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Modified Exponential Product Type Estimators for Estimating Population Mean Using Auxiliary Information

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Abstract

In this paper two exponential product type estimators of population mean of the study variable have been proposed in case of simple random sampling without replacement (SRSWOR) sampling scheme. The large sample properties of the proposed estimators have been evaluated to the first order of approximation. The estimators proposed are found more efficient than the mean per unit estimator, product type estimator of Robson (1957), exponential product type estimator of Bahl and Tuteja (1991) and Onyeka (2013). The theoretical findings of the study have been evaluated and verified empirically using data of two real populations.

Key words: Exponential product estimator; Auxiliary Information; Optimum Value; Efficiency.

1. Introduction

Sampling methods are used to get an overview of the universe by studying a subset. Even if the subset is chosen sufficiently large, it may not fully represent the whole universe meaning thereby that the estimates obtained from this subset may be far away from the true estimates of the universe. To get these sample estimates close and close to the actual parameters of the universe, one may define a second variable called auxiliary variable having high correlation with the variable under study and use some known parameters such as mean, coefficient of variation, median, skewness, kurtosis, *etc.* of this auxiliary variable for the said purpose. This auxiliary variable may have a positive or negative correlation with the study variable. In case of positive correlation, the estimators of Cochran (1940), Kadilar and Cingi (2004), Mishra *et al.* (2017), Hussain *et al.* (2021) *etc.* known as ratio estimators are used while as in case of negative correlation, the estimators of Robson (1957), Murthy (1964), Shukla (1976), Vishwakarma *et al.* (2016) *etc*. known as product type estimators are used. The pioneer work of Bahl and Tuteja (1991) proposed exponential ratio and product type estimators. The significance of exponential estimators lies in estimating the population mean precisely even at low degree of correlation. However, the precision of an estimate may be increased by modifying the conventional/classical estimators. Onyeka (2013) proposed a class of modified exponential product type estimators of population parameter by extending the work of Singh *et al.* (2009). Later, Zaman and Kadilar (2019) and Zaman (2020) also

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Manish Sharma had several close interactions with Hukum Chandra while preparing this article for presentation during the conference. Unfortunately, Hukum passed away on 26 April 2021. Our deepest condolences to the bereaved family. In view of his significant contributions, Hukum has been included as a co-author. This paper is a tribute to Hukum.

contributed to this effort. This paper extends the work of Hussain *et al.* (2021) and proposes modified exponential product type estimators.

Consider a population of N units. A sample of size n is drawn from this population by simple random sampling without replacement (srswor). Let Y_i and X_i denote the study and the auxiliary variables respectively, corresponding to the i^{th} (i =1, 2, …N) unit of population and y_i and x_i denote the corresponding study and auxiliary variables respectively, for the i^{th} (i =1, 2, … n) unit in sample. The formulae and notations used in the paper (See Haq and Shabir, 2014 and John and Inyang, 2015) are as follows:

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ and $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ are the population means, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ are the sample means, $C_y = \frac{S_y}{\bar{Y}}$ and $C_x = \frac{S_x}{\bar{X}}$ are the population coefficient of variation, $S_{yy} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ and $S_{xx} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$ are the population mean squares, $s_{yy} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ and $s_{xx} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ are the sample mean squares of study and auxiliary variable respectively. $\rho = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$ is the correlation coefficient between the auxiliary and study variable, where $S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})$.

Further, $\theta = \frac{a\bar{X}}{2(a\bar{X}+b)}$, $\gamma = \frac{1-f}{n}$, where $f = \frac{n}{N}$ is the sampling fraction.

