Statistics and Applications {**ISSN 2452-7395** (online)}
Volume 17, No. 2, 2019 (New Series), pp 1-6

Row-Column Partial Triallel Cross Designs

M.K. Sharma and Yabebal Ayalew
Department of Statistics, Addis Ababa University, Addis Ababa, Ethiopia

Received: March 17, 2018; Revised: January 20, 2019; Accepted: January 22, 2019

Abstract

In the present paper we are presenting three series of row-column design for partial triallel cross which have been obtained through row- column designs of complete diallel cross method (4). These designs require only $p\ (p-1)/2$ experimental units. *i.e.*, $1/(p-2)^{\text{th}}$ crosses in comparison to complete triallel cross designs. These designs can be used to improve the quantitative traits of economic and nutritional importance in crops and animals. It has been established that the three-way hybrids are more stable than pure lines and single cross hybrids and exhibits individual as well as population buffering mechanisms because of the genetic base.

Key words: Row-column design, partial triallel cross, mating design.

---

1. Introduction

Mating design involving multi–allele cross $m\ (\geq\ 2)$ lines play very important role to study the genetic properties of a set of inbred lines in plant breeding experiments. Most commonly used mating designs are diallel or a two-way cross $(m\ = 2)$. Suppose there are p inbred lines and it is desired to perform a diallel cross experiment involving $v\ =\ p(p-1)/2$ crosses of the type $(i\ \times\ j) = (j\ \times\ i)$ for $i\ , j\ =\ 1,2, \dots, p$, this type of mating design is called complete diallel cross (CDC) method (4) of Griffing (1956) . When we arrange these $v\ =\ p(p-1)/2$ crosses in row-column set up the mating design becomes row-column design for CDC method (4).

Triallel crosses form an important class of mating designs, which are used for studying the genetic properties of a set of inbred lines in plant breeding experiments. For p inbred lines, the number of different crosses for a complete triallel experiment is $3^pC_3 = p(p-1)(p-2)/2$ of the type $(i \times j) \times\ k, i \neq j\ \neq k\ =\ 0, 1, 2, \dots, p-1$. Rawlings and Cockerham (1962) were the first to introduce mating designs for triallel crosses. Triallel cross (TC) experiments are generally conducted using a completely randomized design (CRD) or a randomized complete block (RCB) design as environmental design involving 3^pC_3 crosses.

Even with a moderate number of parents, say $p = 10$, in a TC experiment, the number of crosses becomes unmanageable to be accommodated in homogeneous blocks. For such situations, Hinkelmann (1965) developed partial triallel crosses (PTC) involving only a sample of all possible crosses by establishing a correspondence between PTC and generalized

Correspondence Author: M.K. Sharma
Email: mk_subash@yahoo.co.in

partially balanced incomplete block designs (GPBIBD). Ponnuswamy and Srinivasan (1991) and Subbarayan (1992) obtained PTC using a class of balanced incomplete block (BIB) designs. Let n denote the total number of crosses (experimental units) involved in a triallel experiment. It is desired to compare the lines with respect to their general combining abilities, the specific combining abilities being not included in the model.

Other research workers who contributed in this area are Arora and Aggarwal (1984, 1989), Ceranka *et al*. (1990). More details on TC experiments can be found in Hinkelmann (1975) and Narain (1990).

In this paper we are presenting methods of construction of three series row-column designs for PTC experiments through row-column designs of CDC experiment method (4) of Sharma and Tadesse (2017).

2. Preliminary

Let d be a row-column design with k rows and b columns for CDC method (4) involving p lines and $n = bk$. For the data obtained from d, we postulate the following model.

$$\boldsymbol{y} = \mu + \mathbf{1}_n + \boldsymbol{\Delta}_1'\boldsymbol{g} + \boldsymbol{\Delta}_2'\boldsymbol{\beta} + \boldsymbol{\Delta}_3'\boldsymbol{\gamma} + \boldsymbol{e} \tag{2.1}$$

where $\boldsymbol{y}$ is an $n \times 1$ vector of observed responses, μ is the general mean $\boldsymbol{\beta}$, $\boldsymbol{g}$ and $\boldsymbol{\gamma}$ are column vectors of p gca parameters, k row effects and b column effects, respectively, $\boldsymbol{\Delta}_1'(n \times p)$, $\boldsymbol{\Delta}_2'(n \times k)$, $\boldsymbol{\Delta}_3'(n \times b)$ are the corresponding design matrices, respectively and $\boldsymbol{e}$ denotes the vector of independent random errors having mean $\mathbf{0}$ and covariance matrix $\sigma^2\boldsymbol{I}_n$.

