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Estimation for the Length Biased Log-Logistic Model under Adaptive Progressive Type II Censoring

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Abstract

In this paper, point estimation of shape and scale parameters of length-biased log-logistic distribution under adaptive progressive type II censoring is addressed using Bayesian and non-Bayesian approaches. Maximum Likelihood estimators are proposed and evaluated using Newton-Raphson numerical approximation method. Asymptotic confidence interval and parametric bootstrap confidence intervals are also constructed. Parametric Bayes estimators are proposed under three different loss functions using Markov Chain Monte Carlo iterative method. Credible intervals and Highest Posterior Density region are also constructed. Simulation study for different sample sizes and different censoring schemes is carried out to establish utility of the proposed decision-theoretic strategies. A real dataset has also been analyzed to reinforce the simulated results.

Key words: Length-biased log-logistic distribution; Adaptive progressive type II censoring; Bootstrap confidence intervals; Highest Posterior Density region; Markov Chain Monte Carlo.

AMS Subject Classifications: 62F15, 65C05

1. Introduction

The main purpose of any censoring plan is to reduce time duration while optimizing the total experimental cost. A balanced censoring strategy might take into account the length of the experiment, the number of units involved, and the effectiveness of statistical inference drawn from the study's outcomes. The basic censoring schemes are time and failure censoring. Progressive censoring schemes have flexibility of removing additional live (functioning and good) items at the observed actual failure times. Ng *et. al* (2009) proposed a new and more flexible censoring scheme known as adaptive progressive type II censoring (APT-IIC) scheme which is combination of the type I and progressive type II censoring plans. APT-IIC ensures a desired number of failed observations in a tested sample within a prescribed duration of the experiment.

Consider n test units in a life-test. Let m be the desired counts of failed units in the observed sample. Let $R = (R_1, R_2, \ldots, R_m)$ be the pre-determined intermittent withdrawals

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under progressive censoring scheme such that the experiment span is pre-fixed at time T. Let k be the total number of observed failure times before the pre-determined time T *i.e.* $X_{k:m:n} \leq T \leq X_{k+1:m:n}$; $k = 0, 1, \ldots, m$ where $X_{0:m:n} = 0$ and $X_{m+1:m:n} = \infty$. If the total experiment time exceeds the ideal test time T, then $R_{k+1} = R_{k+2} = \cdots = R_{m-1} = 0$ and $R_m = n - m - \sum_{i=1}^{k} R_i$. In this situation, no surviving units get chance to be removed except at the time of m^{th} failure. This condition helps to accelerate the experiment so that it ends as soon as possible.

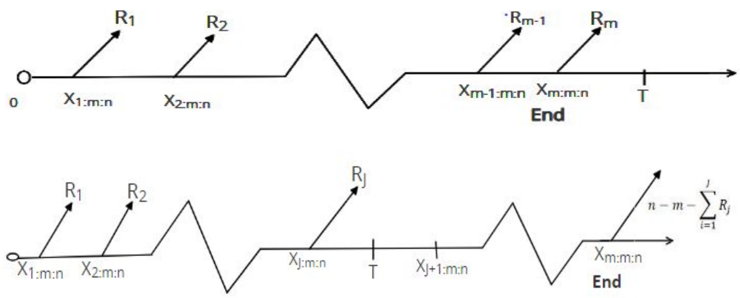

Figure 1: A visual of the Adaptive Progressive Type II censoring scheme

Inferential studies for different life-time models under APT-IIC scheme are undertaken by various authors. Parameter estimation of exponential distribution under APT-IIC has been considered by Ng *et. al* (2009). Burr type XII distribution was considered by Amein (2016) for estimation of unknown parameters under APT-IIC. Sobhi and Soliman (2016) considered classical and Bayes estimation of the exponentiated Weibull model. Similarly, parameter estimation of exponential, generalized exponential, exponentiated exponential, generalized inverted exponential distributions under APT-IIC have been considered by Ng *et. al* (2009), El-Din *et. al* (2017), Ateya and Mohammed (2017) and Soliman *et. al* (2020) respectively. Maximum product spacing and the maximum likelihood estimation of parameters of generalized Rayleigh distribution and Weibull distribution was discussed by Almetwally *et. al* (2019) and Almetwally *et. al* (2020) respectively. Some other distributions under APT-IIC are: Generalized Pareto distribution by Mahmoud *et. al* (2013), Kumaraswamy distribution by Almalki *et. al* (2022), new Weibull-Pareto distribution by EL-Sagheer *et. al* (2018), Kumaraswamy-exponential distribution by Mohan and Chako (2021), Truncated normal distribution by Chen and Gui (2020), generalized Gompertz distribution by Amein *et. al* (2020), extreme value distribution by Ye *et. al* (2014), exponentiated power Lindley by Ahmad *et. al* (2021), exponentiated half-logistic distribution by Xiong and Gui (2021), exponentiated Pareto distribution by Wang and Gui (2021), asymmetric power hazard distribution by El-Morshedy *et. al* (2022).

The present paper focuses on a length biased model which is defined in section 2. Maximum likelihood estimation (MLE) along with Asymptotic Confidence Interval (ACI) is derived in section 3. Section 4 describes Bayes estimation under three loss functions namely squared error loss function (SELF), general entropy loss function (GELF) and linear exponential loss function (LINEX). In addition, the corresponding Bayesian credible intervals (BCI) and highest probability density intervals (HPD) are also calculated. Markov Chain Monte Carlo (MCMC) approximations are detailed in section 5. A real data set illustrates the developed theory in section 6. Concluding remarks are given in section 7.

2. The model

Weighted distributions (WD) were first proposed by Fisher (1934). WD emerge when information from any stochastic process are produced using a predetermined weight function. Compared to the original distributions, WD are more adaptable and as a result, they are helpful in several fields including ecology, biometry, environmental sciences, survivability, and reliability analysis. When the weight function, say $w(x)$, depends on the length of the units of interest, *i.e.* $w(x) = x$, the resulting distribution is termed as *length biased distribution* (LBD). Although LBD does not add any additional parameters to the model, it does provide it more flexibility. There are LB versions of a number of distributions accessible in statistical literature. Patil and Rao (1978) introduced LB versions of many basic distributions such as log-normal, gamma, Pareto, beta. Das and Roy (2011) discussed LB weighted Weibull distribution. Some other works on LB versions of different distributions are: LB weighted generalized Rayleigh distribution (Das and Roy, 2011), LB beta distribution (Mir et.al, 2013), LB Weibull distribution (Pandya *et. al*, 2013), LB exponentiated inverted Weibull (Seenoi *et. al*, 2014), LB weighted Lomax distribution (Ahmad *et. al*, 2016), LB Inverse Rayleigh distribution (Pandey and Kumari, 2016), LB weighted Erlang distribution (Reyad *et. al*, 2017), LB Sushila distribution (Rather and Subramanian, 2018), LB weighted Lomax distributions (Karimi and Nasiri, 2018), LB Erlang–truncated exponential distribution (Rather and Subramanian, 2019) and many more.

