Statistics and Applications {ISSN 2452-7395 (online)}
Volume 16, No. 1, 2018 (New Series), pp 339- 350

Population Estimation when Median of the Study Variable is known

S.K. Yadav[1], Sat Gupta[2] and Sheela Misra[3]
[1]*Department of Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow, U.P., India*
[2]*Department of Mathematics and Statistics, University of North Carolina at Greensboro, Greensboro, NC-27412, USA*
Department of Statistics, [3]University of Lucknow, Lucknow, U.P., India

Received: January 04, 2018; Revised: Feb 03, 2018; Accepted: March 06, 2018

Abstract

This paper considers the problem of estimation of population mean when the population median of the study variable is known. Two classes of mean estimators when using information on median of the study variable have been proposed. The expressions for the biases and mean squared errors have been derived up to the first order of approximation. The proposed estimators are compared theoretically with the mean per unit estimator, usual ratio and regression estimators and also with the Bahl and Tuteja (1991), Srivastava (1967), Reddy (1974) and Subramani (2016) estimators. The theoretical findings are validated through some numerical examples as well. It has been shown that proposed estimators perform better than the competing estimators.

Key words: Study variable, Bias, Ratio estimator, Mean squared error, Simple random sampling, Efficiency

1. Introduction

A common approach to parameter estimation is to pick the corresponding sample statistics. It is expected that such estimators will have the desirable properties a good estimator should have. One of the most important properties is that the estimator should have minimum variance, or minimum mean squared error if it biased. Thus, for estimating population mean, most appropriate estimator, in the absence of other information, is the sample mean. Although the sample mean is unbiased for population mean, it has reasonably large amount of variation. Thus, we search for even biased but more efficient estimators having lesser mean squared error as compared to variance of the sample mean. This is achieved through the use of auxiliary variable, which is highly correlated with the study variable. But the drawback of auxiliary information is that it is collected with additional cost of the survey. Thus, in search of improved estimators of population mean, we think of using known population parameters of study variable without increasing the cost of the survey. Median is a parameter which is easily available without having exact information on every unit of population. In this paper our aim is to search for a mean estimator which may be biased but has smaller mean squared errors as compared to the commonly known mean estimators. This is usually done using additional information either on any parameter of the study variable or using an auxiliary variable which is strongly correlated

Corresponding Author: Sat Gupta
E-mail: sngupta@uncg.edu

with the study variable. We assume the situations where the population mean is unknown but the population median of study variable is known.

Let the finite population under consideration consist of *N* distinct and identifiable units and let (x_i, y_i), $i = 1, 2, ..., n$ be a bivariate sample of size *n* taken from (*X*, Y) using a simple random sampling without replacement (*SRSWOR)* scheme. Here *Y* is our study variable and *X* is an auxiliary variable. Let $\bar{X}$ and $\bar{Y}$ respectively be the population means, and $\bar{x}$ and $\bar{y}$ be the corresponding sample means.

Let us consider two interesting examples given by Subramani (2016) where population mean is estimated using information on the population median of the study variable. Although the median is not known, it can be easily guessed from the given data. The tables have been used with permission of the author.

Example 1: These concerns the estimation of body mass index (BMI) of the 350 patients in a hospital based on a small simple random sample without replacement.

Table1: Body mass index of 350 patients in a hospital

Category	BMI range – kg/m2	Number of patients	Cumulative total
Very severely underweight	less than 15	15	15
Severely underweight	from 15.0 to 16.0	35	50
Underweight	from 16.0 to 18.5	67	117
Normal (healthy weight)	from 18.5 to 25	92	209
Overweight	from 25 to 30	47	256
Obese Class I (Moderately obese)	from 30 to 35	52	308
Obese Class II (Severely obese)	from 35 to 40	27	335
Obese Class III (Very severely obese)	over 40	15	350
	Total	350	350

The median value will be between 18.5 and 25. One may assume that the population median of the BMI is approximately 21.75.

Example 2: This concerns the problem of estimating the blood pressure of the 202 patients in a hospital using the information given in Table 2 below.

