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Statistical Analysis on Optimal Lockdown Schedule by Developing a Multivariate Prediction model SEIRDVIm

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Abstract

Considering the high infection rate and bed scarcity in hospitals amidst the COVID-19 pandemic it is necessary to find out an optimal lockdown schedule for minimizing infection rate as well as maintaining economic sustainability. This paper proposes an effective compartmental model SEIRDVI_m and yields an optimal lockdown schedule using classical and quantum knapsack algorithms. When the available bed count falls below a certain threshold, the city goes into lockdown mode, and vice versa. The R^2 value of SEIRDVIm is 0.8797 and the Mean Squared Error (RMSE) is 34.59. The proposed model yields better results compared to the classical SEIR model. Variation of infected with vaccination rate and effectiveness of vaccination is demonstrated. Using 10 predictors it is found that for 60 days, quantum-assisted lockdown yields a death toll of 15062 compared to 20123 in classical knapsack induced lockdown.

Key words: SEIRDVIm model; Death rate; Knapsack problem; Lockdown schedule; Mean Squared Error; R-squared (R^2).

AMS Subject Classifications: 62K05, 05B05

1. Introduction

In December 2019 the outbreak of the novel severe acute respiratory syndrome Coronavirus called SARS-CoV-2 started locally in Wuhan, China, and rapidly spread all over the world. As reported 65.8 lakh deaths all over the world on 23rd October 2022. For deciding public policy several epidemic models have been used by the nation during the past few

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years, see Ferguson *et al.* (2020). It is important to understand the impact of precautionary measures and medical intervention on multiple variants. The impact of effective vaccination in to fight against COVID-19 is tremendous. Vaccination production and proper distribution are important. Future prediction on pandemic significantly dominates vaccine distribution. For studying the effect of vaccination, additional compartments have been added to the existing models to analyze the effectiveness of vaccination. Matrajt *et al.* (2021) studied the effectiveness of vaccination for allocating vaccines properly. In the past also several analyses have been done on vaccination at the time of previous outbreaks, see Feng *et al.* (2011), Scherer and McLean (2002) and Chowell *et al.* (2019). For studying the spread of a disease in a population, SIR-based epidemic models are widely used, see Cooper *et al.* (2020), Kuhl and Kuhl (2021), and others. An extended SEIR-based model to predict the future trend of COVID-19 has been proposed by Lal *et al.* (2021). The main framework of the study by Davies *et al.* (2020) is the transmission of disease using age-based modeling. In research it is discussed the population behavior a level of caution and sense of safety while considering vaccine efficacy, see Usherwood *et al.* (2021).

This paper proposes an effective compartmental model $\mathrm{SEIRDVI_m}$ and yields an optimal lockdown schedule using classical and quantum knapsack algorithms. When the available bed count falls below a certain threshold, the city goes into lockdown mode, and vice versa. The novel contributions of this research article are as follows:

Proposed a new compartmental model $\mathrm{SEIRDVI_m}$ for designing an optimal lockdown schedule using the quantum knapsack algorithm by maximizing the objective function, available bed capacity and minimizing the death and then converting the objective function into an energy function using binary quadratic model (bqm). Then minimize the same by D-Wave Quantum Annealer.

This paper is represented as follows. Section 1 illustrates the introduction. The newly proposed model SEIRDVIm is discussed in Section 2. Section 3 highlights the result of the evolution of the proposed model with lockdown optimization using classical and quantum knapsack. Section 4 provides the discussion of the work and at last section 5 concludes the paper.

2. Methods

The proposed $\mathrm{SEIRDVI_m}$ model in Figure 1 divides the population into susceptible (S), exposed (E), infected incompletely vaccinated (I_{iv}), infected completely vaccinated (I_{cv}), recovered (R), vaccinated (V), Immunized (I_m), and deceased (D).

$\mathrm{SEIRDVI_m}$ model is described by eight linear differential equations. Variation of eight compartments S, E, I_{iv}, I_{cv}, R, V, I_m, and D with time (t) are depicted in equations 1 to 8. The assumptions of the model are:

