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Construction of Nearly Orthogonal Arrays Mappable to Tight Orthogonal Arrays of Strength Two Using Projective Geometry

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Abstract

Bose and Bush (1952) used projective geometry to construct orthogonal arrays of strength two and three. Mukerjee *et al.* (2014) constructed some series of mappable nearly orthogonal arrays (MNOAs) of strength two by using resolvable orthogonal arrays. This paper proposes a method to construct nearly orthogonal arrays that are mappable to tight orthogonal arrays of strength two by using projective geometry. The method is illustrated through examples and the constructed MNOAs are tabulated for 1-flat, 2-flat, and 3-flat of the projective geometry. Many new arrays are constructed with a better degree of orthogonality.

Key words: Orthogonal arrays; Tight orthogonal array; Mappable nearly orthogonal arrays; Projective geometry.

AMS Subject Classifications: 05B15

1. Introduction

Orthogonal arrays have been widely used in scientific, agricultural, and industrial investigations and in computer experiments. The concept of orthogonal arrays was introduced by Rao (1946). Rao (1947) obtained the upper bound on the maximum number of factors for a symmetric orthogonal array. Bose and Bush (1952) constructed orthogonal arrays of strength two and three by using Galois field, difference schemes and projective geometry. For more details on the construction and applications of orthogonal arrays see Hedayat *et al.* (1999).

Wang and Wu (1992) systematically studied Nearly Orthogonal Arrays (NOAs) and proposed some general combinatorial methods for their construction. Nguyen (1996) proposed an algorithm for constructing NOAs by adding two level columns to the existing OAs. These arrays are economic in run size but sacrifice the column orthogonality.

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Mukerjee *et al.* (2014) introduced the concept of Mappable Nearly Orthogonal Array (MNOA) and developed a method for construction of these arrays by using resolvable orthogonal arrays. In these arrays, each column is orthogonal to a large proportion of the other columns and easily convertible to fully orthogonal array via a mapping of symbols in each column to a possibly smaller set of symbols. The importance and applications of this type of array have been of considerable interest because of their inherent better space filling properties.

Mukerjee *et al.* (2014) have also illustrated through an example how an MNOA with 81 runs, 40 factors each with 9 symbols can achieve stratification on a 9 x 9 grid in 720 out of 780 two-dimensions and on a 3 x 3 grid in the remaining 60 two-dimensions. Thus, having a better space filling properties than an OA with 81 runs, 40 factors each at 3 levels and accommodating more factors than an OA with 81 runs, 10 factors with 9 symbols. An important property of MNOAs is that an another MNOA can be obtained with the same number of runs but less columns after deleting one or more columns from a MNOA Mukerjee *et al.* (2014). However, the main intent is to increase the number of groups for attaining a better degree of orthogonality instead of obtaining other orthogonal array after deleting columns. Li *et al.* (2023) constructed mappable nearly orthogonal arrays with column-orthogonality and enhance the projection uniformity on any one dimension by using the constructed nearly column orthogonal MNOAs and rotation matrices. Singh *et al.* (2023 a) constructed many new mappable nearly orthogonal arrays using difference matrix. Singh *et al.* (2023 b) constructed mappable nearly orthogonal array using projective geometry.

In this paper, we propose a method to construct mappable nearly orthogonal arrays of strength two using projective geometry. The constructed nearly orthogonal arrays are mappable to tight symmetric orthogonal arrays of strength two.

Section 2 gives notations and definitions of orthogonal array, MNOA and projective geometry. In Section 3, the steps of proposed method of construction of MNOA using projective geometry are given. Some newly constructed mappable nearly orthogonal arrays of strength two using proposed method are also given in this section. The constructed MNOAs are listed in Table 3 to Table 5.

2. Preliminaries

The following results and definitions are important for the present study.

2.1. Orthogonal array

An $N \times k$ matrix A, with entries from a set G of $s(\geq 2)$ elements, is called a symmetric orthogonal array of strength t, size N, k constraints and s levels if every $(N \times t)$ submatrix of A contains all possible $(1 \times t)$ row vectors with the same frequency λ. The array is denoted by $OA[N, k, s, t]$ and the number λ is called index of the array. For a symmetric orthogonal array $N = \lambda s^t$.

Theorem 1: (Rao, 1947) In an $OA[N, k, s, t]$ the following inequalities must hold:

$$N-1 \geq \binom{k}{1}(s-1)+\cdots+\binom{k}{u}(s-1)^u \qquad \text{if } t=2u$$

and

$$N-1 \geq \binom{k}{1}(s-1)+\cdots+\binom{k}{u}(s-1)^u+\binom{k-1}{u}(s-1)^{u+1} \qquad \text{if } t=2u+1$$

An orthogonal array is said to be tight orthogonal array if the equality holds in Theorem 1. Theorem 1 gives the lower bound on the minimum number of runs required for the existence of $OA[N,k,s,t]$.

2.2. Mappable nearly orthogonal array

A mappable nearly orthogonal array $MNOA\Big[N, \prod_{i=1}^m s_i^{c_i}, \prod_{i=1}^m \prod_{j=1}^{c_i} r_{ij}\Big]$ is an $N \times \tilde{c}$ arrays whose $\tilde{c}=c_1+c_2+\cdots+c_m$ columns can be partitioned into m disjoint groups of $c_1, c_2, \ldots, c_m$ columns with the following properties:

I. for $i=1,\ldots,m$ every column of the ith group is populated by s_i symbols;

II. any two columns from different groups are orthogonal;

III. for $i=1,\ldots,m$ and for $j=1,\ldots,c_i$ the s_i symbols in the jth column of the ith group can be mapped to a set of $r_{ij} \leq s_i$ symbols such that these mappings convert the array into an orthogonal array $OA[N, \prod_{i=1}^m \prod_{j=1}^{c_i} r_{ij}]$ of strength two.