Recently Pandey *et. al* (2021) introduced LB Log Logistic distribution ($LBLL(\alpha, \beta)$) as a lifetime model. The pdf of ($LBLL(\alpha, \beta)$) is given as

$$f(x;\alpha,\beta) = \frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta}}{\left\{1+\left(\frac{x}{\alpha}\right)^{\beta}\right\}^{2}} \frac{\sin\left(\frac{\pi}{\beta}\right)}{\left(\frac{\pi}{\beta}\right)} \quad \text{for } x,\ \alpha,\ \beta > 0 \tag{1}$$

The corresponding cdf can be obtained as (see Pandey *et. al* (2021))

$$F(x) = \int_{x}^{\infty} f(t;\alpha,\beta)dt$$

$$F(x) = \frac{\sin\left(\frac{\pi}{\beta}\right)}{\left(\frac{\pi}{\beta}\right)} \frac{1}{\beta}\left(\frac{x}{\alpha}\right)^{1-\beta} \log\left(1+\left(\frac{x}{\alpha}\right)^{\beta}\right) - \frac{\left(\frac{x}{\alpha}\right)}{1+\left(\frac{x}{\alpha}\right)^{\beta}} - \left(\frac{1-\beta}{\beta}\right)\left[\left(\frac{x}{\alpha}\right) + \sum_{u=1}^{\infty} \frac{(-1)^{u}\left(\frac{x}{\alpha}\right)^{1+u\beta}}{u\,(1+u\beta)}\right] \quad \text{for } x,\ \alpha,\ \beta > 0 \tag{2}$$

3. Classical point and interval estimation

Under APT-IIC, n, m, R, T be fixed before the experiment begins. Lifetime distribution is assumed to follow pdf $f(x;\Theta)$and corresponding cdf $F(x;\Theta)$, where Θ represents a vector of parameters. The likelihood function under APT-IIC scheme (Ng *et al.*, 2009) is given as

$$L(\Theta;t) = C_J \left(\prod_{i=1}^{m} f\left(t_i;\Theta\right)\right)\left(\prod_{i=1}^{k}\left(1-F\left(t_i;\Theta\right)\right)^{R_i}\right)\left(1-F\left(t_m;\Theta\right)\right)^{n-m-\sum_{i=1}^{k}R_i}$$
$$0 < t_1 < t_2 < \cdots < t_m < \infty \quad (3)$$

where

$$C_J = \prod_{i=1}^{m}\left(n-i+1-\sum_{j=1}^{\max\{i-1,k\}} R_j\right)$$

The likelihood function for $LBLL(\alpha,\beta)$ whose pdf and cdf are given by (1) and (2) respectively, under APT-IIC is given as

$$L\left(\beta,\alpha;t\right) = C_J\left(\prod_{i=1}^{m}\{E_1\}\right)\left(\prod_{i=1}^{J}\{E_2\}^{R_i}\right)\left(\{E_3\}^{n-m-\sum_{i=1}^{k}R_i}\right) \quad (4)$$

where

$$\begin{aligned}
E_1 =& \frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{t_i}{\alpha}\right)^{\beta}}{\left\{1+\left(\frac{t_i}{\alpha}\right)^{\beta}\right\}^2}\frac{\sin\left(\frac{\pi}{\beta}\right)}{\left(\frac{\pi}{\beta}\right)}\\
E_2 =& 1-\frac{\sin\left(\frac{\pi}{\beta}\right)}{\left(\frac{\pi}{\beta}\right)}\frac{1}{\beta}\left(\frac{t_i}{\alpha}\right)^{1-\beta}\log\left(1+\left(\frac{t_i}{\alpha}\right)^{\beta}\right)\\
&+\frac{\left(\frac{t_i}{\alpha}\right)}{1+\left(\frac{t_i}{\alpha}\right)^{\beta}}+\left(\frac{1-\beta}{\beta}\right)\left[\left(\frac{t_i}{\alpha}\right)+\sum_{r=1}^{\infty}\frac{(-1)^r\left(\frac{t_i}{\alpha}\right)^{1+r\beta}}{r\left(1+r\beta\right)}\right]\\
E_3 =& 1-\frac{\sin\left(\frac{\pi}{\beta}\right)}{\left(\frac{\pi}{\beta}\right)}\frac{1}{\beta}\left(\frac{t_m}{\alpha}\right)^{1-\beta}\log\left(1+\left(\frac{t_m}{\alpha}\right)^{\beta}\right)\\
&+\frac{\left(\frac{t_m}{\alpha}\right)}{1+\left(\frac{t_m}{\alpha}\right)^{\beta}}+\left(\frac{1-\beta}{\beta}\right)\left[\left(\frac{t_m}{\alpha}\right)+\sum_{r=1}^{\infty}\frac{(-1)^r\left(\frac{t_m}{\alpha}\right)^{1+r\beta}}{r\left(1+r\beta\right)}\right]
\end{aligned}$$

The corresponding log likelihood function is written as

$$\ln L = const + \sum_{i=1}^{m}\ln\{E_1\} + \sum_{i=i}^{J} R_i\ln\{E_2\} + \left(n-m-\sum_{i=1}^{k}R_i\right)\ln\{E_3\} \quad (5)$$

Partially differentiating (5) with respect to (w.r.t.) α yields

$$\frac{\partial\ln L}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left\{\sum_{i=1}^{m}\ln\{E_1\}\right\} + \frac{\partial}{\partial\alpha}\left\{\sum_{i=i}^{J}R_i\ln\{E_2\}\right\} + \left(n-m-\sum_{i=1}^{k}R_i\right)\frac{\partial}{\partial\alpha}\{\ln\{E_3\}\} \quad (6)$$

$\hat{\alpha}$, mle of α is the value for which $\frac{\partial\ln L}{\partial\alpha}=0$ and $\left.\frac{\partial^2\ln L}{\partial\alpha^2}\right|_{\alpha=\hat{\alpha}} < 0$.

Similarly, partial differentiation of (5) w.r.t. β yields

$$\frac{\partial \ln L}{\partial \beta} = \frac{\partial}{\partial \beta}\left\{\sum_{i=1}^{m} \ln\{E_1\}\right\} + \frac{\partial}{\partial \beta}\left\{\sum_{i=i}^{J} R_i \ln\{E_2\}\right\} + \left(n - m - \sum_{i=1}^{k} R_i\right)\frac{\partial}{\partial \beta}\{\ln\{E_3\}\} \quad (7)$$

$\hat{\beta}$, mle of β, is the value for which $\frac{\partial \ln L}{\partial \beta} = 0$ and $\left.\frac{\partial^2 \ln L}{\partial \beta^2}\right|_{\beta=\hat{\beta}} < 0$.