I. The population is fixed.

II. After being completely vaccinated, a person can become infected with a lower rate of infection.

III. After complete and successful vaccination, immunity may be gained at rate η.

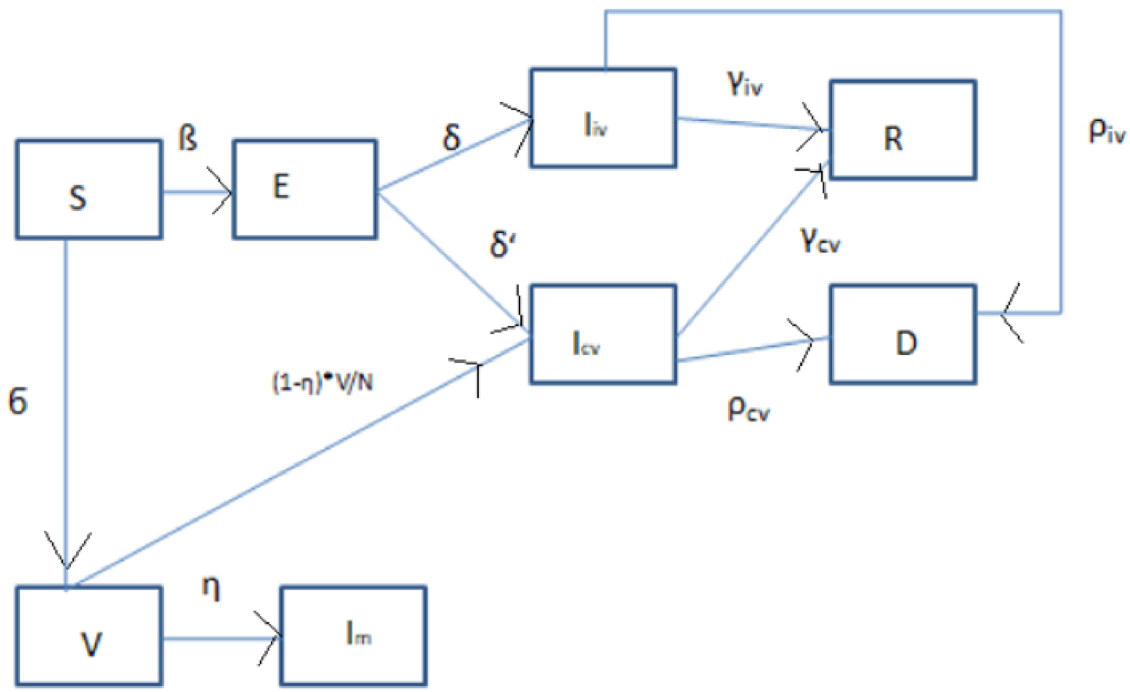

Figure 1: SEIRDVI$_\mathbf{m}$ model

The differential equations 1 to 8 of the model are given below:

$$\frac{dS}{dt} = -\beta S\frac{(I_{IV}+I_{CV})}{N} - \sigma S + \mu I_m \tag{1}$$

$$\frac{dE}{dt} = -\beta S\frac{(I_{IV}+I_{CV})}{N} - \delta E + \delta' E \tag{2}$$

$$\frac{dI_{IV}}{dt} = -\delta E - (1-\alpha)\gamma_{IV} I_{IV} - \alpha\rho_{IV} I_{IV} \tag{3}$$

$$\frac{dI_{CV}}{dt} = -\delta' E - (1-\alpha)\gamma_{CV} I_{CV} - \alpha\rho_{CV} I_{CV} + (1-\eta)\frac{V}{N} \tag{4}$$

$$\frac{dR}{dt} = (1-\alpha)\gamma_{CV} I_{CV} + (1-\alpha)\gamma_{IV} I_{IV} \tag{5}$$

$$\frac{dD}{dt} = \alpha\rho_{CV} I_{CV} + \alpha\rho_{IV} I_{IV} \tag{6}$$

$$\frac{dv}{dt} = \sigma\frac{S}{N} - \frac{V}{N} \tag{7}$$

$$\frac{dI_m}{dt} = \eta\frac{V}{N} - \mu I_m \tag{8}$$

The parameters of the equations are described in Table 1, see Lobinska *et al.* (2022) and Rella *et al.* (2021).

Table 1: Parameters of the model

Parameter	Value
Transmission of disease β	$\{0.0155, 0.18\}$
Infection rate δ	1.1
Infection rate after vaccination δ'	0.5
Death rate after incomplete vaccination ρ_{IV}	0.2
Death rate after complete vaccination ρ_{CV}	0.01
Recovery rate after incomplete vaccination γ_{IV}	0.076
Recovery rate after complete vaccination γ_{CV}	0.79
Fatality rate α	0.05
Vaccination rate σ	$\{0.3, 0.8\}$
Vaccine effectiveness η	$\{0.2, 0.7\}$