2. Existing Estimators of Population Mean

The usual sample mean $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ provides an unbiased estimator of the population mean. The Bias and MSE of $\bar{y}$ are as

Bias ($\bar{y}$) = 0 and

$$V(\bar{y}) = \gamma C_y^2 \bar{Y}^2. \quad (1)$$

When study and auxiliary variables are negatively correlated, Robson (1957) proposed product type estimator as

$$\bar{y}_{RB} = \bar{y}\,\frac{\bar{x}}{\bar{X}}.$$

The estimator $\bar{y}_{RB}$ is biased and is more efficient than the estimator $\bar{y}$, if $\rho < -\frac{C_x}{2C_y}$. The Bias and MSE of the estimator $\bar{y}_{RB}$ are as

Bias ($\bar{y}_{RB}$) = $\gamma\bar{Y}C_{yx}$ and

MSE ($\bar{y}_{RB}$) = $\gamma\bar{Y}^2(C_y^2 + C_x^2 + 2C_{yx})$ respectively. (2)

Bahl and Tuteja (1991) were the pioneer to propose exponential product type estimator as a precise estimator of population mean as

$$\bar{y}_{BT} = \bar{y}\exp\left(\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}}\right).$$

The Bias and MSE of the estimator $\bar{y}_{BT}$ are as

Bias ($\bar{y}_{BT}$) = $\gamma\bar{Y}\left(\frac{1}{2}C_{yx} - \frac{1}{8}C_x^2\right)$ and

MSE $(\bar{y}_{BT}) = \gamma\bar{Y}^2\left(C_y^2 + \frac{C_x^2}{4} + C_{yx}\right)$ respectively. (3)

Onyeka (2013) extended the work which was carried out by Singh *et al.* (2009) and proposed a class of product type estimators as

$$\bar{y}_{NK} = \bar{y}\,\exp\left[\frac{(a\bar{x}+b)-(a\bar{X}+b)}{(a\bar{x}+b)+(a\bar{X}+b)}\right].$$

With the Bias and MSE as

Bias $(\bar{y}_{NK}) = \gamma\bar{Y}\left(\frac{1}{2}\theta C_{yx} - \frac{1}{8}\theta^2 C_x^2\right)$ and

MSE $(\bar{y}_{NK}) = \gamma\bar{Y}^2\left(C_y^2 + \frac{1}{4}\theta^2 C_x^2 + \theta C_{yx}\right)$ respectively. (4)

3. Proposed Exponential product type Estimators of population mean

The modified exponential product type estimators of population mean proposed are as

$$\bar{y}_{\alpha_1} = \bar{y}\,\exp\left(\frac{\bar{x}-\bar{X}}{\alpha_1\bar{x}}\right).$$

$$\bar{y}_{\alpha_2} = \bar{y}\,\exp\left(\frac{\bar{x}-\bar{X}}{\alpha_2\bar{X}}\right).$$

where α_1 and α_2 are the constants to be determined such that the proposed estimators $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ estimate population mean precisely. The Bias and MSE of the proposed estimators $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ to the first order of approximation are as

Bias $(\bar{y}_{\alpha_1}) = \gamma\bar{Y}\frac{1}{\alpha_1}\left(\frac{1}{2\alpha_1}C_x^2 - C_x^2 + C_{yx}\right)$.

Bias $(\bar{y}_{\alpha_2}) = \gamma\bar{Y}\frac{1}{\alpha_2}\left(\frac{1}{2\alpha_2}C_x^2 + C_{yx}\right)$.

MSE $(\bar{y}_{\alpha_1}) = \gamma\bar{Y}^2\left(C_y^2 + \frac{1}{{\alpha_1}^2}C_x^2 + \frac{2}{\alpha_1}C_{yx}\right)$.

MSE $(\bar{y}_{\alpha_2}) = \gamma\bar{Y}^2\left(C_y^2 + \frac{1}{{\alpha_2}^2}C_x^2 + \frac{2}{\alpha_2}C_{yx}\right)$.

In order to find out the expressions for Bias and MSE. Let us consider

$$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}} \text{ and } e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}.$$

We have,

$E(e_0) = E(e_1) = 0$, $E(e_0^2) = \gamma C_y^2$, $E(e_1^2) = \gamma C_x^2$, $E(e_0e_1) = \gamma C_{yx}$.