The solution of the system of nonlinear equations (6)-(7) of the first partial derivatives of log-likelihood function w.r.t. parameters cannot be obtained in closed form . Therefore, to find approximate MLE*s*, a numerical method like Newton-Raphson (N-R) method is used.

Let $\widehat{\lambda} = (\widehat{\alpha}, \widehat{\beta})$ be denoting the *mle* of $\lambda = (\alpha, \beta)$ and $I(\lambda)$ is Fisher's Information matrix, *i.e.*

$$I(\lambda) = -\frac{1}{n}\begin{bmatrix} E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha \partial \beta}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \alpha \partial \beta}\right) & E\left(\frac{\partial^2 \log L}{\partial \beta^2}\right) \end{bmatrix} \quad (8)$$

Matrix elements for $I(\lambda)$, given in (8) are defined under APT-IIC as under,

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = \frac{\partial^2}{\partial \alpha^2}\left\{\sum_{i=1}^{m} \ln\{E_1\}\right\} + \frac{\partial^2}{\partial \alpha^2}\left\{\sum_{i=i}^{J} R_i \ln\{E_2\}\right\} + \left(n - m - \sum_{i=1}^{k} R_i\right)\frac{\partial^2}{\partial \alpha^2}\{\ln\{E_3\}\} \quad (9)$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = \frac{\partial^2}{\partial \beta^2}\left\{\sum_{i=1}^{m} \ln\{E_1\}\right\} + \frac{\partial^2}{\partial \beta^2}\left\{\sum_{i=i}^{J} R_i \ln\{E_2\}\right\} + \left(n - m - \sum_{i=1}^{k} R_i\right)\frac{\partial^2}{\partial \beta^2}\{\ln\{E_3\}\} \quad (10)$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} = \frac{\partial^2}{\partial \alpha \partial \beta}\left\{\sum_{i=1}^{m} \ln\{E_1\}\right\} + \frac{\partial^2}{\partial \alpha \partial \beta}\left\{\sum_{i=i}^{J} R_i \ln\{E_2\}\right\} + \left(n - m - \sum_{i=1}^{k} R_i\right)\frac{\partial^2}{\partial \alpha \partial \beta}\{\ln\{E_3\}\} \quad (11)$$

As it is evident from (9)-(11), the expectation of Hessian matrix is complicated due to presence of mathematically intractable terms. Since the parameter vector λ is unknown, hence using uniqueness property of *mle*, we estimate $I^{-1}(\lambda)$ by $I^{-1}(\widehat{\lambda})$. This provides ACI for the unknown parameters α and β given as

$$\left(\hat{\alpha} - z_{\frac{\xi}{2}}\sqrt{var(\hat{\alpha})}, \hat{\alpha} + z_{\frac{\xi}{2}}\sqrt{var(\hat{\alpha})}\right)$$
$$\left(\hat{\beta} - z_{\frac{\xi}{2}}\sqrt{var\left(\hat{\beta}\right)}, \hat{\beta} + z_{\frac{\xi}{2}}\sqrt{var\left(\hat{\beta}\right)}\right)$$

where $var(\hat{\alpha})$ and $var\left(\hat{\beta}\right)$ are the estimated variances of $\hat{\alpha}$ and $\hat{\beta}$ given by the main diagonal elements of $I^{-1}(\widehat{\lambda})$ and $z_{\frac{\xi}{2}}$ represents the right tail probability for standard normal distribution.

4. Bayesian estimation

Availability of prior information about concerned parameters, enables alternate Bayesian inferential approach. In this paper, Bayes estimators (BE*s*) of the unknown parameters (α, β) are proposed under SELF, GELF and LINEX loss function. In addition, the corresponding

BCI and HPD interval are also calculated. Prior distributions for the unknown independent parameters α and β are taken to be non-informative prior and gamma prior respectively.

$$p(\alpha) = \frac{1}{\alpha}; \quad \alpha > 0$$
$$p(\beta) = \frac{c^d}{\Gamma d}\beta^{d-1}\exp(-c\beta); \quad \beta, c, d > 0$$

where c and d are hyper parameters. Assuming the independence of the scale and shape parameters, the joint prior distribution of α and β is written as

$$p(\alpha, \beta) = \frac{c^d}{\alpha\Gamma d}\beta^{d-1}\exp(-c\beta); \qquad \alpha, \beta, c, d > 0 \tag{12}$$

Joint posterior distribution of α and β is

$$p(\alpha, \beta|x) \propto \left(\prod_{i=1}^{m}\{E_1\}\right)\left(\prod_{i=1}^{J}\{E_2\}^{R_i}\right)\left(\{E_3\}^{n-m-\sum_{i=1}^{k}R_i}\right)\{E_5\} \tag{13}$$

where

$$E_5 = \frac{\beta^{d-1}}{\alpha}\exp(-c\beta)$$

4.1. Marginal posterior distributions

Marginal posterior distribution of unknown parameter α

$$p(\alpha|x, \beta) \propto \int_0^\infty \left(\prod_{i=1}^{m}\{E_1\}\right)\left(\prod_{i=1}^{J}\{E_2\}^{R_i}\right)\left(\{E_3\}^{n-m-\sum_{i=1}^{k}R_i}\right)\{E_5\}\,d\beta \tag{14}$$

Marginal posterior distribution of unknown parameter β

$$p(\beta|x, \alpha) \propto \int_0^\infty \left(\prod_{i=1}^{m}\{E_1\}\right)\left(\prod_{i=1}^{J}\{E_2\}^{R_i}\right)\left(\{E_3\}^{n-m-\sum_{i=1}^{k}R_i}\right)\{E_5\}\,d\alpha \tag{15}$$

4.2. Parametric Bayes estimators under different loss functions

The BE of an unknown parameter depends on the form of the loss function. We obtain the expressions for BEs of unknown parameters under different loss functions. SELF, a symmetric loss function, weighs underestimation (UE) and overestimation (OE) equally. GELF (Calabria and Pulcini, 1996) and LINEX (Varian, 1975) are asymmetric in respect of UE and OE being assigned different degrees of seriousness.