3. Results

The newly proposed SEIRDVI$_\mathrm{m}$ model is used to run for 51 days. The time frame is divided into intervals of five days. SEIRDVI$_\mathrm{m}$ model is used to run for each interval of time for each of the five cities. SEIRDVI$_\mathrm{m}$ model in Figure 2 depicts the variation of incompletely vaccinated people with vaccination rate and effectiveness of vaccination. The vaccination rate has varied from 0.3 to 0.8 with effectiveness 0.2 to 0.7. From Figure 3, the Variation of infection with completely vaccinated with vaccination rate and effectiveness is seen. Figure 4 exhibits the variation of death with a product of vaccination rate and effectiveness. As the product increases total death count decreases. Figure 5 depicts the comparison of the total infected in the simulation result and the actual data value. The registry data of the United States is collected in the period of 1st March 2020 to 23rd March 2020, see Liu *et al.* (2021) and Alamo *et al.* (2020). The data set is used for validation of the model.

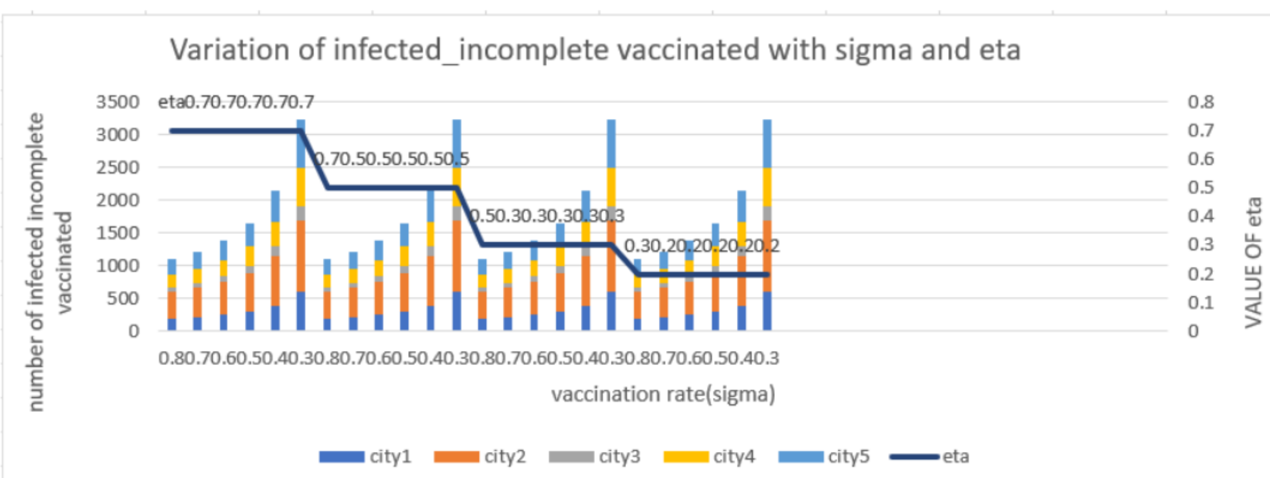

Figure 2: Variation of infected incompletely vaccinated with vaccination rate and effectiveness

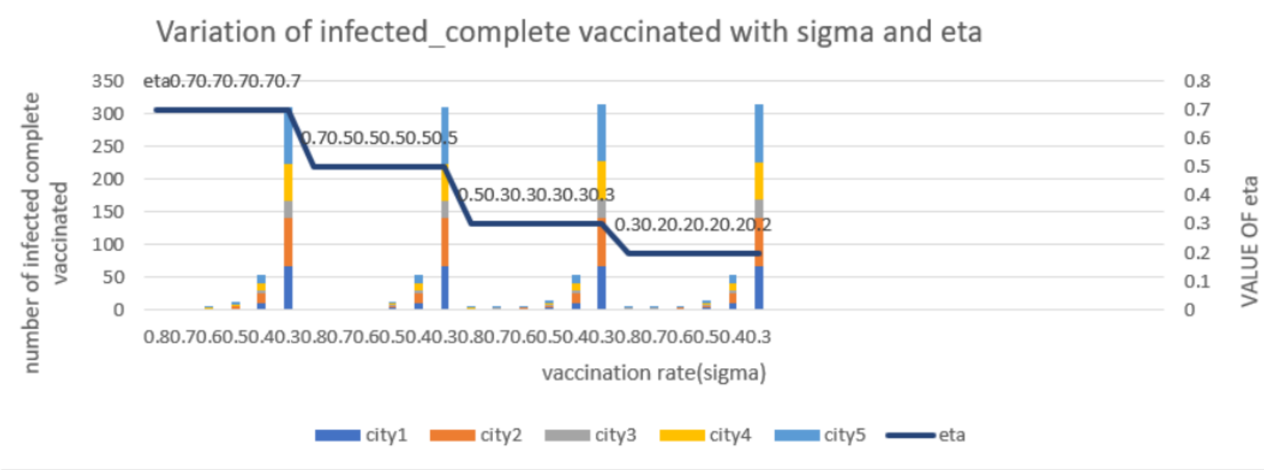

Figure 3: Variation of infection with completely vaccinated with vaccination rate and effectiveness

