874–82.

Chaudhury, M. K., Singh, R., Shukla, R. K., Kumar, M., and Smarandache, F. (2009). A family of estimators for estimating population mean in stratified sampling under non-response. *Pakistan Journal of Statistics and Operation Research*, **5**, 47-54.

Cochran, W. G. (1977). *Sampling Techniques*. 3rd Edition, John Wiley, New York.

Hansen, M. H. and Hurwitz, W. N. (1946). The problem of non-response in sample surveys. *Journal of the American Statistical Association*, **41**, 517-529.

Ismail, M. and Shahbaz, M. Q. (2017). Combination of ratio and regression estimator of population mean in presence of non–response. *Gazi University Journal of Sci*ence, **30**, 634-642.

Khare, B. B. and Srivastava, S. (1997). Transformed ratio-type estimators for the population mean in the presence of non-response. *Communications in Statistics-Theory and Methods,* **26**, 1779- 1791.

Kumar, S., Kour, S. P., and Sharma, V. (2022). Modified exponential estimators using auxiliary information under response and non-response. *Revista Investigación Operacional*, **43**, 491-504.

Onyeka, A. C., Ogbumuo, D. T., and Izunobi, C. (2019). Estimation of population mean in stratified random sampling when using auxiliary information in the presence of non-response. *Far East Journal of Theoretical Statistics*, **55**, 151-167.

Rao, P. S. R. S. (1986). Ratio estimation with sub-sampling the non-respondents. *Survey Methodology*, **12**, 217-230.

Saleem, I., Sanaullah, A., and Hanif, M. (2018). A generalized class of estimators for estimating population mean in the presence of non-response. *Journal of Statistical Theory and Applications*, **17**, 616-626.

Sanaullah, A., Noor-Ul-Amin, M., and Hanif, M. (2015). Generalized exponential-type ratio-cum-ratio and product-cum-product estimators for population mean in the presence of non-response under stratified two-phase random sampling. *Pakistan Journal of Statistics and Operation Research*, **31**, 71-94.

Shabbir, J., Gupta, S., and Ahmed, S. (2019). A generalized class of estimators under two-phase stratified sampling for non-response. *Communications in Statistics-Theory and Methods,* **48**, 3761- 3777.

Singh, H. P. and Kumar, S. (2008). A general family of estimators of finite population ratio, product and mean using two-phase sampling scheme in presence of non-response. *Journal of Statistical Theory and Practice*, **2**, 677-692.

Singh, H. P. and Kumar, S. (2009). A general procedure of estimating the population mean in the presence of non-response under double sampling using auxiliary information. *SORT*, **33**, 71-84.

Singh, N. and Vishwakarma, G. K. (2019). A generalized class of estimator of population mean with the combined effect of measurement errors and non-response in sample survey. *Investigación Operacional*, **40**, 275-285.

Singh, R. and Sharma, P. (2015). Method of estimation in the presence of non-response and measurement errors simultaneously. *Journal of Modern Applied Statistical Methods,* **14**, 107-114.

Singh, R., Singh, R., and Bouza, C. N. (2018). Effect to measurement error and non-response on estimation of population mean. *Revista Investigación Operacional,* **39**, 108-120.

Ozel, G. K. (2016). A new exponential type estimator for the population mean in simple random sampling. *Journal of Modern Applied Statistical Methods*, **15**, 207-214.

Upadhyaya, L. N., Singh, H. P., Chatterjee, S., and Yadav, R. (2011). Improved ratio and product exponential type estimators. *Journal of Statistical Theory and Practice*, **5**, 285–302.

Vishwakarma, G. K., Singh, R., Gupta, P. C., and Pareek, S. (2016). Improved ratio and product type estimators of finite population mean in simple random sampling. *Revista Investigacion Operacional*, **37**, 70–76.

Wani, Z. H., Rizvi, S. E. H., Sharma, M., and Bhat, M. I. J. (2021). Efficient class of combined ratio type estimators for estimating the population mean under non-response. *International Journal of Scientific Research in Mathematical and Statistical Sciences*, **8**, 01-06.