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Markov-Switching GARCH and Mixture of GARCH-type Models for Accuracy in Forecasting

Godfrey Joseph Saqware[1] and Ismail B.[2]
[1]*Deparment of Statistics, Mangalore University, Karnataka, India*
[2]*Department of Statistics, Yenepoya* (*Deemed to be University*), *Mangalore, Karnataka, India*

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Abstract

The stock markets all over the world have been experiencing fluctuations. These fluctuations are due to some political and administrative decisions. For example, in Tanzania, structural transformations in the economic sectors have been happening time after time, which resulted in fluctuations in the stock market. In this paper, the stock market's volatility was modelled using Markov-Switching GARCH (MS GARCH) and the mixture of GARCH type models. The Bayesian Information Criterion (BIC) was employed to get the best GARCH type models with respective conditional distributions. The GARCH (1, 1) with skewed normal distribution, EGARCH (1, 1) with student's *t*-distribution and Glosten, Jagannathan and Runkle-GARCH (GJR GARCH) (1, 1) with generalized error distribution selected for further analysis. The study found that the three-state heterogeneous regime MS GARCH and Mixture of the selected GARCH type models provide the best fit and the dynamic feedback between components for the DSEI All-share stock data. The Bayesian Markov Chain Monte Carlo (MCMC) method resulted in an acceptance rate of 28.7%, which lies between 20% and 50% as the requirement of the rule of thumb. The different sample sizes employed on the Bayesian MCMC technique have also proven the fitted model's powerfulness since all acceptance sampler rate falls within the range. Furthermore, the forecasting results for the next 30, 60, 90, and 120 days have shown a continuous fluctuation in the DSEI All-share Stock Index.

Key words: MS GARCH; GARCH; GJR GARCH; Bayesian MCMC; DES.

1. Introduction

The global economy has been experiencing fluctuations in response to policy directives. Stock Market performance is also affected by the economic and other related instabilities. Stochastic models play essential roles in the forecasting stock market volatility. The famous symmetric models such as Autoregressive Heteroscedasticity (ARCH) (Engle, 1982) and Generalized ARCH (Bollerslev, 1986); and asymmetric models namely Exponential GARCH (Nelson, 1991), Threshold GARCH (Glosten *et al*., 1993), GARCH-M (Hamilton, 1994) and Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) (Baillie *et al*.,1996) were extended from Generalized ARCH model to capture asymmetric characteristics in the stock market. The complexity and uncertainty of the financial time series have resulted in the continuous modification of the GARCH-type model. To handle volatility prediction in the stock market. The best way forward to the question is to allow the GARCH model parameters to vary over time by considering the regime-switching. A single regime is

Corresponding Author: Godfrey Joseph Saqware
Email: godjose70@yahoo.com

inflexible; thus, it's crucial to incorporate regime-switching. The Markov-Switching GARCH (MS GARCH) model is a new approach introduced in more than one decade. It enables a quick adaption to the unconditional volatility variations (Oseifua and Korkpoe, 2018).

This paper will be the first in incorporating the heterogeneous regimes-switching model to the DSEI All-share Index log-returns. The annotated article is distributed into four major sections. The first section covers the introduction; the second section will cover the literature reviews on modelling volatility based on Markov-Switching GARCH models. Part three will formally lay out the materials and methods employed in the study. Section four covers empirical analysis and discussion. Finally, section five concludes the paper.

2. Related Work

The MS GARCH models' history goes back to introducing the mixed normal distribution that was combined with the GARCH-type structure (MN-GARCH) that captures conditional variance and the dynamic feedback between the components(Haas et al., 2004). Recent studies have shown that volatility predictions using GARCH type models failed to capture the stock market volatility's actual variation due to the regime changes and volatility dynamics (Korkpoe and Kawor, 2018). The MS GARCH models are flexible alternatives to GARCH models with fixed parameters. Bayesian inference estimate based on data augmentation has solved the path dependence problem. Furthermore, the model is useful for capturing changes in the dynamics and volatilities in the financial market (Bauwens *et al*., 2014). Based on this perspective, the effective and efficient prediction of the market volatility has been crucial for smooth economic growth.