Let $\boldsymbol{N}_{d1} = \boldsymbol{\Delta}_1\boldsymbol{\Delta}_2'$ be the $p \times k$ incidence matrix of lines vs rows and $\boldsymbol{N}_{d2} = \boldsymbol{\Delta}_1\boldsymbol{\Delta}_3'$ be the $p \times b$ incidence matrix of treatments vs columns and $\boldsymbol{\Delta}_2\boldsymbol{\Delta}_3' = \mathbf{1}_k\mathbf{1}_b$. Let r_{dl} denote the number of times the l^{th} cross appears in the design d, $l = 1, 2, \ldots, n_c$ and similarly s_{di} denote the number of times the i^{th} line occurs in design d, $i = 1, \ldots, p$. Under (3.1), it can be shown that the reduced normal equations for estimating the gca effects of lines, after eliminating the effect of rows and columns, in design d are

$$\boldsymbol{C}_d\widehat{\boldsymbol{g}} = \boldsymbol{Q}_d \tag{2.2}$$

where

$$\boldsymbol{C}_d = \boldsymbol{G}_d - \frac{1}{b}\boldsymbol{N}_{d1}\boldsymbol{N}_{d1}' - \frac{1}{k}\boldsymbol{N}_{d2}\boldsymbol{N}_{d2}' + \frac{\boldsymbol{s}_{d1}\boldsymbol{s}_{d1}'}{\boldsymbol{s}_{d1}'\mathbf{1}}$$

and

$$\boldsymbol{Q}_d = \boldsymbol{T} + \frac{1}{b}\boldsymbol{N}_{d1}\boldsymbol{R} - \frac{1}{k}\boldsymbol{N}_{d2}\boldsymbol{C} + \frac{G}{bk}\boldsymbol{s}_{d1}$$

$\boldsymbol{C}_d$ is a $p \times p$ information matrix of the treatments and $\boldsymbol{G}_d = \boldsymbol{\Delta}_1\boldsymbol{\Delta}_1' = (g_{dii',})$, $\boldsymbol{N}_{d1} = (n_{dij..}) \cdot n_{dij}$ is the number of times line i occurs in row j of d, $N_{d2} = n_{di.t}$, $n_{i.t}$ is the number of times the cross i occurs in column t. $\boldsymbol{s}_{d1}$ is the replication vector of lines in design d. $\boldsymbol{Q}$ is a $p \times 1$ vector of adjusted treatments (crosses) total. $\boldsymbol{T}$ is a $p \times 1$ vector of treatment (line) totals, $\boldsymbol{R}$ is a $k \times 1$ vector of rows totals, $\boldsymbol{C}$ is a $b \times 1$ vector of columns totals, respectively, in design d. G is a grand total of all observations in design d.

Now we will show a connection between row- column CDC designs method (4) and PTC row- column designs through nested balanced incomplete block designs of Preece (1967).

Let us consider a nested balanced incomplete block design d with parameters $v = p, b_1, b_2, k_1, r^*, \ k_2 = 2$, and t. If we identify the treatments of d as lines of a CDC experiment method (4) and perform crosses among the lines appearing in the same sub block of d and considering these t sub blocks as single block, we get a block design d^* for a CDC experiment method (4) involving p lines with $p(p-1)/2$ crosses, each replicated $r = 2b_2/\{p(p-1)\}$ times in $b = b_1/t$ blocks with block size $k = t\,k_2$.

From the above design we can derive the row-column design d^{**} for CDC experiment method (4), if we consider the arrangement of $p(p-1)/2$ crosses in $b = b_1/t$ blocks and $k = t\,k_2$ rows and each cross is replicated $r = 2b_2/\{p(p-1)\}$ times in arrangement. Such a design $d^{**} \in \boldsymbol{D}_1(p, b, k)$; also, for such a design $n_{d*ij} = 0$ or 1for $i = 1, 2, \ldots, p$, $j = 1, 2, \ldots, b$ and

$$\boldsymbol{C}_{d^{**}} = (p-1)^{-1}p(p-3)\left[\boldsymbol{I}_p - \left\{\frac{(9p^2-5p-1)}{p^2(p-3)}\right\}\boldsymbol{I}_p\boldsymbol{I}'_p\right] \tag{2.3}$$

A row-column PTC designs can be derived from d^{**} by attaching i^{th} line with each cross in j^{th} column provided i^{th} line does not appear in j^{th} column, where $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, b$. Hence we get a row-column PTC design d_1 for a PTC experiment involving p lines with $p(p-1)/2$ triallel crosses, each replicated $r = 2b_2/\{p(p-1)\}$ times, and $b = b_1/2$ blocks, each of size $k = 2k_2$. Such a design $d_1 \in D(p, b, k)$; and, for a design d_1, $n_{d^*ij} = 3$ for $i = 1, 2, \ldots, p$, $j = 1, 2, \ldots, b$ and

$$\boldsymbol{C}_{d1} = \frac{p(p-3)}{(p-1)}\left[\boldsymbol{I}_p - \left\{\frac{4p^2+9p-3}{p^2(p-3)}\right\}\boldsymbol{I}_p\boldsymbol{I}'_p\right] \tag{2.4}$$