1. SELF
   BE of the unknown parameter α, the posterior mean, is given as

$$\tilde{\alpha}_{BS} \propto \alpha \iint \left(\prod_{i=1}^{m}\{E_1\}\right)\left(\prod_{i=1}^{J}\{E_2\}^{R_i}\right)\left(\{E_3\}^{n-m-\sum_{i=1}^{k}R_i}\right)\{E_5\}\,d\beta d\alpha \tag{16}$$

BE of unknown parameter β is given as

$$\tilde{\beta}_{BS} \propto \beta \iint \left(\prod_{i=1}^{m} \{E_1\}\right) \left(\prod_{i=1}^{J} \{E_2\}^{R_i}\right) \left(\{E_3\}^{n-m-\sum_{i=1}^{k} R_i}\right) \{E_5\}\, d\alpha d\beta \tag{17}$$

2. GELF
BE of unknown parameter α is given as

$$(\tilde{\alpha}_{BG})^{-q} \propto \iint \alpha^{-q} \left(\prod_{i=1}^{m} \{E_1\}\right) \left(\prod_{i=1}^{J} \{E_2\}^{R_i}\right) \left(\{E_3\}^{n-m-\sum_{i=1}^{k} R_i}\right) \{E_5\}\, d\beta d\alpha \tag{18}$$

BE of unknown parameter β is given as

$$(\tilde{\beta}_{BG})^{-q} \propto \iint \beta^{-q} \left(\prod_{i=1}^{m} \{E_1\}\right) \left(\prod_{i=1}^{J} \{E_2\}^{R_i}\right) \left(\{E_3\}^{n-m-\sum_{i=1}^{k} R_i}\right) \{E_5\}\, d\alpha d\beta \tag{19}$$

3. LINEX
BE of unknown parameter α is given as

$$\tilde{\alpha}_{BLL} \propto \frac{1}{q} \ln \iint e^{-q\alpha} \left(\prod_{i=1}^{m} \{E_1\}\right) \left(\prod_{i=1}^{J} \{E_2\}^{R_i}\right) \left(\{E_3\}^{n-m-\sum_{i=1}^{k} R_i}\right) \{E_5\}\, d\beta d\alpha \tag{20}$$

BE of unknown parameter β is given as

$$\tilde{\beta}_{BLL} \propto \frac{1}{q} \ln \iint e^{-q\beta} \left(\prod_{i=1}^{m} \{E_1\}\right) \left(\prod_{i=1}^{J} \{E_2\}^{R_i}\right) \left(\{E_3\}^{n-m-\sum_{i=1}^{k} R_i}\right) \{E_5\}\, d\alpha d\beta \tag{21}$$

5. Markov Chain Monte Carlo approximation

Markov Chain Monte Carlo (MCMC) technique approximates the complex expressions of posterior distribution and BE*s* which are not available in closed form. We use MCMC iteration such that the Gibbs sampler nests Metropolis-Hastings (M-H) algorithms (Metropolis *et. al*, 1953; Hastings, 1970). Convergence of Markov chain simulation is achieved by choosing a starting value which is nearer to the true value. Initial M simulated variates are omitted to shake off the transient influence of arbitrary initial values. The desired posterior sample is thus the residual set corresponding to position $i, i = M+1, \ldots, N$, for sufficiently large N.
BE*s* of the unknown parameters under SELF are given by

$$\tilde{\alpha}_{BSMC} = \frac{1}{N-M} \sum_{i=M+1}^{N} \alpha_i$$
$$\tilde{\beta}_{BSMC} = \frac{1}{N-M} \sum_{i=M+1}^{N} \beta_i \tag{22}$$

Also, the approximate BE*s* of the unknown parameters under GELF are given by

$$\begin{aligned}
\tilde{\alpha}_{BGMC} &= \left(\frac{1}{N-M}\sum_{i=M+1}^{N}{\alpha_i}^{-q}\right)^{-\frac{1}{q}} \\
\tilde{\beta}_{BGMC} &= \left(\frac{1}{N-M}\sum_{i=M+1}^{N}{\beta_i}^{-q}\right)^{-\frac{1}{q}}
\end{aligned} \tag{23}$$

where $q > 0$ represents OE $\left(\tilde{\alpha}_{BG_1MC}, \tilde{\beta}_{BG_1MC}\right)$ and $q < 0$ represents UE $\left(\tilde{\alpha}_{BG_2MC}, \tilde{\beta}_{BG_2MC}\right)$. The approximate BE*s* of the unknown parameters under LINEX are given by

$$\begin{aligned}
\tilde{\alpha}_{BLMC} &= -\frac{1}{q}\log\left(\frac{1}{N-M}\sum_{i=M+1}^{N}e^{-q\alpha_i}\right) \\
\tilde{\beta}_{BLMC} &= -\frac{1}{q}\log\left(\frac{1}{N-M}\sum_{i=M+1}^{N}e^{-q\beta_i}\right)
\end{aligned} \tag{24}$$

where $q > 0$ represents OE $\left(\tilde{\alpha}_{BL_1MC}, \tilde{\beta}_{BL_1MC}\right)$ and $q < 0$ represents UE $\left(\tilde{\alpha}_{BL_2MC}, \tilde{\beta}_{BL_2MC}\right)$.

6. Simulation study

In this section, foremost data from LBLLD is generated via simulation. Next, APT-IIC samples from the obtained LBLLD data is extracted by following procedure of Balakrishnan and Sandhu (1995) and Ng *et al.* (2009). The algorithm described below resamples according to APT-IIC from continuous lifetime distribution.

1. Set the values of n, m, Θ, T and $R = (R_1, R_2, \ldots, R_m)$, as desired by the sample situation.
2. Simulate m random variables from $U(0,1)$ as $U_1, U_2, \ldots, U_m$.
3. Set $W_i = U_i^{1/(i+R_m+R_{m-1}+\cdots+R_{m-i+1})}$ for $i = 1, 2, \ldots, m$.
4. Set $V_i = 1 - W_m W_{m-1} \cdots W_{m-i+1}$ for $i = 1, 2, \ldots, m$. Then $V_1, V_2, \ldots, V_m$ is the m progressive type II censored sample from $U(0,1)$.
5. Set $X_i = F^{-1}(V_i; \Theta)$ for $i = 1, 2, \ldots, m$, where $F^{-1}(.; \Theta)$ is the quantile function of the lifetime distribution. Thus $X_1, X_2, \ldots, X_m$ represent the required m progressive type II censored sample from the specified distribution $F(.)$.
6. Next, identify the value of k, where $X_{k:m:n} < T < X_{k+1:m:n}$ and discard the sample $X_{k+2:m:n}, \cdots, X_{m:m:n}$.
7. Simulate the first $m-k-1$ order statistics from a truncated distribution considered as $\frac{f(x)}{1-F(x_{k+1:m:n})}$ with sample size $\left(n - \sum_{i=1}^{k} R_i - J - 1\right)$ as $X_{k+2:m:n}, X_{k+3:m:n} \cdots, X_{m:m:n}$.

Two random samples of sizes $n = 30,\ 50$ have been generated from $LBLL\,(\alpha, \beta)$ by setting $\alpha = 1.5$, $\beta = 3.2$. Three different preset values for $T = 4.5, 5, 5.5$ are taken. MLEs are computed from these samples through numerical approximation N-R method in R software. OpenBUGS is utilised for generating posterior samples using MCMC by fixing the hyper parameters at $b = 2, c = 4$. 10000 samples with 2000 samples for burn-in period are generated. We have taken $q = 2$ for OE and $q = -2$ for UE. Bayes estimates under MCMC have been calculated using (22)-(24).