In particular, if $s_i=s, c_i=c$ and $r_{ij}=r$ for every i and j, then a mappable nearly orthogonal array is denoted as $A=MNOA[N,(s^c)^m,(r^c)^m]$.

By property II, in a mappable nearly orthogonal array before mapping, each of the c_i columns in the i group is orthogonal to at least a proportion $\tilde{\pi}=(\tilde{c}-c_i)/(\tilde{c}-1)$ of the other columns. This leads to the following measures of the pre-mapping degree of orthogonality among the columns:

$$\overline{\pi}=\frac{\sum_{i=1}^m c_i\pi_i}{\sum_{i=1}^m c_i}=(\tilde{c}^2-\sum_{i=1}^m c_i^2)/\{\tilde{c}(\tilde{c}-1)\} \tag{1}$$

$$\pi_{min}=min_{1\leq i\leq m}\pi_i=(\tilde{c}-max_{1\leq i\leq m}c_i)/(\tilde{c}-1) \tag{2}$$

if $c_1=c_2=\cdots=c_m=c$, then $\tilde{c}=mc$, where m and c are the number of groups and number of columns respectively and by (1) and (2) we have

$$\overline{\pi}=\pi_{min}=(m-1)c/(mc-1) \tag{3}$$

In this paper, symmetric orthogonal arrays are constructed, so every columns c_i contains same set of symbols.

2.3. Projective geometry

The projective geometry $PG(r,s)$ over Galois field $GF(s)$ of order s, where s is a prime or a power of a prime number, consists of ordered set $(y_0, y_1, \ldots, y_r)$ called points where $y_i, i = 0, 1, \ldots, r$, are elements of $GF(s)$ and not all them are simultaneously zero. The point $(ay_0, ay_1, \ldots ay_r)$ represents the same point as $(y_0, y_1, \ldots, y_r)$, for any $a \in GF(s), (a \neq 0)$. The collection of all those points which satisfy a set of $(r-t)$ linearly independent homogeneous equation with coefficients from $GF(s)$, not all of them are simultaneously zero within the same equation, is said to represents a t-flat in $PG(r,s)$.

In particular a $0-$flat, a $1-$flat,$\ldots, a(r-1)-$flat respectively in $PG(r,s)$ are known as a point, a line, $\ldots$, a hyperplane of $PG(r,s)$. The number of points lying on a t-flats is $(t+1)$.

3. Method of construction

Projective Geometry is a direct representation of orthogonal arrays of strength two. Raghavarao (1971) obtained the orthogonal arrays $OA[s^{(r+1)}, (\frac{s^{(r+1)}-1}{(s-1)}), s, 2]$ using by $PG(r,s)$. Mukerjee *et al.* (2014) constructed mappable nearly orthogonal arrays of strength two using resolvable orthogonal arrays. Here, we give a method to construct new series of mappable nearly orthogonal arrays of strength two using projective geometry. The total number of points in $PG(r,s)$ is $|PG(r,s)| = [\frac{s^{(r+1)}-1}{(s-1)}]$. The $PG(r,s)$ has $p = [(\frac{s^{(r+1)}-1}{s^{t+1})-1}]$ disjoint t-flats if and only if $(t+1)|(r+1)$. An orthogonal array $OA[s^{(r+1)}, p, s^{(t+1)}, 2]$ can be constructed using the disjoint t-flats in $PG(r,s)$. The method of construction is described in the following steps and Theorem 2.

Step I: Consider the orthogonal array $D = OA[s^{(t+1)}, q, s, 2]$; $q = [(s^{(t+1)}-1)/(s-1)]$, obtained from the collection of all points of $PG(t,s)$. The array D is of order $(s^{(t+1)} \times q)$ and each column of D has $s$symbols occurring equally often.

Step II: Replace the s^t occurrences of each of the s symbols in the kth column of D by $s^{(t+1)}$ symbols from the set $s_i = (0, 1, 2, \ldots, (s^{(t+1)}-1))$ is as follows: For $k = 1, 2, \ldots q$, define

$$t_{kh} = \{hs, hs+1, \ldots, hs+(s^t-1)\}, \qquad h = 0, 1, 2, \ldots, (s-1) \tag{4}$$

and replace the s^t occurrences of symbol h by the s^t members of t_{kh} in order as obtained in (4), that is, the first occurrence of h is replaced by hs and second occurrence by $hs+1$ and so on.

Step III: Let R denote the $(s^{(t+1)} \times q)$ array obtained from D after changing symbols of D according to (4), so that each column of R is a permutation of $\{0, 1, \ldots, (s^{(t+1)}-1)\}$ symbols. Let $r(0), r(1), \ldots, r(s^{(t+1)}-1)$ denote the $s^{(t+1)}$ rows of R.