The $\boldsymbol{C}_{d1}$ given by (2.4) is completely symmetric.

$$tr(\boldsymbol{C}_{d1}) = p(p-3) \tag{2.5}$$

And using d_1 each elementary contrast among gca effects is estimated with a variance

$$\left[\frac{2(p-1)}{p(p-3)}\right]\sigma^2 \tag{2.6}$$

3. Method of Construction

Series 1: Let $p = 4m+1, m \geq 1$ be a prime or a prime power and x be a primitive element of the $GF(p)$. Consider the following m initial blocks.

$$\{(x^i, x^{i+2m}), (x^{i+m}, x^{i+3m})\}, \quad i = 0,1,2, \ldots, m-1$$

As shown by Dey et al. (1986), these initial blocks, when developed in the sense of Bose (1939), give rise to a nested balanced incomplete block design with parameters $v = p = 4m + 1,\ k_1 = 4, b_1 = m(4m + 1), k_2 = 2$. By arranging the above m blocks into single block as given below and developing the single block over $mod(p)$.

$$\begin{bmatrix} (x^i, x^{i+2m}) \\ (x^{i+m}, x^{i+3m}) \end{bmatrix},\ \ i = 0,1,2, \dots, m-1$$

We obtained an optimal block design for diallel crosses with minimal number of experimental units with parameters $p = 4m + 1,\ b = 4m + 1,\ k = 2m$, and $r = 1$. This diallel cross design can be converted into row-column partial triallel cross design with parameters $p = 4m + 1,\ b = 4m + 1,\ k = 2m$, and $r = 1$ by attaching i^{th} line with the crosses in j^{th} block in which i^{th} line does not appear at all, where $i, j = 1, 2, \dots, 4m + 1$.

Example 1: Let $m = 2$. We get the following two columns.

$$\begin{bmatrix} (1,2) & (x, 2x) \\ (2x + 1, x + 2) & (2x + 2, x + 1) \end{bmatrix}$$

Now we convert the both columns in single column as given below.

$$\begin{bmatrix} (1,2) \\ (2x + 1, x + 2) \\ (x, 2x) \\ (2x + 2, x + 1) \end{bmatrix}$$

where x is a primitive element of $GF(32)$ and the elements of $GF(32)$ are $0, 1, 2, x, x + 1, x + 2,\ 2x, 2x + 1, 2x + 2$. Adding successively the non-zero elements of $GF(32)$ to the contents of the single column, the CDC design method (4) is obtained with parameters $p = 9, b = 9, k = 4$ and $r = 1$, where the lines have been relabeled 1-9, using the correspondence $0 \to 1,\ \ 1 \to 2, 2 \to 3, x \to 4, x + 1 \to 5, x + 2 \to 6, 2x \to 7, 2x + 1 \to 8, 2x + 2 \to 9$:

Now attaching the $1, 2, \dots, 9$ elements, respectively, with the crosses of the 9 blocks because these elements do not appear in the respective blocks of the above design. Considering rows as row blocks, we obtain row-column design for triallel cross with parameters $p = 9, b = 9, k = 4$ and $r = 1$, which fulfill the all conditions for PTC design.

The design is given below.

Row-column partial triallel cross design

	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9
R_1	$(2 \times 3)1$	$(1 \times 3)2$	$(1 \times 2)3$	$(5 \times 6)4$	$(4 \times 5)5$	$(4 \times 5)6$	$(8 \times 9)7$	$(7 \times 9)8$	$(7 \times 8)9$
R_2	$(6 \times 8)1$	$(4 \times 9)2$	$(5 \times 7)3$	$(3 \times 7)4$	$(3 \times 7)5$	$(1 \times 8)6$	$(3 \times 5)7$	$(1 \times 6)8$	$(2 \times 4)9$
R_3	$(4 \times 7)1$	$(5 \times 8)2$	$(6 \times 9)3$	$(2 \times 8)4$	$(2 \times 8)5$	$(3 \times 9)6$	$(1 \times 4)7$	$(2 \times 5)8$	$(3 \times 6)9$
R_4	$(5 \times 9)1$	$(6 \times 7)2$	$(4 \times 8)3$	$(1 \times 9)4$	$(1 \times 9)5$	$(2 \times 7)6$	$(2 \times 6)7$	$(3 \times 4)8$	$(1 \times 5)9$

Series 2: Let $p = 6m+1, m \geq 1$ be a prime or a prime power and x be a primitive element of the Galois field of order p, $GF(p)$. Consider the initial blocks

$$\{(x^i, x^{i+3m}), (x^{i+m}, x^{i+4m}), (x^{i+2m}, x^{i+5m})\}, \quad i = 0,1,2,\dots,m-1$$

Dey et al. (1986) showed that these initial blocks, when developed give a solution of a nested incomplete block design with parameters $v = p = 6m+1, b_1 = m(6m+1), k_1 = 6, k_2 = 2, \lambda_2 = 1$.