Table 2: Blood pressure of 202 patients in a hospital

Category	Systolic, mmHg	Number of patients	Cumulative No. of patients
Hypotension	< 90	10	10
Desired	90–119	112	122
Pre-hypertension	120–139	40	162
Stage 1 Hypertension	140–159	20	182
Stage 2 Hypertension	160–179	13	195
Hypertensive Emergency	≥ 180	7	202
	Total	202	202

The median value will be between 90 and 119. One may assume the population median to be approximately 104.5.

2. Review of Existing Estimators

The most commonly used estimator of population mean $\bar{Y}$ is the mean per unit estimator, the sample mean $\bar{y}$, given by

$$t_o = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \,. \quad (1)$$

It is an unbiased estimator and its variance is given by

$$V(t_0) = \frac{1-f}{n} S_y^2 = \frac{1-f}{n} \bar{Y}^2 C_y^2 , \quad (2)$$

where $C_y = \frac{S_y}{\bar{Y}}$, $S_y^2 = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{y}_i - \bar{Y})^2$, $f = \frac{n}{N}$.

Cochran (1940) utilized a positively correlated auxiliary variable and proposed the following ratio estimator

$$t_1 = \bar{y}\frac{\bar{X}}{\bar{x}} \,. \quad (3)$$

Cochran (1940) showed that it is a biased estimator of population mean and he derived the expressions for bias and mean squared error, up to the first order of approximation, given respectively by

$$B(t_1) = \frac{1-f}{n}\bar{Y}[C_x^2 - C_{yx}] \text{ and} \quad (4)$$

$$MSE(t_1) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + C_x^2 - 2C_{yx}], \quad (5)$$

where, $C_x = \frac{S_x}{\bar{X}}$, $S_x^2 = \frac{1}{N-1}\sum_{i-1}^{N}(X_i - \bar{X})^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{x}_i - \bar{X})^2$, $\rho_{yx} = \frac{Cov(x,y)}{S_x S_y}$,

$Cov(x,y) = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})$, and $C_{yx} = \rho_{yx} C_y C_x$.

Watson (1937) proposed the usual linear regression estimator of population mean given by,

$$t_2 = \bar{y} + \beta_{yx}(\bar{X} - \bar{x}), \quad (6)$$

where β_{yx} is the regression coefficient of Y on X.

He showed that it is an unbiased estimator of population mean. The variance of the above estimator, up to the first order of approximation, is given by,

$$V(t_2) = \frac{1-f}{n}\bar{Y}^2 C_y^2 (1-\rho_{yx}^2) \,. \quad (7)$$

Bahl and Tuteja (1991) utilized a positively correlated auxiliary variable and proposed the following exponential ratio type estimator of population mean,

$$t_3 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]. \quad (8)$$

It is a biased estimator and the bias and the mean squared error of this estimator, up to the first order of approximation, are given respectively by

$$B(t_3) = \frac{1-f}{8n}\bar{Y}[3C_x^2 - 4C_{yx}] \text{ and} \quad (8)$$

$$MSE(t_3) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \frac{C_x^2}{4} - C_{yx}]. \quad (9)$$

Srivastava (1967) proposed the following generalized type estimator of population mean,

$$t_4 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}. \quad (10)$$

It is a biased estimator and the bias and the mean squared error of this estimator, up to the first order of approximation, are given respectively by

$$B(t_4) = \frac{1-f}{n}\bar{Y}[\frac{\alpha(\alpha-1)}{2}C_x^2 + \alpha C_{yx}] \text{ and}$$

$$MSE(t_4) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \alpha^2 C_x^2 + 2\alpha C_{yx}].$$

The minimum value of bias and $MSE(t_4)$ for optimum value of $\alpha = -C_{yx}/C_x^2$ are respectively given by

$$B(t_4) = \frac{1-f}{n}\bar{Y}C_{yx} \quad (11)$$

$$MSE_{\min}(t_4) = \frac{1-f}{n}\bar{Y}^2 C_y^2(1-\rho_{yx}^2). \quad (12)$$

Reddy (1974) proposed the following class of ratio type estimators of population mean,

$$t_5 = \bar{y}\left[\frac{\bar{X}}{\bar{X}+\alpha(\bar{x}-\bar{X})}\right]. \quad (13)$$