Table 1 represents the APT-IIC schemes which we have used in simulation. Estimated values of scale parameter α and shape parameter β with the associated mean square error (MSE) of MLEs and BEs under three loss functions for different combinations of (n, T) are presented in Tables (2)-(4) respectively. Following inferences based on these tables:

1. For unknown scale parameter α, Bayes estimates give values which are closer to true values for all the three values of T. For some censoring schemes, MLEs also give better estimates in terms of minimum MSEs. Among Bayes estimates, LINEX OE gives values with higher precision for most of the censoring scheme.
2. Same pattern can be seen for the unknown shape parameter β also. Among Bayes estimates, LINEX UE gives better values as they have minimum MSEs than others.

Tables (5)-(6) represent the LL, UL and AL of ACI and BCI, HPD1 and HPD2 of the parameters under study for the three selected values of T respectively. The following relationship is obtained for both unknown parameters

$$HPD1_{AL} < HPD2_{AL} < BCI_{AL} < ACI_{AL}$$

This is true for all the three values of T. Here, HPD1 refers 89%HPD and HPD2 refers 95%HPD intervals. AL of all intervals are decreased as we increase the value of m for different censoring schemes.

Table 1: Progressive type II censoring schemes used in simulation

n	m	**CS**	**R**
30	**10**	**CS[1]**	2*10
		CS[2]	0*3, 5*4, 0*3
	20	**CS[3]**	0*5, 1*10, 0*5
		CS[4]	0*8, 2*5, 0*7
50	**25**	**CS[5]**	1*25
		CS[6]	0*10, 5*5, 0*10
	35	**CS[7]**	0*20, 1*15
		CS[8]	1*15, 0*20

Table 2: Different point estimates of unknown parameters for $T = 4.5$

n	m	CS		MLE		Bayes Estimates											
						SELF		GELF				LINEX					
								α		β		α		β			
				α	β	α	β	OE	UE	OE	UE	OE	UE	OE	UE		
30	10	CS[1]	Est.	1.4192	3.4185	1.4434	3.352	1.4348	1.4463	3.3245	3.3612	1.4352	1.4515	3.292	3.4153		
			MSE	0.0354	0.2904	0.0033	0.0235	0.0043	0.0029	0.0159	0.0264	0.0043	0.0024	0.0088	0.047		
		CS[2]	Est.	1.1054	3.0748	1.1285	3.1237	1.1212	1.1311	3.0949	3.1334	1.123	1.1344	3.0655	3.1866		
			MSE	0.1933	0.266	0.138	0.0063	0.1435	0.1362	0.0115	0.0049	0.1422	0.1337	0.0185	0.0008		
	20	CS[3]	Est.	1.3804	3.5995	1.4535	4.3541	1.4522	1.454	4.3415	4.3583	1.4522	1.4548	4.3179	4.391		
			MSE	0.0414	0.4477	0.0022	1.3323	0.0023	0.0021	1.3034	1.342	0.0023	0.002	1.2501	1.4188		
		CS[4]	Est.	1.3321	3.5215	1.399	3.8636	1.3976	1.3994	3.8548	3.8666	1.3977	1.4002	3.8412	3.8864		
			MSE	0.0551	0.3772	0.0102	0.4406	0.0105	0.0101	0.429	0.4445	0.0105	0.01	0.4113	0.4714		
50	25	CS[5]	Est.	1.5107	3.7999	1.6384	3.8142	1.6371	1.6388	3.8077	3.8164	1.6369	1.6399	3.7977	3.831		
			MSE	0.0178	0.5217	0.0192	0.3774	0.0188	0.0193	0.3694	0.3801	0.0188	0.0196	0.3574	0.3984		
		CS[6]	Est.	1.2228	3.3717	1.1115	2.8917	1.1097	1.1121	2.8876	2.893	1.1102	1.1129	2.8839	2.8995		
			MSE	0.0938	0.3066	0.1509	0.0951	0.1524	0.1504	0.0976	0.0943	0.152	0.1499	0.1	0.0903		
	35	CS[7]	Est.	1.3492	3.9822	1.2338	3.4561	1.2332	1.234	3.4536	3.457	1.2333	1.2342	3.4504	3.4619		
			MSE	0.0397	0.8116	0.0709	0.0656	0.0712	0.0708	0.0644	0.0661	0.0711	0.0706	0.0627	0.0686		
		CS[8]	Est.	1.7086	3.9886	1.731	3.7049	1.7302	1.7313	3.7017	3.7059	1.73	1.732	3.6971	3.7127		
			MSE	0.0575	0.7673	0.0534	0.255	0.053	0.0535	0.2518	0.256	0.0529	0.0538	0.2472	0.2629		

Table 3: Different point estimates of unknown parameters for $T = 5$

n	m	CS		MLE		Bayes Estimates									
						SELF		GELF				LINEX			
								α		β		α		β	
				α	β	α	β	OE	UE	OE	UE	OE	UE	OE	UE
30	10	CS[1]	Est.	1.4348	3.3993	1.2226	3.1913	1.2148	1.2252	3.1674	3.1993	1.2163	1.229	3.1417	3.2437
			MSE	0.0383	0.2994	0.077	0.0006	0.0814	0.0755	0.0016	0.0006	0.0805	0.0735	0.0039	0.0026
		CS[2]	Est.	1.0874	2.9197	1.2066	3.1374	1.1966	1.2099	3.105	3.1482	1.1986	1.2147	3.0721	3.209
			MSE	0.2049	0.279	0.0861	0.0043	0.0921	0.0842	0.0094	0.003	0.0909	0.0814	0.0167	0.0005
	20	CS[3]	Est.	1.3757	3.4025	1.3706	3.2105	1.368	1.3715	3.2034	3.2128	1.3683	1.373	3.1955	3.2257
			MSE	0.0443	0.276	0.0168	0.0002	0.0174	0.0165	0.0001	0.0003	0.0174	0.0161	0.0001	0.0008
		CS[4]	Est.	1.3289	3.3174	1.3549	2.9123	1.3516	1.356	2.9064	2.9143	1.352	1.3578	2.9009	2.9238
			MSE	0.0588	0.2396	0.0211	0.0828	0.022	0.0208	0.0863	0.0817	0.0219	0.0202	0.0895	0.0763
50	25	CS[5]	Est.	1.5069	3.6342	1.6052	3.916	1.604	1.6056	3.9096	3.9181	1.6039	1.6064	3.8995	3.9326
			MSE	0.0185	0.3164	0.0111	0.5127	0.0108	0.0112	0.5037	0.5158	0.0108	0.0113	0.4894	0.5368
		CS[6]	Est.	1.2098	3.1185	1.146	3.2756	1.1445	1.1465	3.2698	3.2775	1.1448	1.1472	3.2629	3.2883
			MSE	0.1025	0.159	0.1253	0.0058	0.1264	0.1249	0.005	0.0061	0.1262	0.1245	0.0041	0.0079
	35	CS[7]	Est.	1.3446	3.843	1.3981	3.1858	1.3972	1.3984	3.1836	3.1866	1.3973	1.3989	3.1811	3.1905
			MSE	0.0402	0.5686	0.0104	0.0002	0.0106	0.0103	0.0003	0.0002	0.0106	0.0102	0.0004	0.0001
		CS[8]	Est.	1.7291	3.7901	1.7277	4.1016	1.7271	1.7279	4.0981	4.1028	1.727	1.7284	4.092	4.1113
			MSE	0.0684	0.4845	0.0518	0.813	0.0516	0.0519	0.8066	0.8151	0.0515	0.0521	0.7957	0.8305