Moreover, the era of fast-growing technology and computer applications resulted in gaps in the modelling and forecasting volatility. The MS GARCH models with regime-switching have shown the best forecasting performances based on the management perspective compared to forecasting based on a single regime (Ardia *et al*., 2016). The MS GARCH model provides a better evaluation of volatility by imposing the higher volatility component in each state, which results in the dynamic structure regime that reacts to the various species of shocks (Alemohammad *et al*., 2016). The MS GARCH with the two-regime has exhibited the best in-sample performance with an inverted leverage effect in low and high volatility regimes and their volatility dynamics (Ardia *et al*., 2019). The regime-switching models revealed a better volatility forecast than the constant-variance or a single-regime GARCH (Bibi and Ghezal, 2018). Thus, the earlier researchers have tried to model volatility without defining clearly the process of obtaining conditional distributions. The study involves selecting the conditional distribution and applying the three-state heterogeneous MS GARCH and the Mixture of GARCH-type models to the stock data.

3. Materials and Methods

3.1. The Markov-switching GARCH models

The method allows the regime-switching in the conditional variance process. I*f* $\mathrm{I}_{t-1} \equiv \{r_{t-1},\, i > 0\}$ is the information set denoted by I_{t-1} for the observation up to $t-1$. The general Markov Switching specification is given by

$$r_t \setminus (s_t = k, \mathrm{I}_{t-1}) \sim D(0, h_{t,k}, \xi_k) \tag{1}$$

where $D(0, h_{t,k}, \xi_k)$ refers to a continuous distribution with mean zero, $h_{t,k}$ is the time-varying variance and ξ_k is the additional shape parameter and k is the number of regimes. The Stochastic variable s_t defined under the discrete space $\{1,2,...,K\}$ characterizes the Markov-Switching GARCH Models.

3.1.1. Markov-switching ARCH model

The ARCH model (Engle, 1982) that incorporates k the regime can be written as:-

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} r_{t-1}^2 \; ; k = 1, 2, 3, ..., K \tag{2}$$

The $\alpha_{0,k}, \alpha_{1,k}, \beta_k \geq 0$ is required for the positivity while in each regime for the covariance-stationarity $\alpha_{1,k} < 1$.

3.1.2. Markov-switching GARCH model

The GARCH model (Bollerslev, 1986) that incorporates the k regimes Markov-Switching is given by

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} r_{t-1}^2 + \beta_k h_{k,t-1} ; k = 1, 2, 3, ..., K \tag{3}$$

The $\alpha_{0,k}, \alpha_{1,k}, \beta_k \geq 0$ is required for the positivity while in each regime for the covariance-stationarity $\alpha_{1,k} + \beta_k < 1$.

3.1.3. Markov-switching EGARCH model

The Exponential GARCH Model (Nelson, 1991) that incorporates the k regimes is given by

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} \left(\left| \eta_{k,t-1} \right| - E\left(\left| \eta_{k,t-1} \right| \right) \right) + \alpha_{2,k} \eta_{k,t-1} + \beta_k \ln(h_{k,t-1}) \; ; \\ k = 1, 2, 3, ..., K \tag{4}$$

The model considers the leverage effects, where the past negative values influence conditional volatility compared to the previous positive values. The covariance stationary in each regime to be achieved requires $\beta_k < 1$.

3.1.4. Markov-switching GJR GARCH model

The GJR GARCH model (Glosten *et al.*, 1993) captures as well the asymmetric conditional volatility. The GJR GARCH model that incorporates the k regimes Markov-Switching is expressed by

$$h_{k,t} = \alpha_{0,k} + \left(\alpha_{1,k} + \alpha_{2,k}\tau\left\{r_{t-1} < 0\right\}\right)r_{t-1}^{2} + \beta_k h_{k,t-1} \;;\; k = 1,2,3,\ldots,K \tag{5}$$

To ensure positivity, $\alpha_{0,k}, \alpha_{1,k} > 0$ and $\alpha_{2,k}, \beta_k \geq 0$, whereas for the covariance stationarity

$$\alpha_{1,k} + \alpha_{2,k} E\left[\eta_{k,t}^{2}\tau\left\{\eta_{k,t} < 0\right\}\right] + \beta_k < 1 \tag{6}$$

3.1.5. Markov-switching TGARCH model

The Threshold GARCH model of Zakoian (1994) included conditional volatility as the dependent variable instead of the conditional variance. The model in (7) incorporates Markov-Switching.

$$h_{k,t}^{1/2} \equiv \alpha_{0,k} + \left(\alpha_{1,k}\tau\left\{r_{t-1} \geq 0\right\} - \alpha_{2,k}\tau\left\{r_{t-1} < 0\right\}\right)r_{t-1} + \beta_k h_{k,t-1}^{1/2} \quad ;k = 1,2,3,\ldots,K \tag{7}$$