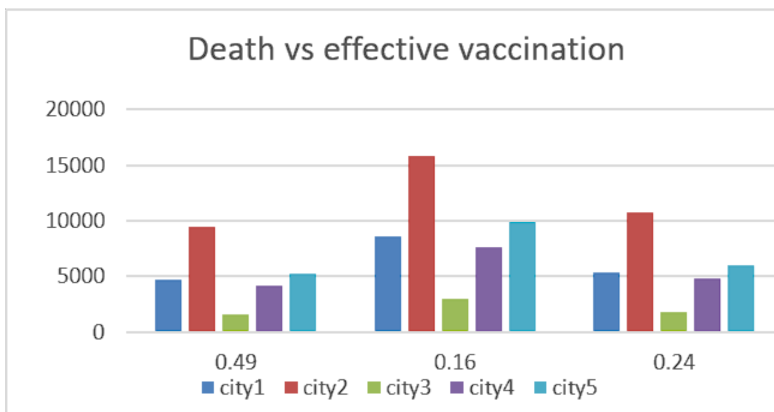

Figure 4: Variation of Death with effective vaccination

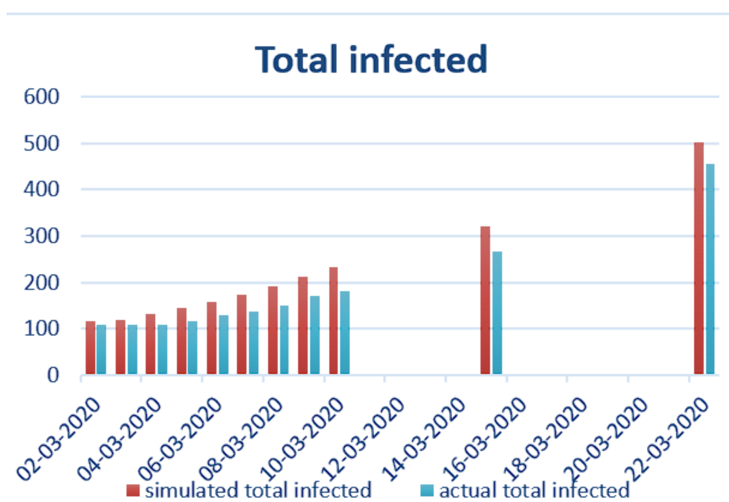

Figure 5: Comparison on total infected with actual data and simulated results

In order to analyze the model, R-squared (R^2) and Mean Squared Error (RMSE) are used for comparison. R-squared (R^2) is a statistical measure of how close the simulation result matches the actual data. The higher the value of R-squared (R^2), the better the model fits the actual data. Equations 9 and 10 describe R-squared (R^2) and Mean Squared Error (RMSE).

$$R^2 = \frac{\sum(simulated\ result - actual\ value)^2}{\sum(actual\ value - Mean\ value)^2} = 0.8797 \tag{9}$$

The RMSE value calculates the error between the simulated result value and the real data. The more the RMSE value closes to 0, the better the result, see Lucas (2014).

$$RMSE = \frac{\sqrt{\sum_{i=1}^{N}(simulated\ value\ i - actual\ value\ i)^2}}{N} = 34.59 \tag{10}$$

Table 2: Comparison of Models

Parameter	Classical SEIR, Liu *et al.* (2021)	Proposed Model SEIRDVIm
R^2	0.60624	0.8979
RMSE	4132.2348	34.59

3.1. Model 1: Lockdown using classical knapsack

Lockdown state is represented by 0 and open state is represented by 1. Figure 6 depicts the scenario when the city is in open or closed states.