It is a biased estimator and the bias and the mean squared error of this estimator, up to the first order of approximation are given respectively by

$$B(t_5) = \frac{1-f}{n}\bar{Y}[\alpha^2 C_x^2 - \alpha C_{yx}] \text{ and } \quad MSE(t_5) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \alpha^2 C_x^2 - 2\alpha C_{yx}].$$

The optimum value of the characterizing scalar is given by $\alpha = C_{yx}/C_x^2$. The estimator is unbiased for this optimum value of α and the minimum MSE is given by

$$MSE_{\min}(t_5) = \frac{1-f}{n}\bar{Y}^2 C_y^2 (1-\rho_{yx}^2) \,. \tag{14}$$

Subramani (2016) utilized the population median of the study variable and proposed the following ratio estimator of population mean of the study variable

$$t_6 = \bar{y}\left(\frac{M}{m}\right). \tag{15}$$

The bias and the mean squared error respectively of the above estimator, up to the first order of approximation, are given by

$$B(t_6) = \bar{Y}\left[\frac{1-f}{n}(C_m^2 - C_{ym}) - \frac{Bias(m)}{M}\right] \text{and} \tag{16}$$

$$MSE(t_6) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + R_6^2 C_m^2 - 2R_6 C_{ym}], \tag{17}$$

where, $R_6 = \frac{\bar{Y}}{M}$, $C_m = \frac{S_m}{M}$, $S_m^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(m_i - M)^2$, $S_{ym} = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{y}_i - \bar{Y})(m_i - M)$ and $C_{ym} = \frac{S_{ym}}{\bar{Y}M}$.

For the study of the modified ratio type estimators of population mean of the study variable, one may refer to Abid *et al.* (2016), Subramani (2013), Subramani and Kumarapandiyan (2012, 2013), Tailor and Sharma (2009), Yan and Tian (2010), Yadav et al. (2014, 2015), and Yadav et al. (2016).

3. Proposed Estimators

Motivated by Srivastava (1967), Reddy (1974) and Subramani (2016), we propose two classes of ratio type estimators of population mean using the information on population median of the study variable.

$$t_{p_1} = \bar{y}\left(\frac{M}{m}\right)^{\alpha}, \text{ and} \tag{18}$$

$$t_{p_2} = \bar{y}\left[\frac{M}{M+\alpha(m-M)}\right], \tag{19}$$

where α is a characterizing scalar to be determined such that the mean squared error of the proposed estimator is minimum.

To study the properties of the proposed estimators, the following approximations have been used: $\bar{y} = \bar{Y}(1+e_0)$ and $m = M(1+e_1)$ such that $E(e_0) = 0$, $E(e_1) = \frac{\bar{M}-M}{M} = \frac{Bias(m)}{M}$ and $E(e_0^2) = \frac{1-f}{n}C_y^2$, $E(e_1^2) = \frac{1-f}{n}C_m^2$, $E(e_0 e_1) = \frac{1-f}{n}C_{ym}$, where, $\bar{M} = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n} m_i$ (average of sample medians), m_i = sample median of i[th] sample ($i = 1, 2, ..., {}^N C_n$).

Using the above approximations, t_{p_1} can be expressed as

$$t_{p_1} = \bar{Y}(1+e_0)\left(\frac{M}{M(1+e_1)}\right)^{\alpha}$$
$$= \bar{Y}(1+e_0)(1+e_1)^{-\alpha}.$$

Expanding the second term of above equation, and retaining terms up to the first order of approximations, we get,

$$t_{p_1} = \bar{Y}(1+e_0-\alpha e_1-\alpha e_0 e_1+\frac{\alpha(\alpha-1)}{2}e_1^2) \text{ and}$$

$$t_{p_1}-\bar{Y} = \bar{Y}(e_0-\alpha e_1-\alpha e_0 e_1+\frac{\alpha(\alpha-1)}{2}e_1^2). \tag{20}$$