Table 4: Different point estimates of unknown parameters for $T = 5.5$

n	m	CS		MLE		Bayes Estimates									
						SELF		GELF				LINEX			
								α		β		α		β	
				α	β	α	β	OE	UE	OE	UE	OE	UE	OE	UE
30	10	CS[1]	Est.	1.4335	3.3875	1.376	2.8097	1.3612	1.3808	2.7892	2.8165	1.3627	1.3893	2.7722	2.849
			MSE	0.0371	0.2799	0.0155	0.1526	0.0194	0.0143	0.169	0.1473	0.019	0.0124	0.1833	0.1235
		CS[2]	Est.	1.0831	2.8149	1.0673	2.661	1.0635	1.0686	2.6443	2.6666	1.0646	1.0703	2.632	2.6918
			MSE	0.211	0.3229	0.1872	0.2908	0.1905	0.1861	0.309	0.2848	0.1896	0.1847	0.3228	0.2586
	20	CS[3]	Est.	1.352	3.2312	1.4541	3.854	1.4519	1.4548	3.8415	3.8582	1.452	1.4562	3.8222	3.8866
			MSE	0.0519	0.1687	0.0021	0.428	0.0023	0.0021	0.4118	0.4335	0.0023	0.0019	0.3874	0.4717
		CS[4]	Est.	1.3035	3.1502	1.294	3.197	1.2917	1.2947	3.1901	3.1994	1.292	1.2959	3.1823	3.212
			MSE	0.0668	0.1693	0.0425	0.0001	0.0434	0.0421	0.0002	0.0001	0.0433	0.0416	0.0004	0.0002
50	25	CS[5]	Est.	1.4992	3.5115	1.6836	3.5613	1.6821	1.684	3.556	3.5631	1.6819	1.6852	3.5488	3.5738
			MSE	0.0169	0.2139	0.0337	0.1306	0.0332	0.0339	0.1268	0.1319	0.0331	0.0343	0.1217	0.1398
		CS[6]	Est.	1.1935	2.9376	1.3022	3.168	1.3008	1.3027	3.1637	3.1694	1.301	1.3035	3.1591	3.1769
			MSE	0.1116	0.1741	0.0391	0.0011	0.0397	0.0389	0.0014	0.001	0.0396	0.0386	0.0017	0.0006
	35	CS[7]	Est.	1.3395	3.7328	1.3375	3.5832	1.3369	1.3378	3.5806	3.584	1.337	1.3381	3.577	3.5894
			MSE	0.0442	0.4306	0.0264	0.1469	0.0266	0.0263	0.1449	0.1475	0.0266	0.0262	0.1421	0.1517
		CS[8]	Est.	1.7275	3.6091	1.7821	3.7001	1.7812	1.7824	3.6971	3.7011	1.7811	1.7831	3.6926	3.7076
			MSE	0.0681	0.2788	0.0796	0.2502	0.0791	0.0797	0.2471	0.2512	0.079	0.0801	0.2427	0.2578

Table 5: Different interval estimates for scale parameters

n	m	CS		T=4.5				T=5				T=5.5			
				ACI	BCI	HPD1	HPD2	ACI	BCI	HPD1	HPD2	ACI	BCI	HPD1	HPD2
30	10	CS[1]	LL	0.855	1.262	1.299	1.268	0.86	1.066	1.088	1.062	0.857	1.150	1.193	1.151
			UL	1.982	1.621	1.589	1.625	2.009	1.382	1.347	1.378	2.009	1.602	1.560	1.603
			AL	1.126	0.359	0.290	0.357	1.149	0.316	0.259	0.316	1.152	0.452	0.367	0.452
		CS[2]	LL	0.630	1.010	1.002	1.000	0.590	1.039	1.060	1.030	0.567	1.003	1.000	1.000
			UL	1.580	1.295	1.230	1.267	1.584	1.388	1.347	1.376	1.598	1.201	1.139	1.174
			AL	0.949	0.285	0.228	0.267	0.994	0.349	0.287	0.346	1.030	0.198	0.139	0.174
	20	CS[3]	LL	1.025	1.382	1.391	1.382	0.999	1.276	1.292	1.276	0.959	1.362	1.376	1.360
			UL	1.735	1.525	1.508	1.524	1.752	1.466	1.447	1.465	1.744	1.544	1.523	1.542
			AL	0.709	0.143	0.117	0.142	0.752	0.190	0.155	0.189	0.784	0.182	0.147	0.182
		CS[4]	LL	0.986	1.327	1.338	1.326	0.960	1.250	1.271	1.246	0.920	1.206	1.223	1.205
			UL	1.677	1.468	1.453	1.467	1.697	1.462	1.442	1.458	1.686	1.382	1.364	1.380
			AL	0.690	0.141	0.115	0.141	0.736	0.212	0.171	0.212	0.766	0.176	0.141	0.175
50	25	CS[5]	LL	1.185	1.563	1.576	1.567	1.166	1.535	1.548	1.534	1.146	1.603	1.620	1.602
			UL	1.835	1.713	1.697	1.716	1.847	1.674	1.662	1.673	1.852	1.762	1.747	1.760
			AL	0.649	0.150	0.121	0.149	0.681	0.139	0.114	0.139	0.706	0.159	0.127	0.158
		CS[6]	LL	0.923	1.039	1.054	1.037	0.887	1.078	1.092	1.075	0.852	1.233	1.248	1.231
			UL	1.522	1.185	1.172	1.182	1.532	1.215	1.203	1.212	1.534	1.371	1.359	1.368
			AL	0.598	0.146	0.118	0.145	0.644	0.137	0.111	0.137	0.682	0.138	0.111	0.137
	35	CS[7]	LL	1.122	1.192	1.199	1.193	1.11	1.344	1.353	1.342	1.098	1.292	1.300	1.289
			UL	1.576	1.277	1.267	1.277	1.578	1.454	1.442	1.452	1.58	1.385	1.376	1.382
			AL	0.453	0.085	0.068	0.084	0.468	0.110	0.089	0.110	0.482	0.093	0.076	0.093
		CS[8]	LL	1.413	1.670	1.681	1.673	1.410	1.677	1.685	1.676	1.389	1.719	1.729	1.719
			UL	2.003	1.791	1.779	1.793	2.047	1.779	1.767	1.777	2.065	1.844	1.830	1.843
			AL	0.590	0.121	0.098	0.120	0.637	0.102	0.082	0.101	0.675	0.125	0.101	0.124