Now arranging the above initial m blocks into single block as given below and developing $mod(p)$, will yield an optimal CDC design method (4) with parameters $p = 6m+1, b = 6m+1, k = 3m$, and $r = 1$.

$$\begin{bmatrix} (x^i, x^{i+3m}) \\ (x^{i+m}, x^{i+4m}) \\ (x^{i+2m}, x^{i+5m}) \end{bmatrix}, \quad i = 0,1,2,\dots,m-1$$

This design can be converted into row-column partial triallel cross design with parameters $p = 6m+1, k = 3m, b = 6m+1$ and $r = 1$ by the procedure described above in Series 1.

Example 2: Let $m = 2$. Then we get the following two initial blocks.

$$\begin{bmatrix} (1,12) & (2,11) \\ (4,9) & (8,5) \\ (3,10) & (6,7) \end{bmatrix}$$

Now we arrange these two blocks in a single block as given below.

$$\begin{bmatrix} (1,12) \\ (2,11) \\ (4,9) \\ (8,5) \\ (3,10) \\ (6,7) \end{bmatrix}$$

Now developing the above block $mod(13)$, we obtain optimal CDC design with parameters $p = 13, k = 6, b = 13$ and $r = 1$. Following the procedure of Example 1, we can obtain row-column design for PTC with parameters $p = 13, b = 13, k = 7$ and $r = 1$.

Series3: Let $p = 2m+1, m \geq 2$ be a prime or a prime power, then cyclically developing the following m columns

$$(0, 2m), (1, 2m-1), (2, 2m-2), \dots, (m-1, m+1) \; mod(2m+1)$$

yields an optimal CDC design method (4) with parameters $p = 2m+1, k = m, b = 2m+1$. A row-column PTC design with parameters $p = 2m+1, b = 2m+1$, and $k = m$ can be obtained by the procedure described in Example 1.

Example 3: Let $m = 3$. Then $p = 7$ and developing the following columns $mod(7)$

$$\begin{bmatrix}(0,6)\\(1,5)\\(2,4)\end{bmatrix}$$

yields optimal CDC design with parameters $p = 7, b = 7$, and $k = 3$ and $r = 1$. A PTC design with parameters $p = 7, b = 7$, $k = 3$ and $r = 1$ can be obtained by the procedure given in Example 2.

Note: The m columns form a nested balanced incomplete block design with parameters

$$v = p = 2m + 1, b_1 = m, k_1 = m(2m + 1), k_2 = 2, \lambda_2 = 1$$

Block designs for CDC method (4), with $p \leq 13$, which can be obtained from NBIB designs of Morgan et al. (2001), are listed below in Table 2. Using these designs row-column partial triallel cross designs can also be constructed.

Table 1: Block design for complete diallel crosses method (4) with $p \leq 13$ generated by using NBIB designs of Morgan, Preece and Rees (2001)

S. No.	p	b	k	Source
1	7	7	6	MPR 2
2	9	18	4	MPR5w
3	9	9	8	MPR8
4	11	11	5	MPR 14
5	13	39	4	MPR 20w
6	13	26	6	MPR21
7	13	13	12	MP23

References

Arora, B.S. and Aggarwal, K.R. (1984). Confounded triallel experiments and their applications. *Sankhya*, **B46**, 54-63.

Arora, B.S. and Aggarwal, K.R. (1989). Triallel experiments with reciprocal effects. *Journal of the Indian Society of Agricultural Statistics*, **41**, 91-103.

Bose, R.C. (1939). On the construction of balanced incomplete block designs. *Annals of Eugenics*, **9**, 353-399.

Ceranka, B., Chudzik, H., Dobek, A. and Kielczewska, H. (1990). Estimation of parameters for triallel crosses compared in block designs. *Statistics and Applications*, **2**, 27-35.

Dey, A., Das, U.S., and Banerjee, A.K. (1986). On nested balanced incomplete block designs. *Calcutta Statistical Association Bulletin*, **35**, 161-167.

Griffing, B. (1956). Concepts of general and specific combining ability in relation to diallel crossing system. *Australian Journal Biological Sciences*, **9**, 463-493.

Hinkelmann, K. (1965). Partial triallel crosses. *Sankhya*, **A27**, 173-196.

Hinkelmann, K. (1975). Design of genetical experiments. In *A Survey of Statistical Design and Linear Models*, Ed. J.N. Srivastava, Amsterdam, North Holland, 243-269.