Taking expectation on both sides and putting the values of various expectations, we can get the bias of the estimator t_{p_1}, up to the first order of approximation, as given by

$$B(t_{p_1}) = \bar{Y}\left[\frac{1-f}{n}\frac{\alpha(\alpha-1)}{2}C_m^2-\alpha\frac{Bias(m)}{M}-\alpha\frac{1-f}{n}C_{ym}\right].$$

Squaring on both sides of (19) and taking expectation, we get the approximate mean squared error of t_{p_1} as given by

$$MSE(t_{p_1}) = \bar{Y}^2 E(e_0^2+\alpha^2 e_1^2-2\alpha e_0 e_1).$$

Putting the values of various expectations, we have

$$MSE(t_{p_1}) = \frac{1-f}{n}\bar{Y}^2[C_y^2+\alpha^2 C_m^2-2\alpha C_{ym}] \tag{21}$$

which is minimum for $\alpha_{opt} = C_{ym}/C_m^2$.

The corresponding bias and mean squared error of t_{p_1} respectively are,

$$B(t_{p_1}) = -\bar{Y}\left[\frac{1-f}{n}\left(\frac{C_{ym}^2}{2C_m^2}+\frac{C_{ym}}{2}\right)+\frac{C_{ym}}{C_m^2}\frac{Bias(m)}{M}\right] \tag{22}$$

$$MSE_{\min}(t_{p_1}) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2-\frac{C_{ym}^2}{C_m^2}\right]. \tag{23}$$

Similarly the bias and mean squared error respectively of the other estimator t_{p_2}, up to the first order of approximation, are given by,

$$B(t_{p_2}) = \bar{Y}\left[\frac{1-f}{n}\alpha C_m^2-\alpha\frac{1-f}{n}C_{ym}-\alpha\frac{Bias(m)}{M}\right] \text{ and}$$

$$MSE(t_{p_2}) = \frac{1-f}{n}\bar{Y}^2[C_y^2+\alpha^2 C_m^2-2\alpha C_{ym}].$$

The *MSE* is minimum for $\alpha_{opt} = C_{ym} \big/ C_m^2$, and the corresponding bias and mean squared error of t_{p_2} are respectively given by

$$B(t_{p_2}) = \bar{Y}\left[\frac{1-f}{n}\left(C_{ym} - \frac{C_{ym}^2}{C_m^2}\right) - \frac{C_{ym}}{C_m^2}\frac{Bias(m)}{M}\right] \tag{24}$$

$$MSE_{\min}(t_{p_2}) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 - \frac{C_{ym}^2}{C_m^2}\right] \tag{25}$$

It is worth noticing that the minimum mean squared errors of both classes of estimators are same, up to the first order of approximation but the expressions for biases of both the proposed estimators are different. We denote this common minimum mean square error by $MSE_{\min}(t_p)$.

4. Estimator with Estimated Optimum $\hat{\alpha}$

For situation when values of C_{ym} and C_m or their good guessed values are not available, the alternative is to replace them in the optimum α_{opt} by their estimates $\hat{C}_{ym}$ and $\hat{C}_m$ based on sample values and get the estimated optimum value of α denoted by $\hat{\alpha}$ as,

$$\hat{\alpha} = \hat{C}_{ym} \big/ \hat{C}_m^2 = \frac{s_{ym}}{\bar{y}m} \bigg/ \frac{s_m^2}{m^2} = \frac{s_{ym}.m}{s_m^2 \bar{y}}.$$

Using the following

$\bar{y} = \bar{Y}(1+e_0)$, $m = M(1+e_1)$, $s_{ym} = \sigma_{ym}(1+e_2)$ and $s_m^2 = \sigma_m^2(1+e_3)$ such that $E(e_0) = 0$, $E(e_1) = \frac{\bar{M}-M}{M} = \frac{Bias(m)}{M}$, $E(e_2) = E(e_3) = 0$ and $E(e_0^2) = \frac{1-f}{n}C_y^2$, $E(e_1^2) = \frac{1-f}{n}C_m^2$, $E(e_0 e_1) = \frac{1-f}{n}C_{ym}$, $E(e_1 e_2) = \frac{1-f}{n}\frac{\mu_{21}}{\sigma_{ym}M}$ and $E(e_1 e_3) = \frac{1-f}{n}\frac{\mu_{12}}{\sigma_m^2 M}$,