Step IV: Consider an orthogonal array $A = OA[s^{(r+1)}, p, s^{(t+1)}, 2]$ obtained from $p = [(s^{(r+1)}-1)/(s^{(t+1)}-1)]$ disjoint t-flats of $PG(r,s)$. Let $0, 1, \ldots, (s^{(t+1)}-1)$ denote the symbols in the ith column of orthogonal array $A = [a_{li}]; l = 1, 2, \ldots, s^{(r+1)}$ and $i = 1, 2, \ldots, p$.

Step V: Construct the following arrays using the array A and step III. Write

$$A = [A_1 : A_2 : \ldots : A_p],$$

where $A_i(i = 1, 2, \ldots, p)$ is of order $(s^{r+1} \times 1)$ with s^{t+1} symbols. Replace the s^{t+1} symbols $\{0, 1, 2, \ldots, (s^{t+1} - 1)\}$ of A_i by the rows $r(0), r(1), r(2), \ldots\ldots\ldots, r(s^{(t+1)} - 1)$ of R respectively and denote it by T_i. Then

1. T_i is of order $s^{(r+1)} \times q$, having rows $r(a_{1i}), r(a_{2i}), ..., r(a_{s^{(r+1)}i})$, for i=1, 2, $\ldots, p$.
2. $T = [T_1 : T_2 : \cdots : T_p]$, of order $s^{(r+1)} \times (pq)$ with symbols 0,1,2,...,$(s^{(t+1)} - 1)$, is the pre-mapping array.

Step VI: For the post mapping array, $s^{(t+1)}$ symbols are mapped to s symbols as follows: For i=1,2,...,p, consider T_i and use reverse mapping of (4) as

$$\{hs, hs + 1, \ldots\ldots, hs + (s^t - 1)\} \to h = 0, 1, 2, \ldots, (s - 1)$$

in each of the q columns of T_i, to get B_i of order $(s^{(r+1)} \times q)$ with s symbols $\{0, 1, \ldots, (s - 1)\}$ in every column. Write $B = [B_1 : B_2 : \cdots : B_p]$, then B is the post mapping array $B = OA[s^{(r+1)}, ((s)^q)^p]$. Thus, the mappable nearly orthogonal array

$$MNOA[s^{(r+1)}, \{(s^{(t+1)})^q\}^p, \{(s)^q\}^p]$$

is constructed and we have the following result.

Theorem 2: For given r, s and t where $(t+1)|(r+1)$, an $MNOA[s^{(r+1)}, \{(s^{(t+1)})^q\}^p, \{(s)^q\}^p]$ can always be constructed by using $PG(t, s)$ and p distinct t-flats of $PG(r, s)$.

The method of construction is illustrated through the following examples.

Example 1: Let $t = 1,\ r = 3$ and $s = 2$ in $PG(r, s)$. Using step I, we obtain the array $D = OA[4, 3, 2, 1]$ of order (4×3) as

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Replace two symbols of set $s = (0, 1)$ in each column of D by four symbols of set $s_i = (0, 1, 2, 3)$ as described in step II to obtain R, so that each column of R is a permutation of four symbols $0, 1, 2, 3$. The array R is

$$R = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

Now, consider the orthogonal array $A = OA[16, 5, 4, 2]$ obtained by using the five disjoint 1-flat of $PG(3, 2)$ as given in step IV. The orthogonal array A is given below:

$$A^T = \begin{bmatrix} 0 & 1 & 2 & 2 & 0 & 3 & 3 & 1 & 0 & 2 & 2 & 1 & 3 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 & 2 & 1 & 2 & 2 & 3 & 3 & 0 & 3 & 3 & 1 & 0 & 1 \\ 0 & 2 & 2 & 1 & 2 & 0 & 3 & 0 & 3 & 0 & 3 & 1 & 2 & 1 & 1 & 3 \\ 0 & 2 & 0 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 0 & 3 & 3 & 1 & 1 \\ 0 & 1 & 3 & 0 & 2 & 2 & 1 & 3 & 3 & 1 & 2 & 2 & 0 & 1 & 3 & 0 \end{bmatrix}$$

Divide the columns of A into 5 groups, each group consisting of a single column denoted by A_i and replace the entries of A_i by rows of R as described in Step V to get T_i for $i = 1, 2, \ldots, 5$ each of order 16×3. The pre-mapping array $T = [T_1 : T_2 : T_3 : T_4 : T_5]$ is given in Table 1.

Table 1: Pre-mapping array using $PG(3, 2)$ and $t = 1$

Group 1	Group 2	Group 3	Group 4	Group 5
0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
2 1 1	0 0 0	1 2 3	1 2 3	2 1 2
1 2 3	2 1 2	1 2 3	0 0 0	3 3 1
1 2 2	1 2 3	2 1 2	1 2 3	0 0 0
0 0 0	1 2 3	1 2 3	2 1 2	1 2 3
3 3 1	2 1 2	0 0 0	1 2 3	1 2 3
3 3 1	1 2 3	3 3 1	0 0 0	2 1 2
2 1 2	1 2 3	0 0 0	3 3 1	3 3 1
0 0 0	3 3 1	3 3 1	1 2 3	3 3 1
1 2 3	3 3 1	0 0 0	2 1 2	2 1 2
1 2 3	0 0 0	3 3 1	3 3 1	1 2 3
2 1 2	3 3 1	2 1 2	0 0 0	1 2 3
3 3 1	3 3 3	1 2 3	3 3 1	0 0 0
0 0 0	2 1 2	2 1 2	3 3 1	2 1 2
3 3 1	0 0 0	2 1 2	2 1 2	3 3 1
2 1 2	2 1 2	3 3 1	2 1 2	0 0 0

For post mapping, map the symbols $(0, 1, 2, 3)$ to $(0, 1)$ using the reverse mapping of (4). The post-mapping array is a symmetric tight orthogonal array $OA[16, (2^3)^5]$ of strength two and it is given in Table 2.