Writing the estimator $\bar{y}_{\alpha_1}$ in terms of e_i's $(i = 1, 2)$, therefore

$$\bar{y}_{\alpha_1} = \bar{Y}(1+e_0)\,\exp\left[\frac{(1+e_1)\bar{X}-\bar{X}}{\alpha_1(1+e_1)\bar{X}}\right]$$

$$\Rightarrow\ \bar{y}_{\alpha_1} = \bar{Y}(1+e_0)\,\exp\left[\left(\frac{e_1}{\alpha_1}\right)(1+e_1)^{-1}\right]$$

$$\Rightarrow \ \bar{y}_{\alpha_1} = \bar{Y}(1+e_0)\ \exp\left[\left(\frac{e_1}{\alpha_1}\right)(1-e_1+e_1^2-e_1^3+\cdots)\right]. \quad (5)$$

Solving equation (5) and retaining the terms upto second degree only, the resulting expression as

$$\bar{y}_{\alpha_1} = \bar{Y}\left(1+e_0+\frac{e_1}{\alpha_1}-\frac{e_1^2}{\alpha_1}+\frac{e_1^2}{2{\alpha_1}^2}+\frac{e_0e_1}{\alpha_1}\right)$$

$$\Rightarrow \ \bar{y}_{\alpha_1}-\bar{Y} = \bar{Y}\left(e_0+\frac{e_1}{\alpha_1}-\frac{e_1^2}{\alpha_1}+\frac{e_1^2}{2{\alpha_1}^2}+\frac{e_0e_1}{\alpha_1}\right). \quad (6)$$

Taking expectation on both sides of equation (6) for obtaining Bias $(\bar{y}_{\alpha_1})$ as

$$\text{E}\,(\bar{y}_{\alpha_1}-\bar{Y}) = \text{E}\left[e_0+\frac{e_1}{\alpha_1}-\frac{e_1^2}{\alpha_1}+\frac{e_1^2}{2{\alpha_1}^2}+\frac{e_0e_1}{\alpha_1}\right]$$

$$\Rightarrow \ \text{E}\,(\bar{y}_{\alpha_1}-\bar{Y}) = \frac{1}{2{\alpha_1}^2}\text{E}\,(e_1^2)-\frac{1}{\alpha_1}\text{E}\,(e_1^2)+\frac{1}{\alpha_1}\text{E}\,(e_0e_1)$$

$$\Rightarrow \ \text{Bias}\,(\bar{y}_{\alpha_1}) = \gamma\bar{Y}\frac{1}{\alpha_1}\left(\frac{1}{2\alpha_1}C_x^2-C_x^2+C_{yx}\right). \quad (7)$$

Squaring equation (6) on both sides and then taking expectation for obtaining MSE $(\bar{y}_{\alpha_1})$ as

$$\text{E}\,(\bar{y}_{\alpha_1}-\bar{Y})^2 = \text{E}\left[e_0+\frac{e_1}{\alpha_1}-\frac{e_1^2}{\alpha_1}+\frac{e_1^2}{2{\alpha_1}^2}+\frac{e_0e_1}{\alpha_1}\right]^2. \quad (8)$$

Solving equation (8) and retaining the terms up to second degree only, the expression for MSE $(\bar{y}_{\alpha_1})$ is obtained as

$$\text{MSE}\,(\bar{y}_{\alpha_1}) = \gamma\bar{Y}^2\left(C_y^2+\frac{1}{{\alpha_1}^2}C_x^2+\frac{2}{\alpha_1}C_{yx}\right). \quad (9)$$

Now writing the estimator $\bar{y}_{\alpha_2}$in terms of e_i's $(i=1,2)$, therefore

$$\bar{y}_{\alpha_2} = \bar{Y}(1+e_0)\ \exp\left[\frac{(1+e_1)\bar{X}-\bar{X}}{\alpha_2\bar{X}}\right]$$

$$\Rightarrow \ \bar{y}_{\alpha_2} = \bar{Y}(1+e_0)\ \exp\left(\frac{e_1}{\alpha_2}\right)$$

$$\Rightarrow \ \bar{y}_{\alpha_2} = \bar{Y}(1+e_0)\left(1+\frac{e_1}{\alpha_2}+\frac{e_1^2}{2\alpha_2^2}+\frac{e_1^3}{6\alpha_2^3}+\cdots\right). \quad (10)$$