To ensure positivity, $\alpha_{0,k}, \alpha_{1,k} > 0$; $\alpha_{2,k} > 0$ and $\beta_k \geq 0$, while for the covariance stationarity

$$\alpha_{1,k}^{2} + \beta_k^{2} - 2\beta_k\left(\alpha_{1,k} + \alpha_{2,k}\right)E\left[\eta_{k,t}\tau\left\{\eta_{k,t} < 0\right\}\right] - \left(\alpha_{1,k}^{2} - \alpha_{2,k}^{2}\right)E\left[\eta_{k,t}^{2}\tau\left\{\eta_{k,t} < 0\right\}\right] < 1 \tag{8}$$

3.2. Conditional distributions

The specification of the model to be completed requires conditional distributions. The commonly used conditional distributions are Normal distribution, Student's *t*-distribution and the generalized error distribution.

Normal distribution: The probability distribution function for the normal distribution is given by

$$f_N(r) \equiv \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\eta^2}, \qquad \eta \in \sim \tag{9}$$

Student's *t* distribution: The probability distribution function for the Student's *t*-distribution is given by

$$f_S(\eta;V) \equiv \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{(v+1)\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1 + \frac{\eta^2}{(v-2)}\right)^{-\left(\frac{v+1}{2}\right)}, \qquad \eta \in \sim \tag{10}$$

The $\Gamma(\bullet)$ is the Gamma function, and $v > 2$ for the existence of the second moment.

Generalized error distribution: The probability distribution function for the generalized error distribution (GED) is given by

$$f_{GED}(\eta;V) \equiv \frac{Ve^{-\frac{1}{2}\left|\frac{\eta}{\lambda}\right|^{v}}}{\lambda 2^{\left(1+\frac{1}{v}\right)}\Gamma\left(1/v\right)}, \quad \lambda \equiv \left(\frac{\Gamma\left(1/v\right)}{4^{1/v}\Gamma\left(3/v\right)}\right)^{1/2}, \quad \eta \in {}^{\sim} \ \ and \ \ v > 0 \tag{11}$$

Skewed distributions: Recently, the unimodal standardized distributions introduced skewness in estimating the EGARCH, GJR GARCH, and TGARCH models (Trottier and Ardia, 2016).

3.3. Model estimation

The estimation of the MS GARCH and the Mixture of the GARCH type models can either based on the Bayesian Markov Monte Carlo (MCMC) or Maximum Likelihood (ML) methods. The two methods require evaluation under the maximum likelihood function.

3.3.1. The maximum likelihood method

Let $\psi = \left(\theta_1, \xi_1, \ldots, \theta_K, \xi_K, \mathrm{P}\right)$ be the vector of the model parameters whose likelihood function is given by

$$L\left(\psi \,|\, \mathrm{I}_T\right) \equiv \prod_{t=1}^{T} f\left(r_t \,|\, \psi, \mathrm{I}_{t-1}\right) \tag{12}$$

where $f\left(r_t \,|\, \psi, \mathrm{I}_{t-1}\right)$ refers to the density function of r_t given the past observations, I_{t-1} is the information set and ψ the model parameters.

The MS GARCH model for the conditional density of r_t is given by

$$f\left(r_t \,|\, \psi, \mathrm{I}_{t-1}\right) = \sum_{i=1}^{K} \sum_{j=1}^{K} p_{i,j} z_{i,t-1} f_D\left(r_t \,|\, s_t = j, \psi, \mathrm{I}_{t-1}\right) \tag{13}$$

where $z_{i,t-1} \equiv P\left(s_{t-1} = i \,|\, \psi, \mathrm{I}_{t-1}\right)$ refers to the filter probability of state i and time $t-1$.

Moreover, for the Mixture of the GARCH type models, the conditional density function for r_t becomes

$$f\left(r_t \,|\, \psi, \mathrm{I}_{t-1}\right) = \sum_{i=1}^{K} w_j f_D\left(r_t \,|\, s_t = j, \psi, \mathrm{I}_{t-1}\right) \tag{14}$$

Combining the two, the conditional density of the r_t in-state or component $s_t = k$ given ψ and I_{t-1} is denoted by $f_D\left(r_t \,|\, s_t = k, \psi, \mathrm{I}_{t-1}\right)$.

3.3.2. Bayesian Markov Monte Carlo (MCMC) method

The MCMC estimation requires a combination of the likelihood with a truncated prior $f(\psi)$ to building kernel of the posterior distribution $f\left(\psi \mid \mathrm{I}_T\right)$. The unknown form of posterior distribution should be approximated based on the simulation techniques. The random-walk Metropolis sampler generates draws of the posterior distribution (Vihola, 2012).