Thus, using projective geometry $PG(3, 2)$, the required mappable nearly orthogonal array $MNOA[16, (4^3)^5, (2^3)^5]$ is constructed, which is mappable to fully symmetric tight orthogonal array of strength two.

Example 2: Let $t = 1,\ r = 3$ and $s = 3$ in $PG(r, s)$. Using step I, we obtain the array $D = OA[9, 4, 3, 2]$ of order (9×4) as

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & 2 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 1 & 2 & 0 & 2 \\ 2 & 0 & 2 & 2 \\ 2 & 1 & 0 & 1 \\ 2 & 2 & 1 & 0 \end{bmatrix}$$

Replace three symbols of set $s = (0, 1, 2)$ in each column of D by nine symbols of set $s_i = (0, 1, 2, 3, 4, 5, 6, 7, 8)$ as described in step II to obtain R, so that each column of R is a

Table 2: Post-mapping array using $PG(3,2)$ and $t = 1$

Group 1	Group 2	Group 3	Group 4	Group 5
0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
1 0 1	0 0 0	0 1 1	0 1 1	1 0 1
0 1 1	1 0 1	0 1 1	0 0 0	1 1 0
0 1 1	0 1 1	1 0 1	0 1 1	0 0 0
0 0 0	0 1 1	0 1 1	1 0 1	0 1 1
1 1 0	1 0 1	0 0 0	0 1 1	0 1 1
1 1 0	0 1 1	1 1 0	0 0 0	1 0 1
1 0 1	0 1 1	0 0 0	1 1 0	1 1 0
0 0 0	1 1 0	1 1 0	0 1 1	1 1 0
0 1 1	1 1 0	0 0 0	1 0 1	1 0 1
0 1 1	0 0 0	1 1 0	1 1 0	0 1 1
1 0 1	1 1 0	1 0 1	0 0 0	0 1 1
1 1 0	1 1 0	0 1 1	1 1 0	0 0 0
0 0 0	1 0 1	1 0 1	1 1 0	1 0 1
1 1 0	0 0 0	1 0 1	1 0 1	1 1 0
1 0 1	1 0 1	1 1 0	1 0 1	0 0 0

permutation of nine symbols $0, 1, 2, 3, 4, 5, 6, 7, 8$. The array R is

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 3 & 3 & 6 \\ 2 & 6 & 6 & 3 \\ 3 & 1 & 4 & 4 \\ 4 & 4 & 6 & 1 \\ 5 & 7 & 1 & 7 \\ 6 & 3 & 8 & 8 \\ 7 & 5 & 2 & 5 \\ 8 & 8 & 5 & 2 \end{bmatrix}$$

Now, consider the orthogonal array $A = OA[81, 10, 9, 2]$ obtained by using the ten disjoint 1-flat of $PG(3,3)$ as given in step IV. The orthogonal array A is given in Table 6 in Annexure.

Divide the columns of A into 10 groups, each group consisting of a single column denoted by A_i and replace the entries of A_i by rows of R as described in Step V to get T_i for $i = 1, 2, \ldots, 10$ each of order 81×4. The pre-mapping array $T = [T_1 : T_2 : T_3 : T_4 : T_5 : T_6 : T_7 : T_8 : T_9 : T_{10}]$ is given in Table 7 in Annexure.

For post mapping, map the symbols $(0, 1, 2, 3, 4, 5, 6, 7, 8,)$ to $(0, 1, 2)$ using the reverse mapping of (4). The post-mapping array is a symmetric tight orthogonal array $OA[81, (3^4)^{10}]$ of strength two and it is given in Table 8 in Annexure. Thus, using projective geometry $PG(3,3)$, the required mappable nearly orthogonal array $MNOA[81, (9^4)^{10}, (3^4)^{10}]$ is constructed, which is mappable to fully symmetric tight orthogonal array of strength two.

Similarly, we can construct many more design using our proposed method, some of them are listed in the following tables along with the corresponding values of $\overline{\pi}$.

It may be noted here that all values in the last column of the above tables are obtained

Table 3: Some tight nearly orthogonal arrays based on t-flat and $PG(r,s)$ for $t=1$