Solving (10) and retaining the terms up to the second degree only, the resulting expression is as

$$\bar{y}_{\alpha_2} = \bar{Y}\left(1+e_0+\frac{e_1}{\alpha_2}+\frac{e_1^2}{2\alpha_2^2}+\frac{e_0e_1}{\alpha_2}\right)$$

$$\Rightarrow \ \bar{y}_{\alpha_2}-\bar{Y} = \bar{Y}\left(e_0+\frac{e_1}{\alpha_2}+\frac{e_1^2}{2\alpha_2^2}+\frac{e_0e_1}{\alpha_2}\right). \quad (11)$$

Using the same procedure for finding Bias $(\bar{y}_{\alpha_2})$ and MSE $(\bar{y}_{\alpha_2})$ from equation (11) as applied to equation (6) for finding Bias $(\bar{y}_{\alpha_1})$ and MSE $(\bar{y}_{\alpha_1})$, we have

$$\text{Bias}\,(\bar{y}_{\alpha_2}) = \gamma\bar{Y}\frac{1}{\alpha_2}\left(\frac{1}{2\alpha_2}C_x^2+C_{yx}\right). \quad (12)$$

$$\text{MSE}\,(\bar{y}_{\alpha_2}) = \gamma\bar{Y}^2\left(C_y^2+\frac{1}{{\alpha_2}^2}C_x^2+\frac{2}{\alpha_2}C_{yx}\right). \quad (13)$$

The expressions obtained (7), (9), (12) and (13) are the required expressions.

Optimum value of α_1 and α_2

Now differentiating the equations (9) and (13) partially with respect to α_1 and α_2 respectively and equating to zero, the optimal value of α_1 and α_2 is found to be $\frac{-C_x}{\rho C_y} = \eta$ (say). The value of η can be obtained quite accurately from some previous survey or from the experience of the researcher (See Reddy (1973, 1974), Singh and Vishwakarma (2008), Singh and Kumar (2008), Singh and Kapre (2010)). Substituting the value of η value in equations (7) and (12), the expressions for Bias ($\bar{y}_{\alpha_1}$) and Bias ($\bar{y}_{\alpha_2}$) at the optimal condition are obtained as

$$\text{Bias}\,(\bar{y}_{\alpha_1}) = \gamma\bar{Y}\left(C_{yx} - \frac{1}{2}\rho^2 C_y^2\right). \tag{14}$$

$$\text{Bias}\,(\bar{y}_{\alpha_2}) = -\frac{1}{2}\gamma\bar{Y}\rho^2 C_y^2. \tag{15}$$

Now substituting the optimal value of α_1 and α_2 in equations (9) and (13), the minimum value of MSE ($\bar{y}_{\alpha_1}$) and MSE ($\bar{y}_{\alpha_2}$) is obtained as

$$\text{MSE}_{\min}(\bar{y}_{\alpha_i}) = \gamma\bar{Y}^2 C_y^2\,(1-\rho^2): \quad i = 1,2 \tag{16}$$

Special cases: The proposed product type estimators $\bar{y}_{\alpha_i}$ (i=1, 2) can be used as an alternative to product type estimator of Robson (1957), exponential product type estimator of Bahl and Tuteja (1991) under the conditions as

(i) $\alpha_1 = \alpha_2 = 1$, the MSE of the proposed estimators $\bar{y}_{\alpha_i}$ (i=1, 2) is same as that of the MSE of product type of Robson (1957).

(ii) $\alpha_1 = \alpha_2 = 2$, the MSE of the proposed estimators $\bar{y}_{\alpha_i}$ (i=1, 2) is same as the MSE of the exponential product type estimator of Bahl and Tuteja (1991).

4. Theoretical Efficiency Comparison

The efficiency comparisons of the study are done using the MSE of the proposed estimators $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ and that of the existing estimators $\bar{y}$, $\bar{y}_{RB}$, $\bar{y}_{BT}$ and $\bar{y}_{NK}$ considered.