Furthermore, for the Bayesian estimation, the likelihood function is combined with a prior $f(\psi)$ in building kernel of the posterior distribution $f\left(\psi \mid \mathrm{I}_T\right)$. The build prior based on the independent diffuse priors is done as follows

$$
\begin{aligned}
& f(\boldsymbol{\psi})=f\left(\boldsymbol{\theta}_1, \xi_1\right) \ldots f\left(\boldsymbol{\theta}_K, \xi_K\right) f(\mathbf{P}) \\
& f\left(\boldsymbol{\theta}_k, \boldsymbol{\xi}_k\right) \propto f\left(\boldsymbol{\theta}_k\right) f\left(\boldsymbol{\xi}_k\right) \mathrm{I}\left\{\left(\boldsymbol{\theta}_k, \boldsymbol{\xi}_k\right) \in CSC_k\right\} \quad (k=1,2,3, \ldots, K) \\
& f\left(\boldsymbol{\theta}_k\right) \propto f_N\left(\boldsymbol{\theta}_k ; \boldsymbol{\mu \theta}_k, \; diag\left(\boldsymbol{\sigma}^{\mathbf{2}} \boldsymbol{\theta}_{\mathbf{k}}\right)\right) \mathrm{I}\left\{\boldsymbol{\xi}_{k, 1} \in PC_k\right\} \quad (k=1,2,3, \ldots, K) \\
& f\left(\boldsymbol{\xi}_k\right) \propto f_N\left(\boldsymbol{\xi}_k ; \mu \boldsymbol{\xi}_k, \; diag\left(\sigma^2 \boldsymbol{\xi}_k\right)\right) \mathrm{I}\left\{\xi_{k, 1}>0, \xi_{k, 2}>2\right\} \quad (k=1,2,3, \ldots, K) \\
& f(\mathbf{P}) \propto \prod_{i=1}^K\left(\prod_{j=1}^K p_{i, j}\right) \mathrm{I}\left\{0<p_{i, i}<1\right\}
\end{aligned} \tag{15}
$$

where CSC_k and PC_k denotes the covariance stationarity and the positivity conditions for the regime k , respectively.

3.4. Data specification

The website hosted at https://www.investing.com/indices/tanzania-all-share is the source of the data for this investigation. The data contain information such as stock day open, low, high and close prices. The dataset ranged from 15/08/2009 to 20/1/2020 with a total of 2067 observations. Tanzania DSEI All-share Index has the market capitalization-weighted index with 1000 base reference. The index includes all stocks listed at the Dar Es Salaam Stock Exchange categorized into Commercial Banks, Cement Companies, Tanzania Breweries companies, Tanzania Cigarette Company and Liquefied Natural Gas and Oil companies.

4. Empirical Results and Discussions

4.1. Descriptive statistics for the log-returns

The summary statistics reported in Table 1 below shows that the mean log-return is the positive value of 0.0004 and a standard deviation of 0.0175. The most significant price drop is –32.09%, and the largest price increase is 32.81%. Data exhibit a positive skewness and a very large excess kurtosis. The Jarque-Bera test for normality has shown that the data is not normally distributed ($p < 0.05$). The suitable distributions for this kind of data are the skewed conditional distributions.

Table 1: Summary statistics

Statistic	Mean	Min	Max	SD	Skewness	Kurtosis	JB	JB(*p*-value)
Value	0.0004	–0.321	0.3281	0.0175	0.3847	168.563	3647640	2.20E-16

The computations of the log-returns for the DSEI All-share closing price is given by

$r_t = \log P_t - \log P_{t-1}$, where r_t is the daily log-return, while P_t *and* P_{t-1} are the stock prices for time t *and* $t-1$ respectively.

4.2. Time series of the DSEI all-share stock index

The sharp decline in the DSEI All-share stock Index at different periods was a result of various factors. The drop observed almost every year since DSEI All-share Stock Market started its operation in August 2011. The Central Bank of Tanzania (BOT) merged some banks because of bankruptcy in 2018/2019. Moreover, the closure and liquidation non-performing banks aimed at stabilizing the banking system. The current President of the United Republic of Tanzania Hon. Dr John Pombe Magufuli has tried to support the economy; nevertheless, some companies failed to survive since he came into power in October 2015. The log-returns exhibited a continuous and frequent period of high and low volatility since the stock market started operation in August 2011. Figure 1 below shows a time series plot for DSEI All-share Stock Index.