r	s	$D = OA[s^{t+1}, q, s, 2]$	$MNOA[s^{r+1}, \{(s^{t+1})^q)^p\}, \{(s)^q\}^p]$	$\overline{\pi}$
3	2	$D = OA[4, 3, 2, 2]$	$MNOA[16, \{(4)^3\}^5, \{(2)^3\}^5)]^*$	0.8571
5	2	$D = OA[4, 3, 2, 2]$	$MNOA[64, \{(4)^3\}^{21}, \{(2)^3\}^{21})]$	0.9677
7	2	$D = OA[4, 3, 2, 2]$	$MNOA[256, \{(4)^3\}^{85}, \{(2)^3\}^{85})]$	0.9921
3	3	$D = OA[9, 4, 3, 2]$	$MNOA[81, \{(9)^4\}^{10}, \{(3)^4\}^{10})]^*$	0.9230
5	3	$D = OA[9, 4, 3, 2]$	$MNOA[729, \{(9)^4\}^{91}, \{(3)^4\}^{91})]$	0.9890
7	3	$D = OA[9, 4, 3, 2]$	$MNOA[6561, \{(9)^4\}^{820}, \{(3)^4\}^{820})]$	0.9990
3	4	$D = OA[16, 5, 4, 2]$	$MNOA[256, \{(16)^5\}^{17}, \{(4)^5\}^{17})]^*$	0.9523
5	4	$D = OA[16, 5, 4, 2]$	$MNOA[4096, \{(16)^5\}^{273}, \{(4)^5\}^{273})]$	0.9970
7	4	$D = OA[16, 5, 4, 2]$	$MNOA[65536, \{(16)^5\}^{4369}, \{(4)^5\}^{4369})]$	0.9998
3	5	$D = OA[25, 6, 5, 2]$	$MNOA[625, \{(25)^6\}^{26}, \{(5)^6\}^{26})]$	0.9677
5	5	$D = OA[25, 6, 5, 2]$	$MNOA[15625, \{(25)^6\}^{651}, \{(5)^6\}^{651})]$	0.9987
3	9	$D = OA[81, 10, 9, 2]$	$MNOA[6561, \{(81)^{10}\}^{82}, \{(9)^{10}\}^{82})]$	0.9890
5	9	$D = OA[81, 10, 9, 2]$	$MNOA[531441, \{(81)^{10}\}^{6643}, \{(9)^{10}\}^{6643})]$	0.9998
3	25	$D = OA[125, 6, 5, 2]$	$MNOA[15625, \{(125)^6\}^{126}, \{(25)^6\}^{126})]$	0.9923

Table 4: Some tight nearly orthogonal arrays based on t-flat and $PG(r,s)$ for $t=2$

r	s	$D = OA[s^{t+1}, q, s, 2]$	$MNOA[s^{r+1}, \{(s^{t+1})^q)^p\}, \{(s)^q\}^p]$	$\overline{\pi}$
5	2	$D = OA[8, 7, 2, 2]$	$MNOA[64, \{(8)^7\}^9, \{(2)^7\}^9)]^*$	0.9032
8	2	$D = OA[8, 7, 2, 2]$	$MNOA[512, \{(8)^7\}^{73}, \{(2)^7\}^{73})]$	0.9882
11	2	$D = OA[8, 7, 2, 2]$	$MNOA[4096, \{(8)^7\}^{585}, \{(2)^7\}^{585})]$	0.9985
5	3	$D = OA[27, 13, 3, 2]$	$MNOA[729, \{(27)^{13}\}^{28}, \{(3)^{13}\}^{28})]$	0.9669
8	3	$D = OA[27, 13, 3, 2]$	$MNOA[19683, \{(27)^{13}\}^{757}, \{(3)^{13}\}^{757})]$	0.9987
5	4	$D = OA[64, 21, 4, 2]$	$MNOA[4096, \{(64)^{21}\}^{65}, \{(4)^{21}\}^{65})]$	0.9853
8	4	$D = OA[64, 21, 4, 2]$	$MNOA[262144, \{(64)^{21}\}^{4161}, \{(4)^{21}\}^{4161})]$	0.9997
5	5	$D = OA[125, 31, 5, 2]$	$MNOA[15625, \{(125)^{31}\}^{126}, \{(5)^{31}\}^{126})]$	0.9920
5	9	$D = OA[729, 91, 9, 2]$	$MNOA[531441, \{(729)^{91}\}^{730}, \{(9)^{91}\}^{730})]$	0.9986

by using equation (3) and the MNOAs marked with $*$ are same as those obtained by Mukerjee *et al.* (2014).

4. Conclusion

In this paper, a method is proposed to construct, mappable nearly orthogonal arrays (MNOAs) using projective geometry. The constructed MNOAs are mappable to tight orthogonal arrays of strength two. It is observed that some new MNOAs are constructed with higher values of degree of orthogonality $\overline{\pi}$ and are therefore useful as better space filling designs.

Table 5: Some tight nearly orthogonal arrays based on t-flat and $PG(r, s)$ for $t = 3$

r	s	$D = OA[s^{t+1}, q, s, 2]$	$MNOA[s^{r+1}, \{(s^{t+1})^q)^p\}, \{(s)^q\}^p]$	$\overline{\pi}$
7	2	$D = OA[16, 15, 2, 2]$	$MNOA[256, \{(16)^{15}\}^{17}, \{(2)^{15}\}^{17})]^*$	0.9448
11	2	$D = OA[16, 15, 2, 2]$	$MNOA[4096, \{(16)^{15}\}^{273}, \{(2)^{15}\}^{273})]$	0.9965
7	3	$D = OA[81, 40, 3, 2]$	$MNOA[6561, \{(81)^{40}\}^{82}, \{(3)^{40}\}^{82})]$	0.9881