where, $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(m_i - M)^r (y_i - \bar{Y})^s$, and expressing $\hat{\alpha}$ in terms of e_i's ($i = 0,1,..,3$), we have,

$$\hat{\alpha} = \frac{\sigma_{ym}(1+e_2).M(1+e_1)}{\sigma_m^2(1+e_3)\bar{Y}(1+e_0)}$$

$$= \frac{\sigma_{ym}M(1+e_1+e_3+e_1e_3)}{\sigma_m^2\bar{Y}(1+e_0+e_3+e_0e_3)}$$

$$= \frac{C_{ym}(1+e_1+e_3+e_1e_3)(1+e_0+e_3+e_0e_3)^{-1}}{C_m^2}.$$

After simplification and up to the first order of approximation, we have

$$\hat{\alpha} = \frac{C_{ym}}{C_m^2}(1 - e_0 + e_1 - e_2 + e_3 + e_0^2 + e_2^2 - e_0e_1 + e_0e_2 - e_0e_3 - e_1e_2 - e_2e_3) \tag{26}$$

From equation (20), putting the value of $\hat{\alpha}$ in (26) in terms of $e_i's$, up to the first order of approximation, we have

$$t_{p_1(e)} - \bar{Y} = \bar{Y}\left[e_0 - \frac{C_{ym}}{C_m^2}e_1 - \frac{C_{ym}}{C_m^2}(e_1^2 - e_1e_2 + e_1e_3)\right] \tag{27}$$

Taking expectations on both sides of above equation and putting values of different expectations, we get bias of $t_{p_1(e)}$ as,

$$B(t_{p_1(e)}) = -\frac{1-f}{n}\bar{Y}\frac{C_{ym}}{C_m^2}\left[\frac{Bias(m)}{M} + \left(C_m^2 - \frac{\mu_{21}}{\sigma_{ym}M} + \frac{\mu_{12}}{\sigma_m^2 M}\right)\right] \tag{28}$$

Squaring on both sides of (27), retaining the terms in $e_i's$ up to second order and taking expectations, we get the MSE of $t_{p_1(e)}$ as,

$$MSE(t_{p_1(e)}) = \bar{Y}^2 E\left[e_0 - \frac{C_{ym}}{C_m^2}e_1\right]^2$$

$$= \bar{Y}^2 E\left[e_0^2 + \frac{C_{ym}^2}{C_m^4}e_1^2 - 2\frac{C_{ym}}{C_m^2}e_0e_1\right]$$

$$= \bar{Y}^2\left[E(e_0^2) + \frac{C_{ym}^2}{C_m^4}E(e_1^2) - 2\frac{C_{ym}}{C_m^2}E(e_0e_1)\right].$$

Putting values of different expectations in above equation, we have

$$MSE(t_{p_1(e)}) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 - \frac{C_{ym}^2}{C_m^2}\right] \tag{29}$$

which is same as mean square error of t_{p_1} for the optimum α_{opt}, that is, the estimator $t_{p_1(e)}$ based on estimated optimum $\hat{\alpha}$ attains the same mean square error as that of the estimator t_{p_1} based on optimum α_{opt}. Same is true for t_{p_2} as well.

5. Efficiency Comparison

In this section the proposed classes of estimators are compared theoretically with other estimators of population mean considered here and the conditions under which the proposed estimators perform better.

From equation (23) and equation (2), we have

$V(t_0) - MSE_{\min}(t_p) > 0$ if

$\frac{C_{ym}^2}{C_m^2} > 0$, or if $C_{ym}^2 > 0$.

Thus we can see that the proposed estimators are always better than the usual mean per unit estimator of population mean.

