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Resolvability of a BIB Design of Takeuchi (1962)

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Abstract

A $(3, 3, 9)$-resolvable solution of a BIB design with parameters: $v = 21, b = 35, r = 15, k = 9, \lambda = 6$, and listed as T47 in the Table of Takeuchi (1962), is obtained. The resolvable solution is obtained by decomposing the incidence matrix into incidence matrices of smaller BIB designs.

Key words: Balanced incomplete block design; Resolvable solution; Circulant matrix.

AMS Subject Classifications: 62K10; 05B05

1. The solution

Let the incidence matrix $\mathbf{N}$ of a balanced incomplete block (BIB) design may be decomposed into submatrices as $\mathbf{N} = [\mathbf{N}_1|\mathbf{N}_2|\ldots|\mathbf{N}_t]$ such that each row sum of $\mathbf{N}_i$ $(1 \leq i \leq t)$ is α_i. Then the design is $(\alpha_1, \alpha_2, \ldots, \alpha_t)$-resolvable [see Kageyama (1976)]. If $\alpha_1 = \alpha_2 = \cdots = \alpha_t = \alpha$ then the design is α-resolvable.

The following solution of a BIB design with parameters: $v = 21, b = 35, r = 15, k = 9, \lambda = 6$ using the method of differences may be found in Takeuchi (1962):
$[0_0, 1_0, 2_0, 4_0, 0_1, 1_1, 2_1, 4_1, 2_2]$; $[0_0, 6_0, 5_0, 3_0, 6_2, 4_2, 3_2, 2_2, 0_1]$; $[0_1, 6_1, 5_1, 3_1, 6_2, 4_2, 3_2, 2_2, 0_0]$;
$[0_0, 2_0, 6_0, 1_1, 3_1, 4_1, 1_2, 2_2, 4_2]$; $[1_0, 3_0, 4_0, 0_1, 2_1, 6_1, 1_2, 2_2, 4_2]$ mod 7.

The incidence matrix $\mathbf{N}$ of the design may be decomposed into block submatrices as follows:

$$\mathbf{N} = (\mathbf{N}_1|\mathbf{N}_2|\mathbf{N}_3)=$$

$$\left(\begin{array}{c|c|ccc}
\beta+\beta^3+\beta^4 & \mathbf{I}_7+\beta^2+\beta^6 & \mathbf{I}_7+\beta+\beta^2+\beta^4 & \mathbf{I}_7+\beta^3+\beta^5+\beta^6 & \mathbf{I}_7 \\
\mathbf{I}_7+\beta^2+\beta^6 & \beta+\beta^3+\beta^4 & \mathbf{I}_7+\beta+\beta^2+\beta^4 & \mathbf{I}_7 & \mathbf{I}_7+\beta^3+\beta^5+\beta^6 \\
\beta+\beta^2+\beta^4 & \beta+\beta^2+\beta^4 & \beta^2 & \beta^2+\beta^3+\beta^4+\beta^6 & \beta^2+\beta^3+\beta^4+\beta^6
\end{array}\right)$$

where $\mathbf{I}_7$ is the identity matrix of order 7 and $\beta = circ(0\ 1\ 0 \ldots 0)$ is a permutation circulant matrix of order 7 such that $\beta^7 = \mathbf{I}_7$. Since each row sum of the block matrices $\mathbf{N}_1, \mathbf{N}_2$ and $\mathbf{N}_3$ are 3, 3 and 9 respectively, the BIB design is $(3, 3, 9)$-resolvable. The resolvable solution is given below in Table 1.

Further repeating above solution of the BIB design three times, we obtain a 9-resolvable solution of BIB design with parameters: $v = 21, b = 105, r = 45, k = 9, \lambda = 18$.

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This solution may be considered new as this is not reported in the tables of Kageyama (1973), Kageyama and Mohan (1983) and Subramani (1990).

Table 1: (3,3,9)-resolvable solution of the BIB design T47

Replication I	Replication II
(4, 5, 7, 8, 9, 13, 18, 20, 21)	(1, 2, 6, 11, 12, 14, 18, 20, 21)
(1, 5, 6, 9, 10, 14, 15, 19, 21)	(2, 3, 7, 8, 12, 13, 15, 19, 21)
(2, 6, 7, 8, 10, 11, 15, 16, 20)	(1, 3, 4, 9, 13, 14, 15, 16, 20)
(1, 3, 7, 9, 11, 12, 16, 17, 21)	(2, 4, 5, 8, 10, 14, 16, 17, 21)
(1, 2, 4, 10, 12, 13, 15, 17, 18)	(3, 5, 6, 8, 9, 11, 15, 17, 18)
(2, 3, 5, 11, 13, 14, 16, 18, 19)	(4, 6, 7, 9, 10, 12, 16, 18, 19)
(3, 4, 6, 8, 12, 14, 17, 19, 20)	(1, 5, 7, 10, 11, 13, 17, 19, 20)

Replication III		
(1, 4, 6, 7, 8, 11, 13, 14, 20)	(1, 8, 9, 10, 12, 16, 18, 19, 20)	(1, 2, 3, 5, 8, 16, 18, 19, 20)
(1, 2, 5, 7, 8, 9, 12, 14, 21)	(2, 9, 10, 11, 13, 17, 19, 20, 21)	(2, 3, 4, 6, 9, 17, 19, 20, 21)
(1, 2, 3, 6, 8, 9, 10, 13, 15)	(3, 10, 11, 12, 14, 15, 18, 20, 21)	(3, 4, 5, 7, 10, 15, 18, 20, 21)
(2, 3, 4, 7, 9, 10, 11, 14, 16)	(4, 8, 11, 12, 13, 15, 16, 19, 21)	(1, 4, 5, 6, 11, 15, 16, 19, 21)
(1, 3, 4, 5, 8, 10, 11, 12, 17)	(5, 9, 12, 13, 14, 15, 16, 17, 20)	(2, 5, 6, 7, 12, 15, 16, 17, 20)
(2, 4, 5, 6, 9, 11, 12, 13, 18)	(6, 8, 10, 13, 14, 16, 17, 18, 21)	(1, 3, 6, 7, 13, 16, 17, 18, 21)
(3, 5, 6, 7, 10, 12, 13, 14, 19)	(7, 8, 9, 11, 14, 15, 17, 18, 19)	(1, 2, 4, 7, 14, 15, 17, 18, 19)

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