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ANNEXURE

Table 6: Orthogonal array $A = [81, 10, 9, 2]$

Rows 1 to 27										Rows 28 to 54										Rows 55 to 81									
0	0	0	0	0	0	0	0	0	0	3	0	3	3	3	3	3	3	3	3	6	0	6	6	6	6	6	6	6	6
0	1	1	2	3	4	5	6	7	8	3	1	8	4	7	5	2	1	0	6	6	1	4	0	1	3	7	2	8	5
0	2	2	3	4	5	6	7	8	1	3	2	4	7	5	2	1	0	6	8	6	2	0	1	3	7	2	8	5	4
0	3	3	4	5	6	7	8	1	2	3	3	7	5	2	1	0	6	8	4	6	3	1	3	7	2	8	5	4	0
0	4	4	5	6	7	8	1	2	3	3	4	5	2	1	0	6	8	4	7	6	4	3	7	2	8	5	4	0	1
0	5	5	6	7	8	1	2	3	4	3	5	2	1	0	6	8	4	7	5	6	5	7	2	8	5	4	0	1	3
0	6	6	7	8	1	2	3	4	5	3	6	1	0	6	8	4	7	5	2	6	6	2	8	5	4	0	1	3	7
0	7	7	8	1	2	3	4	5	6	3	7	0	6	8	4	7	5	2	1	6	7	8	5	4	0	1	3	7	2
0	8	8	1	2	3	4	5	6	7	3	8	6	8	4	7	5	2	1	0	6	8	5	4	0	1	3	7	2	8
1	0	1	1	1	1	1	1	1	1	4	0	4	4	4	4	4	4	4	4	7	0	7	7	7	7	7	7	7	7
1	1	5	3	8	7	0	4	6	2	4	1	7	1	5	8	6	3	2	0	7	1	6	5	0	2	4	8	3	1
1	2	3	8	7	0	4	6	2	5	4	2	1	5	8	6	3	2	0	7	7	2	5	0	2	4	8	3	1	6
1	3	8	7	0	4	6	2	5	3	4	3	5	8	6	3	2	0	7	1	7	3	0	2	4	8	3	1	6	5
1	4	7	0	4	6	2	5	3	8	4	4	8	6	3	2	0	7	1	5	7	4	2	4	8	3	1	6	5	0
1	5	0	4	6	2	5	3	8	7	4	5	6	3	2	0	7	1	5	8	7	5	4	8	3	1	6	5	0	2
1	6	4	6	2	5	3	8	7	0	4	6	3	2	0	7	1	5	8	6	7	6	8	3	1	6	5	0	2	4
1	7	6	2	5	3	8	7	0	4	4	7	2	0	7	1	5	8	6	3	7	7	3	1	6	5	0	2	4	8
1	8	2	5	3	8	7	0	4	6	4	8	0	7	1	5	8	6	3	2	7	8	1	6	5	0	2	4	8	3
2	0	2	2	2	2	2	2	2	2	5	0	5	5	5	5	5	5	5	5	8	0	8	8	8	8	8	8	8	8
2	1	3	6	4	1	8	0	5	7	5	1	0	8	2	6	1	7	4	3	8	1	2	7	6	0	3	5	1	4
2	2	6	4	1	8	0	5	7	3	5	2	8	2	6	1	7	4	3	0	8	2	7	6	0	3	5	1	4	2
2	3	4	1	8	0	5	7	3	6	5	3	2	6	1	7	4	3	0	8	8	3	6	0	3	5	1	4	2	7
2	4	1	8	0	5	7	3	6	4	5	4	6	1	7	4	3	0	8	2	8	4	0	3	5	1	4	2	7	6
2	5	8	0	5	7	3	6	4	1	5	5	1	7	4	3	0	8	2	6	8	5	3	5	1	4	2	7	6	0
2	6	0	5	7	3	6	4	1	8	5	6	7	4	3	0	8	2	6	1	8	6	5	1	4	2	7	6	0	3
2	7	5	7	3	6	4	1	8	0	5	7	4	3	0	8	2	6	1	7	8	7	1	4	2	7	6	0	3	5
2	8	7	3	6	4	1	8	0	5	5	8	3	0	8	2	6	1	7	4	8	8	4	2	7	6	0	3	5	1

Table 7: Pre-mapping array using $PG(3,3)$and $t = 1$

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group7	Group 8	Group 9	Group 10
0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
0000	1336	1336	2663	3144	4461	6388	6388	7525	8852
0000	2663	2663	3144	4461	5717	7525	7525	8852	1336
0000	3144	3144	4461	5717	6388	8852	8852	1336	2663
0000	4461	4461	5717	6388	7525	1336	1336	2663	3144
0000	5717	5717	6388	7525	8852	2663	2663	3144	4461
0000	6388	6388	7525	8852	1336	3144	3144	4461	5717
0000	7525	7525	8852	1336	2663	4461	4461	5717	6388
0000	8852	8852	1336	2663	3144	5717	5717	6388	7525
1336	0000	1336	1336	1336	1336	1336	1336	1336	1336
1336	1336	5763	3144	8852	7525	4461	4461	6388	2663
1336	2663	3144	8852	7525	0000	6388	6388	2663	5717
1336	3144	8852	7525	0000	4461	2663	2663	5717	3144
1336	4461	7525	0000	4461	6388	5717	5717	3144	8852
1336	5717	0000	4461	6388	2663	3144	3144	8852	7525
1336	6388	4461	6388	2663	5717	8852	8852	7525	0000
1336	7525	6388	2663	5717	3144	7525	7525	0000	4461
1336	8852	2663	5717	3144	8852	0000	0000	4461	6388
2663	0000	2663	2663	2663	2663	2663	2663	2663	2663
2663	1336	3144	6388	4461	1336	0000	0000	5717	7525
2663	2663	6388	4461	1336	8852	5717	5717	7525	3144
2663	3144	4461	1336	8852	0000	7525	7525	3144	6388
2663	4461	1336	8852	0000	5717	3144	3144	6388	4461
2663	5717	8852	0000	5717	7525	6388	6388	4461	1336
2663	6388	0000	5717	7525	3144	4461	4461	1336	8852
2663	7525	5717	7525	3144	6388	1336	1336	8852	0000
2663	8852	7525	3144	6388	4461	8852	8852	0000	5717