(i) Efficiency comparison of $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ when the values of α_1 and α_2 coincide with its optimal value

Solving the expressions (1), (2), (3), (4) and (16), the conditions obtained are as

$$\text{MSE}_{\min}(\bar{y}_{\alpha_i}) < V(\bar{y})$$

$$\Rightarrow \gamma\bar{Y}^2 C_y^2\,(1-\rho^2) < \gamma C_y^2\bar{Y}^2,\ if\ \rho^2\bar{Y}^2 > 0. \tag{17}$$

$$\text{MSE}_{\min}(\bar{y}_{\alpha_i}) < \text{MSE}(\bar{y}_{RB})$$

$$\Rightarrow \gamma\bar{Y}^2 C_y^2\,(1-\rho^2) < \gamma\bar{Y}^2(C_y^2 + C_x^2 + 2C_{yx}),\ if\ (\rho C_y + C_x)^2 > 0. \tag{18}$$

$$\text{MSE}_{\min}(\bar{y}_{\alpha_i}) < \text{MSE}(\bar{y}_{BT})$$

$$\Rightarrow \gamma\bar{Y}^2 C_y^2\ (1-\rho^2) < \gamma\bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} + C_{yx}\right),\ if\ (2\rho C_y + C_x)^2 > 0. \quad (19)$$

$$\text{MSE}_{\min}(\bar{y}_{\alpha_i}) < \text{MSE}\ (\bar{y}_{NK})$$

$$\Rightarrow \gamma\bar{Y}^2 C_y^2\ (1-\rho^2) < \gamma\bar{Y}^2 \left(C_y^2 + \frac{1}{4}\theta^2 C_x^2 + \theta C_{yx}\right),\ if\ (2\rho C_y + \theta\ C_x)^2 > 0. \quad (20)$$

Therefore, under the conditions (17) to (20), the proposed estimators $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ will be more efficient than the product type estimators $\bar{y}$, $\bar{y}_{RB}$, $\bar{y}_{BT}$ and $\bar{y}_{NK}$ considered in this study.

(ii) Efficiency comparison of $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ when the value of α_1 and α_2 does not coincide with its optimal value

When the equations (1), (2), (3), (4) and (9), (13) were solved, the following conditions were obtained

$$\text{MSE}\ (\bar{y}_{\alpha_1}) < \text{V}\ (\bar{y}),\ if\ \alpha_1 > \left(\frac{-C_x^2}{2C_{yx}}\right) \quad (21)$$

$$\text{MSE}\ (\bar{y}_{\alpha_2}) < \text{V}\ (\bar{y}),\ if\ \alpha_2 > \left(\frac{-C_x^2}{2C_{yx}}\right) \quad (22)$$

$$\text{MSE}\ (\bar{y}_{\alpha_1}) < \text{V}\ (\bar{y}_{RB}),\ if$$

$$min\left(1, \frac{-C_x^2}{2C_{yx}+C_x^2}\right) < \alpha_1 < max\left(1, \frac{-C_x^2}{2C_{yx}+C_x^2}\right), \frac{C_{yx}}{C_x^2} < -\frac{1}{2} \quad (23)$$

$$Or\ \alpha_1 > 1, -\frac{1}{2} \leq \frac{C_{yx}}{C_x^2} < 0.$$

$$\text{MSE}\ (\bar{y}_{\alpha_2}) < \text{V}\ (\bar{y}_{RB}),\ if$$

$$min\left(1, \frac{-C_x^2}{2C_{yx}+C_x^2}\right) < \alpha_2 < max\left(1, \frac{-C_x^2}{2C_{yx}+C_x^2}\right), \frac{C_{yx}}{C_x^2} < -\frac{1}{2} \quad (24)$$

$$Or\ \alpha_2 > 1, -\frac{1}{2} \leq \frac{C_{yx}}{C_x^2} < 0.$$

$$\text{MSE}\ (\bar{y}_{\alpha_1}) < \text{V}\ (\bar{y}_{BT}),\ if$$

$$min\left(2, \frac{-2C_x^2}{4C_{yx}+C_x^2}\right) < \alpha_1 < max\left(2, \frac{-2C_x^2}{4C_{yx}+C_x^2}\right), \frac{C_{yx}}{C_x^2} < -\frac{1}{4} \quad (25)$$