Table 6: Different interval estimates for shape parameters

n	m	CS		T=4.5				T=5				T=5.5			
				ACI	**BCI**	**HPD1**	**HPD2**	**ACI**	**BCI**	**HPD1**	**HPD2**	**ACI**	**BCI**	**HPD1**	**HPD2**
30	10	CS[1]	**LL**	1.846	2.880	2.954	2.874	1.837	2.764	2.823	2.744	1.831	2.444	2.486	2.431
			UL	4.990	3.855	3.742	3.844	4.960	3.658	3.553	3.633	4.943	3.207	3.109	3.190
			AL	3.144	0.975	0.788	0.970	3.123	0.894	0.730	0.889	3.112	0.763	0.623	0.759
		CS[2]	**LL**	1.738	2.663	2.709	2.649	1.674	2.659	2.716	2.646	1.633	2.343	2.376	2.333
			UL	4.411	3.631	3.497	3.612	4.165	3.685	3.550	3.668	3.996	3.019	2.925	3.004
			AL	2.673	0.968	0.788	0.963	2.491	1.026	0.834	1.022	2.362	0.676	0.549	0.671
	20	CS[3]	**LL**	2.426	3.972	4.050	3.970	2.312	2.972	3.007	2.970	2.212	3.501	3.564	3.494
			UL	4.772	4.740	4.668	4.737	4.492	3.456	3.401	3.453	4.250	4.217	4.141	4.209
			AL	2.345	0.768	0.618	0.767	2.179	0.484	0.394	0.483	2.037	0.716	0.577	0.715
		CS[4]	**LL**	2.382	3.567	3.615	3.567	2.265	2.702	2.732	2.696	2.169	2.957	2.999	2.950
			UL	4.660	4.162	4.096	4.162	4.369	3.126	3.076	3.118	4.131	3.440	3.392	3.430
			AL	2.277	0.595	0.481	0.595	2.104	0.424	0.344	0.422	1.962	0.483	0.393	0.480
50	25	CS[5]	**LL**	2.665	3.561	3.606	3.562	2.561	3.660	3.702	3.652	2.484	3.344	3.375	3.339
			UL	4.934	4.074	4.020	4.074	4.706	4.171	4.116	4.161	4.538	3.787	3.734	3.780
			AL	2.269	0.513	0.414	0.512	2.145	0.511	0.414	0.509	2.054	0.443	0.359	0.441
		CS[6]	**LL**	2.412	2.720	2.745	2.710	2.254	3.052	3.096	3.053	2.142	2.984	3.008	2.981
			UL	4.331	3.069	3.028	3.058	3.982	3.506	3.459	3.506	3.732	3.355	3.310	3.351
			AL	1.918	0.349	0.283	0.348	1.727	0.454	0.363	0.453	1.589	0.371	0.302	0.370
	35	CS[7]	**LL**	2.971	3.305	3.330	3.301	2.875	3.053	3.069	3.052	2.799	3.426	3.454	3.430
			UL	4.993	3.605	3.573	3.601	4.810	3.321	3.288	3.319	4.666	3.740	3.707	3.743
			AL	2.022	0.300	0.243	0.300	1.935	0.268	0.219	0.267	1.867	0.314	0.253	0.313
		CS[8]	**LL**	2.971	3.534	3.557	3.531	2.834	3.907	3.943	3.904	2.711	3.531	3.556	3.533
			UL	5.006	3.878	3.839	3.875	4.745	4.299	4.258	4.294	4.507	3.874	3.835	3.874
			AL	2.035	0.344	0.282	0.344	1.910	0.392	0.315	0.390	1.796	0.343	0.279	0.341

7. Real data illustration

A real dataset is taken from Teza (2015, ch 4). Data describes the mechanical properties such as initial rate of absorption, water absorption, dry density and compressive strength of 50 units of clay bricks and fly ash bricks. This data set has also been analysed by Nagamani *et. al* (2021) for estimating common scale parameter of two logistic populations.

In the present paper, we have taken the data on compressive strength of fly ash bricks to illustrate the proposed method. The uncensored data are composed of 50 observations (3.62, 4.74, 9.88, 5.93, 6.09, 6.94, 6.32, 5.30, 5.14, 4.55, 4.03, 7.36, 3.57, 3.98, 4.03, 4.74, 7.32, 3.23, 5.38, 7.18, 6.07, 3.62, 6.64, 5.58, 5.23, 3.95, 5.86, 5.58, 6.97, 5.05, 4.35, 4.55, 4.79, 4.03, 4.74, 7.58, 3.62, 6.01, 3.99, 6.04, 4.74, 7.21, 3.61, 5.69, 7.21, 6.40, 3.55, 8.70, 4.35, 7). Table (7) indicates that LBLLD is suitable for the given data set based on negative log likelihood and three information criteria.

Table 7: Fitting of data to three different distributions

Sr no.	Reliability model	-LogL	AIC	BIC	AICC
1.	Logistic β=scale α=location	91.350	186.701	190.525	186.956
2.	Log logistic β=shape α=scale	89.547	183.094	186.918	183.349
3.	LBLL β=shape α=scale	89.493	182.986	186.81	183.241

Further two APT-II censored samples for $T = 3.99, 7.18$ are extracted with $n = 50, m = 30, R = (0^{*10}, 2^{*10}, 0^{*10})$. The censored samples thus obtained are (3.23, 3.55, 3.57, 3.61, 3.62, 3.62, 3.62, 3.95, 3.98, 3.99, 4.03, 4.03, 4.03, 4.35, 4.35, 4.55, 4.55, 4.74, 4.74, 4.74, 4.74, 4.79, 5.05, 5.14, 5.23, 5.30, 5.38, 5.58, 5.58, 5.69) and (3.23, 3.55, 3.57, 3.61, 3.62, 3.62, 3.62, 3.95, 3.98, 3.99, 4.03, 4.35, 4.55, 4.74, 5.05, 5.05, 5.14, 5.23, 5.30, 5.30, 5.38, 5.58, 5.58, 5.58, 5.69, 5.93, 6.07, 6.40, 6.97, 7.18). MLEs and Bayes estimates of the unknown parameters are given in Table (8) for the selected values of T. LL, UL, and AL of different confidence and credible intervals for the unknown scale and shape parameters are given in Table (9) for the selected values of T. Among Bayesian intervals, HPD1 interval has shortest length.