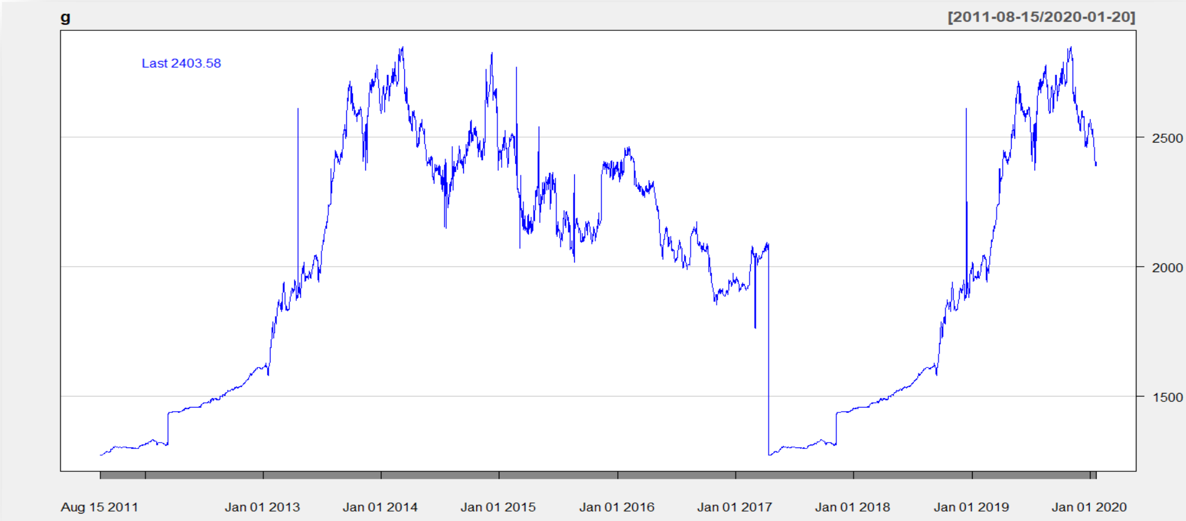

Figure 1: Time series plot of the DSEI series

The plots of the DSEI All-share Index log-returns have also revealed a presence of volatility clustering. The prolonged stock instability observed between 2013 and 2018. The fluctuation has resulted in the collapse and closure of different companies and merging of the key players in the DSEI All-share Stock Market. Figure 2 below shows the plotting of the log-returns series for the DSEI All-share Index.

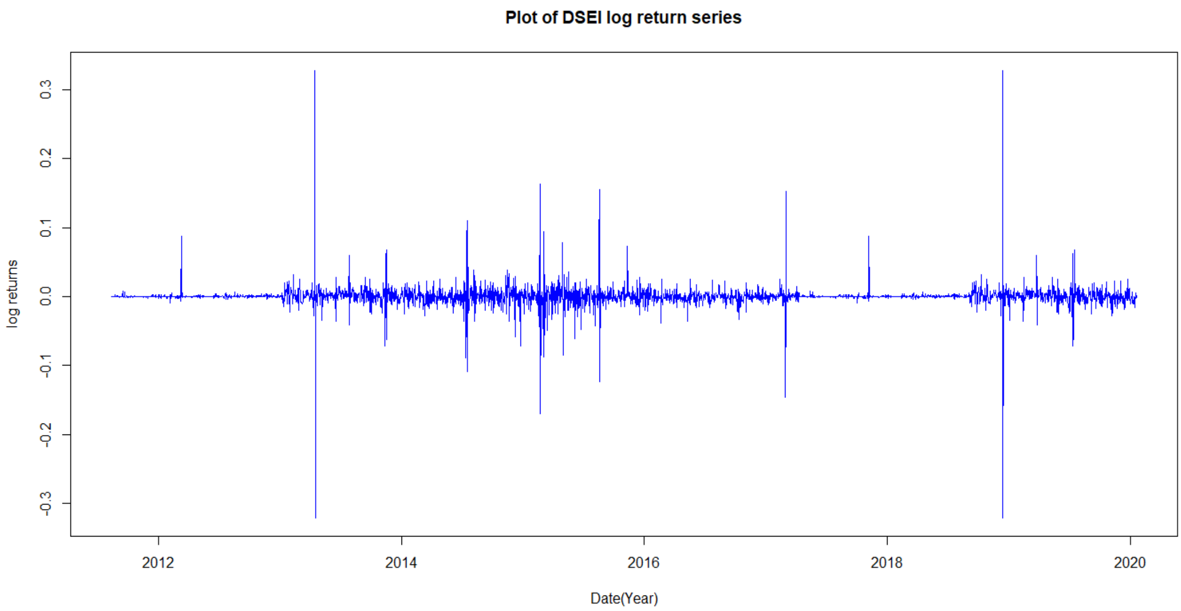

Figure 2: Plot of the DSEI log-return series

The Augmented Dickey-Fuller test confirmed that the return series is stationary since the p-value is less than 5% (p-value = 0.01). Moreover, the GARCH model building has been done by first confirming the (G) ARCH effects in the stock data. The ARCH-LM test gave a

$\chi^2 = 1097.3$ with 10 degrees of freedom and p-value less than 5%. Therefore, we conclude the presence of (G) ARCH in the log-return series.