$$Or\ \alpha_1 > 2, -\frac{1}{4} \leq \frac{C_{yx}}{C_x^2} < 0\ .$$

$$\text{MSE}\ (\bar{y}_{\alpha_2}) < \text{V}\ (\bar{y}_{BT}),\ if$$

$$min\left(2, \frac{-2C_x^2}{4C_{yx}+C_x^2}\right) < \alpha_2 < max\left(2, \frac{-2C_x^2}{4C_{yx}+C_x^2}\right), \frac{C_{yx}}{C_x^2} < -\frac{1}{4} \quad (26)$$

$$Or\ \alpha_2 > 2, -\frac{1}{4} \leq \frac{C_{yx}}{C_x^2} < 0.$$

$$\text{MSE}\ (\bar{y}_{\alpha_1}) < \text{V}\ (\bar{y}_{NK}),\ if$$

$$min\left(\frac{\theta}{2}, \frac{-2C_x^2}{2C_{yx}+\theta C_x^2}\right) < \alpha_1 < max\left(\frac{\theta}{2}, \frac{-2C_x^2}{2C_{yx}+\theta C_x^2}\right), \frac{C_{yx}}{C_x^2} < -\frac{\theta}{4}. \quad (27)$$

$$Or\ \alpha_1 > \frac{\theta}{2}, -\frac{\theta}{4} \leq \frac{C_{yx}}{C_x^2} < 0.$$

$$\text{MSE}\,(\bar{y}_{\alpha_2}) < \text{V}\,(\bar{y}_{NK}),\ if$$

$$min\left(\frac{\theta}{2}, \frac{-2C_x^2}{2C_{yx}+\theta C_x^2}\right) < \alpha_2 < max\left(\frac{\theta}{2}, \frac{-2C_x^2}{2C_{yx}+\theta C_x^2}\right), \frac{C_{yx}}{C_x^2} < -\frac{\theta}{4}. \quad (28)$$

$$or\ \alpha_2 > \frac{\theta}{2}, -\frac{\theta}{4} \leq \frac{C_{yx}}{C_x^2} < 0.$$

Under the conditions (21) to (28) the estimators $\bar{y}_{\alpha_i}$ (i = 1, 2) are more efficient than the estimators $\bar{y}$, $\bar{y}_{RB}$, $\bar{y}_{BT}$ and $\bar{y}_{NK}$.

5. Numerical Study

The numerical study of the present work is done using data of two populations P1 and P2 where the study and auxiliary variable are negatively correlated. The population P1 is from Onyeka (2013) where the study variable (Y) is the percentage of hives affected by disease and the auxiliary variable (X) is the date of flowering of a particular summer species (number of days from January 1). The population P2 has been taken from Gujarati (2004) where the study variable (Y) is the average miles per gallon and the auxiliary variable (X) is the top speed, miles per hour. The data regarding the populations taken is given in Table 1. The performance of the product type estimators $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ has been compared with the sample mean estimator $\bar{y}$ and the product type estimators $\bar{y}_{RB}$, $\bar{y}_{BT}$ and $\bar{y}_{NK}$.

Table-1 Summary statistics of the populations P1 and P2.

Parameter		N	n	$\bar{Y}$	$\bar{X}$	ρ	C_y	C_x	C_{yx}
Population	**P1**	10	4	52	200	− 0.94	0.1562	0.0458	− 0.00673
	P2	81	13	33.83457	112.4568	− 0.69	0.2972	0.1256	− 0.02576

From Table-1 it can be seen that among P1 and P2, the population P1 has higher correlation than P2. For the population P2, the coefficient of variation of auxiliary and study variable is higher than the population P1.

Table 2: Range of α_1 and α_2 for $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ to be more efficient than the estimators considered.

Estimator	Range of α_1 and α_2 for Population			
	P1		**P2**	
$\bar{y}$	$\alpha_1 > 0.156$	$\alpha_2 > 0.156$	$\alpha_1 > 0.306$	$\alpha_2 > 0.306$
$\bar{y}_{RB}$	$\alpha_1 \in (0.185, 1)$	$\alpha_2 \in (0.185, 1)$	$\alpha_1 \in (0.441, 1)$	$\alpha_2 \in (0.441, 1)$
$\bar{y}_{BT}$	$\alpha_1 \in (0.169, 2)$	$\alpha_2 \in (0.169, 2)$	$\alpha_1 \in (0.362, 2)$	$\alpha_2 \in (0.362, 2)$
$\bar{y}_{NK}$	$\alpha_1 \in (0.169, 0.338)$	$\alpha_2 \in (0.169, 0.338)$	$\alpha_1 \in (0.250, 0.723)$	$\alpha_2 \in (0.250, 0.723)$
η **(optimum value)**	η = 0.312		η = 0.613	