From equation (23) and equation (5), we have

$$MSE(t_1) - MSE_{\min}(t_p) > 0 \text{ if } C_x^2 - 2C_{yx} + \frac{C_{ym}^2}{C_m^2} > 0 \text{, or if } C_x^2 + \frac{C_{ym}^2}{C_m^2} > 2C_{yx}.$$

From equation (23) and equation (7), we have

$$MSE(t_2) - MSE_{\min}(t_p) > 0 \text{ if } \quad \frac{C_{ym}^2}{C_m^2} - C_y^2 \rho_{yx}^2 > 0.$$

From equation (23) and equation (9), we have,

$$MSE(t_3) - MSE_{\min}(t_p) > 0 \text{ if}$$

$$\frac{C_x^2}{4} - C_{yx} + \frac{C_{ym}^2}{C_m^2} > 0 \text{, or if}$$

$$\frac{C_x^2}{4} + \frac{C_{ym}^2}{C_m^2} > C_{yx}.$$

From equation (23) and equation (12), we have,

$$MSE(t_4) - MSE_{\min}(t_p) > 0 \text{, if } \frac{C_{ym}^2}{C_m^2} - C_y^2 \rho_{yx}^2 > 0.$$

Under the same conditions, proposed estimators are also better than Reddy (1974) estimator of population mean using auxiliary information.

From equation (23) and equation (16), we have,

$$MSE(t_6) - MSE_{\min}(t_p) > 0 \text{, if}$$

$$R_6^2 C_m^2 - 2R_6 C_{ym} + \frac{C_{ym}^2}{C_m^2} > 0 \text{, or if}$$

$$R_6^2 C_m^2 + \frac{C_{ym}^2}{C_m^2} > 2R_6 C_{ym}.$$

6. Numerical Study

To validate the theoretical findings, we have considered two natural populations given in Mukhopadhyay (2005) and Koutsoyiannis (1977). In Population 1, the study variable is the quantity of raw materials in lakhs of bales, and the number of laborers is the auxiliary variable, in thousands, for 20 jute mills. Population 2 represents amount of rent in dollars (as study variable) and number of rooms in the house (as auxiliary variable) in 20 houses of an area in London. Table 3 present the parameter values along with various constants. Table 4 presents biases of various estimators, and Table 5 presents mean squared errors of some of the existing estimators and the proposed estimators. Percentage relative efficiencies are presented in Table 6.

Table 3: Parameter values and various constants for the three populations

Parameter	Population 1	Population 2
N	20	20
n	5	5
${}^{N}C_{n}$	15504	15504
$\bar{Y}$	41.5	15.35
$\bar{M}$	40.0552	14.5
M	40.5	14.9
$\bar{X}$	441.95	3.05
R_6	1.0247	1.03
C_y^2	0.008338	0.05299
C_x^2	0.007845	0.24300
C_m^2	0.006606	0.04876
C_{ym}	0.005394	0.04920
C_{yx}	0.005275	0.1015
ρ_{yx}	0.6522	0.8900
ρ_{ym}	0.8154	0.9700

Table 4: Bias of various estimators

Estimator	Population 1	Population 2
t_1	0.1067	0.3259
t_3	0.0019	0.0930
t_4	0.0054	0.2336
t_5	0	0
t_6	0.5061	0.4111
t_{p_1}	0.3743	0.3590
t_{p_2}	–0.3474	0.4148

Table 5: Mean squared error of various estimators

Estimator	Population 1	Population 2
t_0	2.1500	1.8732
t_1	1.4000	1.2899
t_2	1.2400	0.3760
t_3	1.3000	0.4345
t_4	1.2400	0.3760
t_5	1.2400	0.3760
t_6	1.0900	0.1195
t_{p_1}, t_{p_2}	0.9800	0.1187

From Table 4, it may be noted that there is less absolute bias with the proposed estimator t_{p_2} as compared to t_{p_1}

Table 6: PRE of the proposed estimator t_{p_1} or t_{p_2} with respect to existing estimators

Estimator	Population 1	Population-2
t_0	219.3878	1578.1000
t_1	151.0204	1086.7000
t_2	126.5306	316.7600
t_3	132.6531	366.0500
t_4	126.5306	316.7600
t_5	126.5306	316.7600
t_6	111.2245	100.6700