4.3. Model estimation and selection

This paper extended the study by Haas (2004) with the Markov-Switching GARCH (MSGARCH) and mixture GARCH type model, incorporating three heterogeneous state regimes and conditional distributions. The selected GARCH type models based on the minimum Bayesian Information Criterion (BIC) are; GARCH (1, 1) with the skewed student-t-distribution (sstd), EGARCH (1, 1) with the skewed generalized distribution (sged) and GJR GARCH (1, 1) with the skewed generalized distribution (sged). The study by (Catania et al., 2018) has come up with regimes without telling where they made the GARCH-type models selection with the conditional distributions. Still, in this paper, we made some initial effort before proceeding with model estimation. Table 2 below shows the result of the model selection based on the BIC.

Table 2: BIC values for conditional distributions

Model	GARCH (1,1)			EGARCH (1,1)			GJR GARCH (1,1)		
Distribution	snorm	sstd	sged	snorm	sstd	sged	snorm	sstd	sged
BIC	–6.3832	**–7.8578**	–4.1269	–6.974	–7.9305	**–8.2764**	–6.3403	–7.8551	**–8.097**

4.3.1. Model estimation based on maximum likelihood (ML) method

The estimated parameters depict the difference in the volatility process from one regime to another. The difference in negative past reactions levels of unconditional volatility of $\alpha_{(2,1)} \approx 0.00$, $\alpha_{(2,2)} \approx -0.0148$ and $\alpha_{(3,2)} \approx 0.7213$ for the three-state heterogeneous regimes. The volatility persistence for the model reports $\alpha_{(1,1)} + \frac{1}{2}\alpha_{(2,1)} + \beta_1 \approx 0.904$, $\alpha_{(1,2)} + \frac{1}{2}\alpha_{(2,2)} + \beta_2 \approx 1.0866$ and $\alpha_{(1,3)} + \frac{1}{2}\alpha_{(2,3)} + \beta_3 \approx 0.9990$ in three-states, respectively. The result implies that the first regime characterized by low unconditional volatility, a strong volatility reaction to the past negative log-returns, and the low volatility process persistence. The second and third regimes are characterized by high unconditional volatility, weak volatility reaction to the past negative log-returns, and high volatility. The market participants can categorize regime one as “tranquil market condition” compared to regimes two and three, which has the “turbulent market condition”. Table 3 shows the estimated model summary based on the ML technique.

Table 3: Estimated model summary for ML technique

K	Parameter	Estimate	*t* value	*P*-value
1	$\alpha_{(0,1)}$	0.0000	1.0000E+8	<1e-16
	$\alpha_{(1,1)}$	0.1092	5.43394E+21	<1e-16
	β_1	0.7948	6.91272E+22	<1e-16
	nu_1	3.7560	1.26907E+22	<1e-16
	xi_1	0.9784	6.05698E+21	<1e-16
2	$\alpha_{(0,2)}$	–0.4566	–3.67276E+20	<1e-16
	$\alpha_{(1,2)}$	0.1421	3.03187E+20	<1e-16
	$\alpha_{(2,2)}$	–0.0148	–9.8235E+19	<1e-16
	β_2	0.9519	3.19745E+22	<1e-16
	nu_2	1.2262	3.75161E+23	<1e-16
	xi_2	0.9855	1.41481E+22	<1e-16
3	$\alpha_{(0,3)}$	0.000	3.05331E+21	<1e-16
	$\alpha_{(1,3)}$	0.2527	2.74911E+22	<1e-16
	$\alpha_{(2,3)}$	0.7213	3.26518E+24	<1e-16
	β_3	0.3856	3.64538E+26	<1e-16
	nu_3	0.7000	7.4042E+36	<1e-16
	xi_3	0.9998	1.0202E+8	<1e-16

The stable probabilities of being in the three states are about 32.15%, 63.97%, and 3.88% respectively. The results indicate that the likelihood of being in the three states differs. The unconditional probabilities reports; 4.57%, 40.41% and 20.97 for state 1, state 2, and state 3 respectively. Thus, this implies a high unconditional probability in state two compared to the rest of the states. Moreover, all the three states' smooth probabilities are closer to one; this evidence a sharp increase in the volatility process. Table 4 below shows the stable probabilities, unconditional volatility, and smooth probabilities for the three states.