ACI is shortest classical interval followed by Boot-p and Boot-t. The following relationship can be seen for scale parameter

$$HPD1_{AL} < HPD2_{AL} < BCI_{AL} < ACI_{AL} < Boot - p_{AL} < Boot - t_{AL}$$

AL of intervals are increased amomg classical intervals while it is decreased among Bayesian intervals as we increase the value of T. Similarly, for shape parameter, AL is decreased for

Table 8: MLEs and Bayes estimates under APT-IIC real data for both values of T

(n,m)	T		MLE	Bayes Estimates				
				SELF	GELF		LINEX	
(50,30)	3.99	α	4.2667	4.2484	4.2476	4.2486	4.2461	4.2506
		β	10.4737	9.5466	9.5137	9.5574	9.3386	9.7536
	7.18	α	4.502	4.4906	4.4899	4.4908	4.4886	4.4927
		β	7.6081	7.3946	7.3837	7.3982	7.341	7.448

all intervals with increment in the value of T. For β, we get

$$HPD1_{AL} < HPD2_{AL} < BCI_{AL} < Boot - t_{AL} < Boot - p_{AL} < ACI_{AL}$$

This can be seen in intervals plots (Figure 2-3). MCMC trace plots are presented in Figure 4.

Table 9: Different intervals of unknown parameters under APT-IIC real data

	T		ACI	BOOT-T	BOOT-P	BCI	HPD1	HPD2
α	3.99	LL	3.999	3.945	4.007	4.158	4.171	4.163
		UL	4.533	4.559	4.565	4.341	4.321	4.345
		AL	0.534	0.614	0.558	0.183	0.15	0.182
	7.18	LL	4.103	3.972	4.096	4.405	4.42	4.408
		UL	4.901	4.978	5.008	4.58	4.561	4.581
		AL	0.798	1.006	0.912	0.175	0.141	0.173
β	3.99	LL	7.489	8.25	9.04	8.65	8.806	8.64
		UL	13.458	12.17	13.37	10.44	10.24	10.41
		AL	5.968	3.92	4.33	1.79	1.434	1.77
	7.18	LL	5.471	5.918	6.637	6.942	7.041	6.926
		UL	9.744	8.742	9.984	7.849	7.791	7.826
		AL	4.273	2.824	3.347	0.907	0.75	0.9

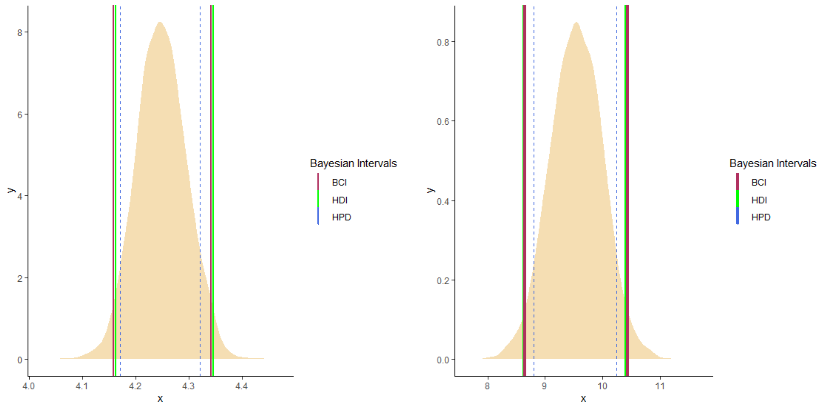

Figure 2: Interval plot for ***T=3.99***

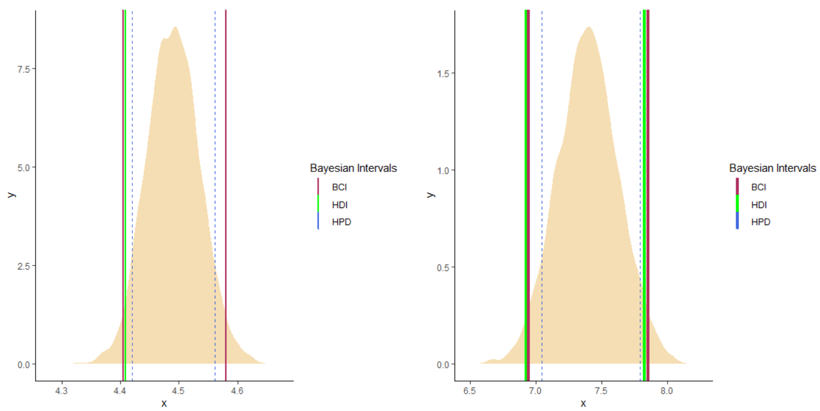

Figure 3: Interval plot for ***T=7.18***

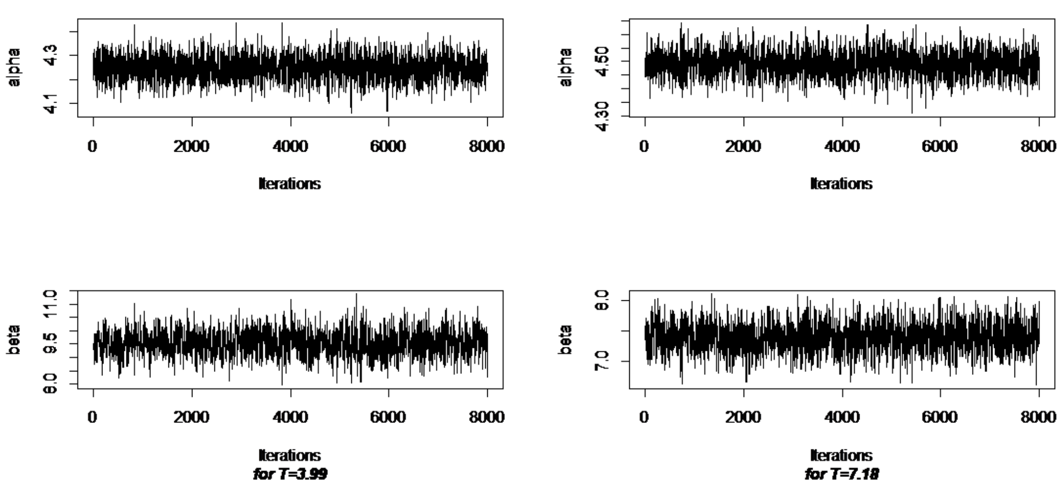

Figure 4: MCMC trace plot for ***T=3.99*** **and** ***T=7.18***

8. Conclusion

In this paper, we have considered the point and interval estimations of the parameters of the LBLLD based on an APT-IIC scheme for Bayes and non-Bayes settings. This censoring scheme allows us to choose the next censoring number taking into account both the previous censoring numbers and previous failure times. The MLEs, the bootstrap confidence intervals and the ACIs based on the observed Fisher information matrix have been discussed. We assume the Jefferys and gamma priors for the unknown scale and shape parameters respectively and provide the Bayes estimators under the assumptions of SELF, GELF and LINEX loss functions. It is also found that when both parameters are unknown, the expressions for Bayes estimates cannot be obtained in explicit form. The Gibbs sampling technique is employed to generate MCMC samples. Credible intervals and HPD intervals have also been constructed. A real life example is discussed to verify the proposed methodology. The performance of different methods is compared via a Monte Carlo simulation.

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Conflict of interest

The authors do not have any financial or non-financial conflict of interest to declare for the research work included in this article.

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