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A Combinatorial Arrangement of Six Elements

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Abstract

A combinatorial arrangement of 15 unordered pairs of six elements in 6 x 6 square with empty diagonals and off-diagonal cells having three disjoint unordered pairs, such that in each row and in each column all the 15 unordered pairs occur once only, is presented. This arrangement is called triangular Room square of order six.

Key words: 6×6 squares with empty diagonals, triangular designs, Room squares

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1. Introduction

Combinatorial arrangements have drawn the attention of mathematicians as long back as Kirkman's School Girl Problem (1850), see Dey (2010), pp.47. Some relevant definitions in the context of the paper follow as:

Block design

An equi-replicate, equi-block size, incomplete block design is an arrangement of v elements into b blocks such that: each block contains k ($<v$) distinct elements and each element occurs in r blocks.

Triangular design

Let there be $v = n(n-1)/2$ elements($n \geq 5$) which are arranged in an $n \times n$ array such that the positions on the principal diagonal are left blank, the $n(n-1)/2$ positions above the principal diagonal are filled with the v elements and the positions below the principal diagonal are also filled with the same v elements in such a manner that the resultant arrangement is symmetric about the principal diagonal. Then, two treatments are called first associates if they belong to the same row or same column of the array, otherwise they are second associates.

Alternatively, a triangular association scheme may be defined as : Let X be a set of n elements, 1,2, . . . , n. Then by $v = n(n-1)/2$ elements are denoted by pairs $(i, j) = (j, i)$, $i \neq j$, $i, j \in X$. Any two elements are first associates if there is an element in common between the pairs,

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otherwise they are second associates. Triangular designs are special class of two associate class partially balanced incomplete block designs.

Resolvable block designs

A block design with parameters v, b, r, k is said to be resolvable, if its blocks can be partitioned into r sets of blocks, each set containing b|r blocks, such that every set contains each treatment precisely once only (see, Dey 2010).

Doubly resolvable designs

Following Stinson (1980) and elsewhere the definition is as follows: Let $R = R_1, R_2, \ldots, R_r$ and $T = T_1, T_2, \ldots, T_r$ be two resolutions of one and the same design. These two resolutions are orthogonal if $|R_i \cap T_j| \leq 1$, $1 \leq i, j \leq r$. When a design has at least two orthogonal resolutions it is called doubly resolvable. Let D be *a* doubly resolvable design with orthogonal resolution classes $R = R_1, R_2, \ldots, R_r$ and $T = T_1, T_2, \ldots, T_r$. Now, form an $r \times r$ array **A**, where the rows are indexed by the elements of R and columns by the elements of T.

The (i, j)-th cell of **A** contains $R_i \cap T_j$. Here, any cell will either be empty or contain a block of D. Obviously this array **A** is row-wise as well as column-wise resolvable.

Here, a combinatorial arrangement of 15 unordered pairs of six elements in 6×6 square with empty diagonals and off-diagonal cells having three disjoint unordered pairs, such that in each row and in each column all the15 unordered pairs occur once only, is presented. This arrangement is called triangular Room square of order six. The results obtained here might be of interest for possible applications in cryptography [see, Chaudhary and Seberry (1998), Zhelezova (2011), Topolova and Zhelezova (2014)]. For terminologies, definitions see Raghavarao and Padgett (2005), Dey (2010).

2. The Arrangement

Given below is an arrangement of 15 unordered pairs of six elements in a 6 × 6 square with empty diagonals and off-diagonal cells having three disjoint unordered pairs, such that in each row and in each column all the 15 unordered pairs occur once only.

Table.1: 6 × 6 square with empty diagonals

-	(12,34,56)	(15,24,36)	(16,23,45)	(14,26,35)	(13,25,46)
(12,34,56)	-	(13,26,45)	(14,25,36)	(15,23,46)	(16,24,35)
(14,25,36)	(16,23,45)	-	(12,35,46)	(13,24,56)	(15,26,34)
(13,26,45)	(15,24,36)	(12,35,46)	-	(16,25,34)	(14,23,56)
(16,24,35)	(13,25,46)	(14,23,56)	(15,26,34)	-	(12,36,45)
(15,23,46)	(14,26,35)	(16,25,34)	(13,24,56)	(12,36,45)	-

Further by the transformation: 12→1, 13→2, 14→3, 15→4, 16→5, 23→6, 24→7, 25→8, 26→9, 34→10, 35 →11, 36→12, 45→13, 46→14, 56→15, the above arrangement may alternatively be viewed as a doubly resolvable, triangular design with empty diagonals. The parameters of this triangular design are: v =15, r = 6, k = 3, b = 30, $n_1 = 8$, $n_2 = 6$, $\lambda_1 = 0$, $\lambda_2 = 2$.

The triangular designs are partially balanced incomplete block designs based on two associate triangular association schemes. For details, see Clatworthy (1973), Dey (2010), Raghavarao and Padgett (2005). Tables of triangular designs may be found in Clatworthy (1973).

A doubly resolvable BIBD (v, 2, 1) is a Room square. There is extensive literature on this topic. A doubly resolvable BIBD (v, k, λ) is a Generalised Room square and vice-versa (Stinson, 2004). Doubly resolvable, BIBD and Group divisible designs are studied by Vanstone (1980). The arrangement obtained above may be known as triangular Room square (TRS) of order six. Analogously, Room Squares based on BIB designs and Group divisible designs may be denoted as BIBRS and GDRS respectively. A special feature of the above triangular Rooms square is that the (i, j)-th cell entry $R_i \cap T_i$ is zero, while $R_i \cap T_j$ is 1.

This triangular design is reported in Clatworthy (1973) as duplicate of T16. A resolvable solution of this triangular design was reported in Sinha (1973). It is not known if the resolvable solution of triangular design T3 given in Sinha and Dey (1982) also has a similar arrangement in 6 × 6 squares with empty principal diagonal.

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