Table-2 contains the range of α_1 and α_2 for the estimators $\bar{y}_{\alpha_1}$ and $\bar{y}_{\alpha_2}$ respectively, to be more precise. The optimum values for P1 and P2 are 0.312 and 0.613 respectively, at this optimum value the proposed estimators are more efficient than all the estimators taken under consideration for study.

Table 3: MSE and Bias of the proposed and the estimators considered.

Estimator	Population			
	P1		P2	
	MSE	|Bias|	MSE	|Bias|
$\bar{y}$	9.8960	0.0000	6.5298	0.0000
$\bar{y}_{RB}$	5.2917	0.0525	3.8878	0.0563
$\bar{y}_{BT}$	7.3812	0.0283	4.9176	0.0324
$\bar{y}_{NK}$	8.5851	0.0136	5.6493	0.0152
$\bar{y}_{\alpha_1}$	1.1519	0.1365	3.4209	0.1022
$\bar{y}_{\alpha_2}$	1.1519	0.0841	3.4209	0.0459

It can be observed from Table-3 that MSE of both the proposed estimators $\bar{y}_{\alpha_1}$and $\bar{y}_{\alpha_2}$ is less than the MSE of all the estimators $\bar{y}$, $\bar{y}_{RB}$, $\bar{y}_{BT}$ and $\bar{y}_{NK}$. The MSE of the proposed estimators for population P1 is less than that of population P2. Further the MSE of $\bar{y}_{RB}$ is less than $\bar{y}_{RB}$ and $\bar{y}_{BT}$ for both the populations P1 and P2. It can be observed on taking the modulus value of the Bias that the proposed estimator $\bar{y}_{\alpha_1}$ has less bias than the estimator $\bar{y}_{\alpha_2}$ for both the datasets.

Table 4: Percent relative efficiency w.r.t the estimators $\bar{y}$, $\bar{y}_{RB}$, $\bar{y}_{BT}$ and $\bar{y}_{NK}$.

Population	Estimator	Percent relative efficiency w.r.t			
		$\bar{y}$	$\bar{y}_{RB}$	$\bar{y}_{BT}$	$\bar{y}_{NK}$
P1	$\bar{y}$	100.0000	53.4731	74.5877	86.7532
	$\bar{y}_{RB}$	187.0094	100.0000	139.4864	162.2371
	$\bar{y}_{BT}$	134.0712	71.6916	100.0000	116.3104
	$\bar{y}_{NK}$	115.2695	61.6382	85.9769	100.0000
	$\bar{y}_{\alpha_i}$	859.1065	459.3888	640.7848	745.2991
P2	$\bar{y}$	100.0000	59.5393	75.3101	86.5157
	$\bar{y}_{RB}$	167.9562	100.0000	126.4879	145.3084
	$\bar{y}_{BT}$	132.7843	79.0589	100.0000	114.8792
	$\bar{y}_{NK}$	115.5860	68.8192	87.0479	100.0000
	$\bar{y}_{\alpha_i}$	190.8796	113.6485	143.7516	165.1406

The findings of Table-4 reveal that the proposed estimators $\bar{y}_{\alpha_i}$ (i = 1, 2) are more efficient than the estimators $\bar{y}$, $\bar{y}_{RB}$, $\bar{y}_{BT}$ and $\bar{y}_{NK}$ considered in the study for both the populati ons P1 and P2.

6. Conclusion

The proposed exponential product type estimators $\bar{y}_{\alpha_i}$(i=1, 2) were found estimating the population mean precisely than the sample mean estimator, product type estimator of Robson (1957), exponential product type estimators of Bahl and Tuteja (1991) and Onyeka (2013) theoretically as well as empirically. Further as per the empirical study conducted, the optimum values of α_1 and α_2 for the proposed estimators were found 0.312 and 0.613 for the populations P1 and P2 respectively.

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