Table 7: Continued

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group7	Group 8	Group 9	Group 10
3144	0000	3144	3144	3144	3144	3144	3144	3144	3144
3144	1336	8852	4461	7525	5717	2663	1336	0000	6388
3144	2663	4461	7525	5717	2663	1336	0000	6388	8852
3144	3144	7525	5717	2663	1336	0000	6388	8852	4461
3144	4461	5717	2663	1336	0000	6388	8852	4461	7525
3144	5717	2663	1336	0000	6388	8852	4461	7525	5717
3144	6388	1336	0000	6388	8852	4461	7525	5717	2663
3144	7525	0000	6388	8852	4461	7525	5717	2663	1336
3144	8852	6388	8852	4461	7525	5717	2663	1336	0000
4461	0000	4461	4461	4461	4461	4461	4461	4461	4461
4461	1336	7525	1336	5717	8852	6388	3144	2663	0000
4461	2663	1336	5717	8852	6388	3144	2663	0000	7525
4461	3144	5717	8852	6388	3144	2663	0000	7525	1336
4461	4461	8852	6388	3144	2663	0000	7525	1336	5717
4461	5717	6388	3144	2663	0000	7525	1336	5717	8852
4461	6388	3144	2663	0000	7525	1336	5717	8852	6388
4461	7525	2663	0000	7525	1336	5717	8852	6388	3144
4461	8852	0000	7525	1336	5717	8852	6388	3144	2663
5717	0000	5717	5717	5717	5717	5717	5717	5717	5717
5717	1336	0000	8852	2663	6388	1336	7525	4461	3144
5717	2663	8852	2663	6388	1336	7525	4461	3144	0000
5717	3144	2663	6388	1336	7525	4461	3144	0000	8852
5717	4461	6388	1336	7525	4461	3144	0000	8852	2663
5717	5717	1336	7525	4461	3144	0000	8852	2663	6388
5717	6388	7525	4461	3144	0000	8852	2663	6388	1336
5717	7525	4461	3144	0000	8852	2663	6388	1336	7525
5717	8852	3144	0000	8852	2663	6388	1336	7525	6388

Table 7: Continued

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group7	Group 8	Group 9	Group 10
6388	0000	6388	6388	6388	6388	6388	6388	6388	6388
6388	1336	4461	0000	1336	3144	7525	2663	8852	5717
6388	2663	0000	1336	3144	7525	2663	8852	5717	4461
6388	3144	1336	3144	7525	2663	8852	5717	4461	0000
6388	4461	3144	7525	2663	8852	5717	4461	0000	1336
6388	5717	7525	2663	8852	5717	4461	0000	1336	3144
6388	6388	2663	8852	5717	4461	0000	1336	3144	7525
6388	7525	8852	5717	4461	0000	1336	3144	7525	2663
6388	8852	5717	4461	0000	1336	3144	7525	2663	8852
7525	0000	7525	7525	7525	7525	7525	7525	7525	7525
7525	1336	6388	5717	0000	2663	4461	8852	3144	1336
7525	2663	5717	0000	2663	4461	8852	3144	1336	6388
7525	3144	0000	2663	4461	8852	3144	1336	6388	5717
7525	4461	2663	4461	8852	3144	1336	6388	5717	0000
7525	5717	4461	8852	3144	1336	6388	5717	0000	2663
7525	6388	8852	3144	1336	6388	5717	0000	2663	4461
7525	7525	3144	1336	6388	5717	0000	2663	4461	8852
7525	8852	1336	6388	5717	0000	2663	4461	8852	3144
8852	0000	8852	8852	8852	8852	8852	8852	8852	8852
8852	1336	2663	7525	6388	0000	3144	5717	1336	4461
8852	2663	7525	6388	0000	3144	5717	1336	4461	2663
8852	3144	6388	0000	3144	5717	1336	4461	2663	7525
8852	4461	0000	3144	5717	1336	4461	2663	7525	6388
8852	5717	3144	5717	1336	4461	2663	7525	6388	0000
8852	6388	5717	1336	4461	2663	7525	6388	0000	3144
8852	7525	1336	4461	2663	7525	6388	0000	3144	5717
8852	8852	4461	2663	7525	6388	0000	3144	5717	1336