Table 4: Results for the three states probabilities and unconditional volatility

State	1	2	3
Stable Probabilities	0.3215	0.6397	0.0388
Unconditional Volatility	0.0457	0.4041	0.2097
Smooth Probability	0.9675	0.9958	0.9994

4.3.2. Model estimation based on Bayesian Markov Chain Monte Carlo (MCMC) method

The estimation model based on Maximum likelihood seems to be not more powerful than the Bayesian MCMC method due to a stuck in the local maximum which may result in unreliable estimates (Billio and Cavicchioli, 2017; Das and Yoo, 2004). The adaptive MCMC estimation always based on the posterior distribution. The MCMC sampler requires decomposition and the Eigenvalue computations, therefore, it largely depends on the Linear Algebra library (Vihola, 2012). The proposed three-state heterogeneous regimes MS GARCH and the Mixture of GARCH type models such as GARCH, EGARCH and GJR GARCH model with skewed normal, Student's *t* and generalized error conditional distributions respectively, has used 10000 iterations, 5000 burn-in phase and ten thinning factors in the estimation of the Bayesian Markov Monte Carlo (MCMC) estimation. The acceptance rate of 28.5% was obtained in the model estimation. The acceptance rate lied within 20%-50% 'rule of thumb' as recommended (Chib and Greenberg, 1995; Roberts and Rosenthal, 2009).

The number of independent and identically distributed sample draws from the posterior distribution is required for the relative numerical efficiency (RNE). This determines how quickly the convergence of the algorithm occurs. The checking of the MCMC sampling scheme for the output quality is necessary (Geweke, 1992; Korkpoe and Kawor, 2018). In the proposed model, we found the values of RNE relatively low (<1), which are considered better for the fast convergence of the MCMC chains. Table 5 below shows the summary of parameter estimated for the three states heterogeneous regimes for the MS GARCH and Mixture of GARCH type models using the Bayesian MCMC method.

The increase of the number of the MCMC draws say 15000, 20000, 30000, 50000, 100000, 500000 and 1000000 for the estimation three state heterogeneous regimes MS GARCH and the Mixture of GARCH type models resulted into the same range of the acceptance rate of (20-50) %. The acceptance rate reveals the consistency of the estimated model. The best model is usually based on the minimum Deviance Information Criterion (Spiegelhalter *et al*., 2002). Moreover, at least 4000 burn-in phase is recommended for the model estimation (Raftery and Lewis, 1992). The thin of every tenth minimizes the posterior draws autocorrelations. The high autocorrelations can result in bias and Monte Carlo standard errors. The number of researchers has raised concern on the appropriate number of thinning, but the thinning number of 10L sounds good (Link and Eaton, 2012; Owen, 2017). Table 6 shows the estimated model summary for the different MCMC sample draws.

Table 5: Estimated model summary for the Bayesian MCMC technique

K	Parameter	Mean	SD	SE	TSSE	RNE
1	$\alpha_{(1,1)}$	0.3509	0.0325	0.001	0.0086	0.0142
	β_1	0.5768	0.024	0.0008	0.0065	0.0138
	nu_1	2.2511	0.0431	0.0014	0.0153	0.0079
	xi_1	0.992	0.0325	0.001	0.003	0.1180
2	$\alpha_{(0,2)}$	–0.0351	0.0427	0.0013	0.0223	0.0036
	$\alpha_{(1,2)}$	0.4177	0.0391	0.0012	0.0056	0.0496
	$\alpha_{(2,2)}$	–0.0983	0.0201	0.0006	0.0019	0.1128
	β_2	0.9955	0.0042	0.0001	0.0024	0.0031
	nu_2	0.7337	0.0273	0.0009	0.0174	0.0025
	xi_2	0.9999	0.0005	0.0000	0.0000	0.3027
3	$\alpha_{(1,3)}$	0.1737	0.0461	0.0015	0.0260	0.0031
	$\alpha_{(2,3)}$	0.0021	0.0018	0.0001	0.0008	0.0048
	β_3	0.7816	0.0194	0.0006	0.0100	0.0038
	nu_3	0.7000	0.0000	0.0000	0.0000	0.0033
	xi_3	14.1021	6.3244	0.200	3.9098	0.0026