7. Results and Conclusion

In the present study we have proposed two classes of population mean estimators utilizing the information on population median of the study variable itself. The expressions for the bias and mean squared error for both the classes of estimators have been obtained up to the first order of approximation. The optimum values of the characterizing scalars which minimize the mean squared error of the proposed classes of estimators are obtained. For these optimum values of the constants, the minimum values of the mean squared errors of the proposed estimators are also obtained. The proposed estimators are compared with the existing estimators under simple random sampling scheme. The conditions under which the proposed estimators perform better than other existing estimators have also been given. It can be seen from Table 5 that the proposed classes of estimators have smaller mean squared error compared to other mentioned competing estimators. It is worth noting that the Subramani (2016) estimator and the proposed estimators, both of which make use of information on known population median of study variable, perform better than other estimators. But the proposed estimators perform slightly better than the Subramani (2016) estimators for these populations. For other populations where m and M differ significantly, the gains may be better. For highly skewed populations, use of population median may provide even more extensive gains. It may be noticed that while the two proposed classes of estimators have same minimum mean squared error, up to first order of approximation, biases of proposed estimators are not same. For the populations we have used here, absolute bias for proposed estimator t_{p_2} is smaller as compared to t_{p_1} .

Acknowledgements: The authors are very much thankful to the editor and anonymous referee for critically reviewing the original submission and giving excellent suggestions, which helped us make significant improvement in the paper.

References

Abid, M., Abbas, N., Sherwani, R.A.K. and Nazir, H. Z. (2016). Improved ratio estimators for the population mean using non-conventional measure of dispersion. *Pakistan Journal of Statistics and Operations Research*, **XII** (**2**), 353-367.

Bahl, S. and Tuteja, R.K. (1991). Ratio and product type exponential estimator. *Information and Optimization Sciences*, **XII** (**I**), 159-163.

Cochran, W. G. (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. *The Journal of Agricultural Science,* **30**, 262-275.

Koutsoyiannis, A. (1977). *Theory of Econometrics*. Palgrave, New York.

Mukhopadhyay, P. (2005). Theory and methods of survey sampling, *PHI Learning, 2nd edition*, New Delhi.

Reddy, V.N. (1974): On a transformed ratio method of estimation. *Sankhya*, C, **36**(1), 59-70.

Singh, D. and Chaudhary, F. S., (1986). Theory and analysis of sample survey designs, *New Age International Publisher*, New Delhi.

Srivastava, S.K. (1967): An estimator using auxiliary information in sample surveys. *Cal. Statist. Assoc. Bull.*, **16**, 62-63.

Subramani, J. (2013). Generalized modified ratio estimator of finite population mean, *Journal of Modern Applied Statistical Methods*, **12** (2), 121–155.

Subramani, J. and Kumarapandiyan, G. (2012). Estimation of population mean using coefficient of variation and median of an auxiliary variable, *International Journal of Probability and Statistics*, **1** (4), 111–118.

Subramani, J. and Kumarapandiyan, G. (2013). A new modified ratio estimator of population mean when median of the auxiliary variable is known, *Pakistan Journal of Statistics and Operation Research*, Vol. **9** (2), 137–145.

Subramani, J. (2016). A new median based ratio estimator for estimation of the finite population mean, *Statistics in Transition New Series*, **17** (4), 1-14.

Tailor, R. and Sharma, B. (2009). A modified ratio-cum-product estimator of finite population mean using known coefficient of variation and coefficient of kurtosis, *Statistics in Transition-New Series*, **10** (1), 15–24.

Watson, D.J. (1937). The estimation of leaf area in field crops, *The Journal of Agricultural Science*, **27** (3), 474-483.

Yadav, S.K; Mishra, S.S. and Shukla, A.K. (2014). Improved Ratio Estimators for Population Mean Based on Median Using Linear Combination of Population Mean and Median of an Auxiliary Variable. *American Journal of Operational Research*, **4** (2), 21-27.

Yadav, S.K; Mishra, S.S. and Shukla, A.K. (2015). Estimation Approach to Ratio of Two Inventory Population Means in Stratified Random Sampling, *American Journal of Operational Research*, **5** (4), 96-101.

Yadav, S. K; Gupta, Sat; Mishra, S. S. and Shukla, A. K. (2016). Modified Ratio and Product Estimators for Estimating Population Mean in Two-Phase Sampling, *American Journal of Operational Research,* **6**, 3, 61-68.