Table 8: Post-mapping array using $PG(3,3)$ and $t = 1$

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group7	Group 8	Group 9	Group 10
0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
0000	0112	0112	0221	1011	1120	2122	2122	2101	2210
0000	0221	0221	1011	1120	1202	2101	2101	2210	0112
0000	1011	1011	1120	1202	2122	2210	2210	0112	0221
0000	1120	1120	1202	2122	2101	0112	0112	0221	1011
0000	1202	1202	2112	2101	2210	0221	0221	1011	1120
0000	2122	2122	2101	2210	0112	1011	1011	1120	1202
0000	2101	2101	2210	0112	0221	1120	1120	1202	2122
0000	2210	2210	0112	0221	1011	1202	1202	2122	2101
0112	0000	0112	0112	0112	0112	0112	0112	0112	0112
0112	0112	1221	1011	2210	2101	1120	1120	2122	0221
0112	0221	1011	2210	2101	0000	2122	2122	0221	1202
0112	1011	2210	2101	0000	1120	0221	0221	1202	1011
0112	1120	2101	0000	1120	2122	1202	1202	1011	2210
0112	1202	0000	1120	2122	0221	1011	1011	2210	2101
0112	2122	1120	2122	0221	1202	2210	2210	2101	0000
0112	2101	2122	0221	1202	1011	2101	2101	0000	1120
0112	2210	0221	1202	1011	2210	0000	0000	1120	2122
0221	0000	0221	0221	0221	0221	0221	0221	0221	0221
0221	0112	1011	2122	1120	0112	0000	0000	1202	2101
0221	0221	2122	1120	0112	2210	1202	1202	2101	1011
0221	1011	1120	0112	2210	0000	2101	2101	1011	2122
0221	1120	0112	2210	0000	1202	1011	1011	2122	1120
0221	1202	2210	0000	1202	2101	2122	2122	1120	0112
0221	2122	0000	1202	2101	1011	1120	1120	0112	2101
0221	2101	1202	2101	1011	2122	0112	0112	2210	0000
0221	2210	2101	1011	2122	1120	2210	2210	0000	1202

Table 8: Continued

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group7	Group 8	Group 9	Group 10
1011	0000	1011	1011	1011	1011	1011	1011	1011	1011
1011	0112	2210	1120	2101	1202	0221	0112	0000	2122
1011	0221	1120	2101	1202	0221	0112	0000	2122	2210
1011	1011	2101	1202	0221	0112	0000	2122	2210	1120
1011	1120	1202	0221	0112	0000	2122	2210	1120	2101
1011	1202	0221	0112	0000	2122	2210	1120	2101	1202
1011	2122	0112	0000	2122	2210	1120	2101	1202	0221
1011	2101	0000	2122	2210	1120	2101	1202	0221	0112
1011	2210	2122	2210	1120	2101	1202	0221	0112	0000
1120	0000	1121	1121	1121	1121	1121	1121	1121	1121
1120	0112	2101	0112	1202	2210	2122	1011	0221	0000
1120	0221	0112	1202	2210	2122	1011	0221	0000	2101
1120	1011	1202	2210	2122	1011	0221	0000	2101	0112
1120	1120	2210	2122	1011	0221	0000	2101	0112	1202
1120	1202	2122	1011	0221	0000	2101	0112	1202	2210
1120	2122	1011	0221	0000	2101	0112	1202	2210	2122
1120	2101	0221	0000	2101	0112	1202	2210	2122	1011
1120	2210	0000	2101	0112	1202	2210	2122	1011	0221
1202	0000	1202	1202	1202	1202	1202	1202	1202	1202
1202	0112	0000	2210	0221	2122	0112	2101	1120	1011
1202	0221	2210	0221	2122	0112	2101	1120	1011	0000
1202	1011	0221	2122	0112	2101	1120	1011	0000	2210
1202	1120	2122	0112	2101	1120	1011	0000	2210	0221
1202	1202	0112	2101	1120	1011	0000	2210	0221	2122
1202	2122	2101	1120	1011	0000	2210	0221	2122	0112
1202	2101	1120	1011	0000	2210	0221	2122	0112	2101
1202	2210	1011	0000	2210	0221	2122	0112	2101	2122

Table 8: Continued

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group7	Group 8	Group 9	Group 10
2122	0000	2122	2122	2122	2122	2122	2122	2122	2122
2122	0112	1120	0000	0112	1011	2101	0221	2210	1202
2122	0221	0000	0112	1011	2101	0221	2210	1202	1120
2122	1011	0112	1011	2101	0221	2210	1202	1120	0000
2122	1121	1011	2101	0221	2210	1202	1120	0000	0112
2122	1202	2101	0221	2210	1202	1120	0000	0112	1011
2122	2122	0221	2210	1202	1120	0000	0112	1011	2101
2122	2101	2210	1202	1120	0000	0112	1011	2101	0221
2122	2210	1202	1120	0000	0112	1011	2101	0221	2210
2101	0000	2101	2101	2101	2101	2101	2101	2101	2101
2101	0112	2122	1202	0000	0221	1120	2210	1011	0112
2101	0221	1202	0000	0221	1120	2210	1011	0112	2122
2101	1011	0000	0221	1120	2210	1011	0112	2122	1202
2101	1120	0221	1120	2210	1011	0112	2122	1202	0000
2101	1202	1120	2210	1011	0112	2122	1202	0000	0221
2101	2122	2210	1011	0112	2122	1202	0000	0221	1120
2101	2101	1011	0112	2122	1202	0000	0221	1120	2210
2101	2210	0112	2122	1202	0000	0221	1120	2210	1011
2210	0000	2210	2210	2210	2210	2210	2210	2210	2210
2210	0112	0221	2101	2122	0000	1011	1202	0112	1120
2210	0221	2101	2122	0000	1011	1202	0112	1120	0221
2210	1011	2122	0000	1011	1202	0112	1120	0221	2101
2210	1120	0000	1011	1202	0112	1120	0221	2101	2122
2210	1202	1011	1202	0112	1120	0221	2101	2122	0000
2210	2122	1202	0112	1120	0221	2101	2122	0000	1011
2210	2101	0112	1120	0221	2101	2122	0000	1011	1202
2210	2210	1120	0221	2101	2122	0000	1011	1202	0112