Table 6: Estimated model summary for different MCMC sample draws

nithin	nburn	nmcmc	Acceptance Rate	DIC
10	5000	15000	27.6%	–22609.639
10	5000	20000	28.3%	–10986.542
10	5000	25000	27.5%	–23339.681
10	5000	30000	28.0%	–21846.993
10	5000	50000	27.4%	–23881.170
10	5000	100000	27.2%	–22595.525
10	5000	500000	26.1%	–19464.122
10	5000	1000000	25.7%	–11562.360

4.3.3. Forecasting of the conditional volatility based on Bayesian MCMC estimated model

The prediction based on 30, 60, 90 and 120 days ahead has shown fluctuations in the DSEI All-share Index log-returns. The study identified the number of future periods of high conditional volatility; March for the 60 days forecasts in (b), April for the 90 days forecasts in (c), and May for the 120 days forecasts in (d) for four months in the year 2020. The results still show instability in the stock market for the next four months. Figure 3 below shows stock volatility for the next four months.

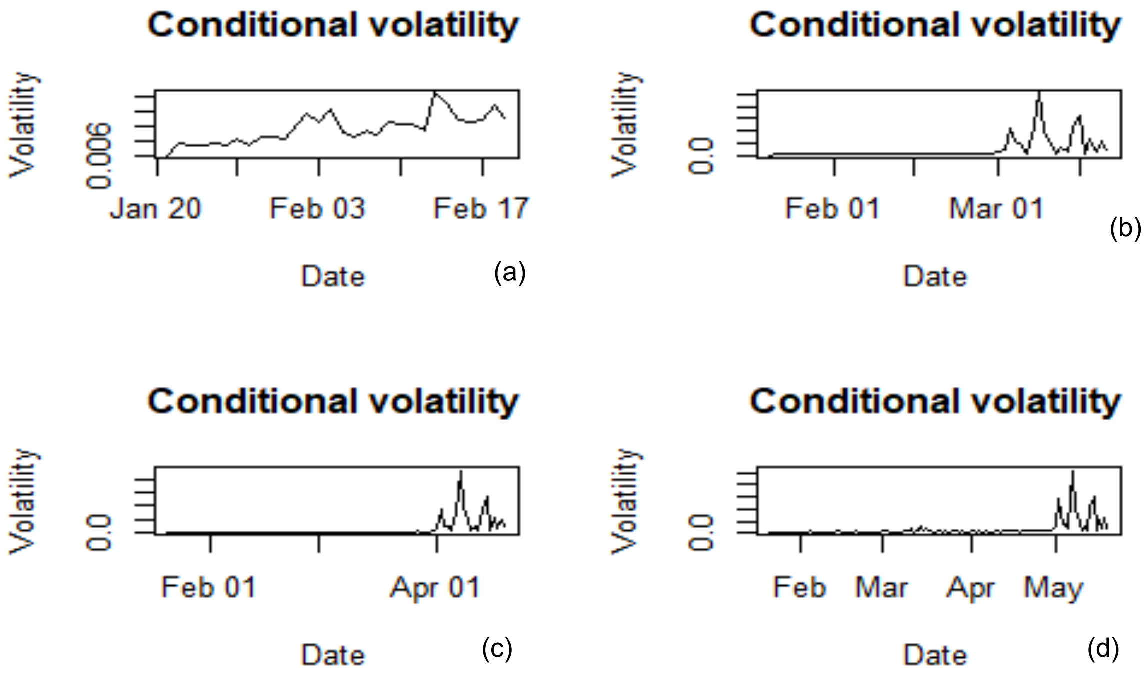

Figure 3: Conditional volatility forecasting

4. Conclusion

The stock market volatility will continue to be topical in finance since traders and investors observe historical data trends for future investments. The insertion of the regime changes become indispensable to model volatility in the stock market. The changing economic condition has caused persistent fluctuations in the stock market across regimes. The study acts as a benchmark for the countries to adopt the best trading policies and strategies to buffer downside. The Central Bank of Tanzania (BOT) reported a decline of shares, trading, market capitalization and underperformance of the Dar Es Salaam Stock Exchange (BOT, 2018).

Moreover, stockbrokers like the five social security funds were joined into Public Service Social Security Fund (PSSSF) that serves public-sector employees and the National Social Security Fund (NSSF) for the private-sector employees and self-employed persons in 2018/2019. The situation has disturbed the performance of the stock market. Eventually, the country's business environment's control and regulation become inevitable for the stock markets' growth.

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