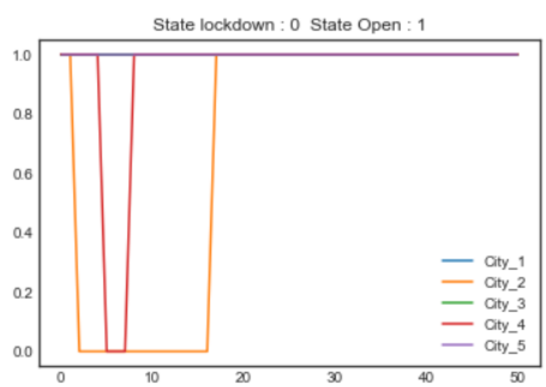

Figure 6: Lockdown in cities

3.1.1. Lockdown and open state for five cities

We are using the same parameters as in the SEIRDVI$_{\mathrm{m}}$ model with no lockdown scenario as in Table 1. The classical knapsack algorithm is applied every five days to obtain the optimal lockdown schedule. We consider bed capacity as cost and the number of infected as weight. The knapsack will contain only the open cities. The cities are selected in such a way that the number of available beds is maximized and death will be minimized. Figure 7 describes the classical knapsack-imposed lockdown schedule. The variation of bed capacity for each city with the number of days in lockdown is portrayed in Figure 8.

Day / City	0	5	10	15	20	25	30	35	40	45	50	55	60
City_1	O	O	O	O	O	O	O	O	O	O	O	O	O
City_2	O	X	X	X	X	O	O	O	O	O	O	O	O
City_3	O	X	X	O	O	O	O	O	O	O	O	O	O
City_4	O	X	X	X	O	X	O	O	O	O	O	O	O
City_5	O	X	X	X	X	X	O	O	O	O	O	O	O

Figure 7: Lockdown schedule using classical knapsack

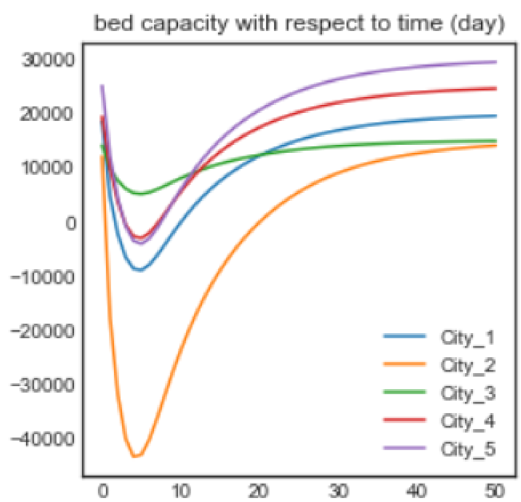

Figure 8: Variation of available bed capacities for five cities with number of days in lockdown

It is reflected in the result that for classical knapsack-imposed lockdown, the total death is 20123 after 60 days, where 356169 have been recovered and 7060317 have been vaccinated.No real data is used to derive the lockdown schedule.

3.2. Model 2: Lockdown using quantum knapsack

For deriving an optimal lockdown schedule in quantum, we need to transfer the objective, i.e. maximizing available bed capacity and minimizing the death into an energy function using a binary quadratic model (bqm). Then we minimize the energy function by D-Wave Quantum Annealer. Lucas (2014) described the quantum algorithm for the knapsack problem. The Quantum algorithm for the knapsack problem is built by using the algorithm Q-Knapsack ($city_{index}$, $city_{GDP}$, $city_{infected}$, $city_{bedCapacity}$) where $city_{GDP}$ represents the GDP of each city, $city_{infected}$ is the number of infected in the city, and $city_{bedCapacity}$ represents the hospital bed capacity of each city. See Annexure for algorithm 1 of quantum knapsack algorithm for generating binary quadratic model, from which lockdown schedule is obtained based on closed and open cities sample set.

For deriving optimal an lockdown schedule using quantum knapsack we are using the same parameters as described in Table 1. Figure 9 depicts the lockdown schedule as time is divided into five-days intervals.

Day City	0	5	10	15	20	25	30	35	40	45	50	55	60
City_1	O	X	X	X	O	X	X	X	X	X	X	X	O
City_2	X	X	X	X	X	X	X	X	X	X	X	X	O
City_3	X	X	X	X	O	X	O	O	X	O	O	X	X
City_4	X	X	X	O	X	O	O	O	X	X	X	X	O
City_5	X	X	X	X	X	O	X	X	O	O	X	O	X

Figure 9: Quantum imposed lockdown schedule

Using the same rule, we are putting the cities in the knapsack such that available bed increases and death decreases. The cities that are not in knapsack need to be in lockdown. Algorithm 2 describes the quantum algorithm for lockdown. Figure 10 shows the variation of bed capacity for each city with a number of days in lockdown.

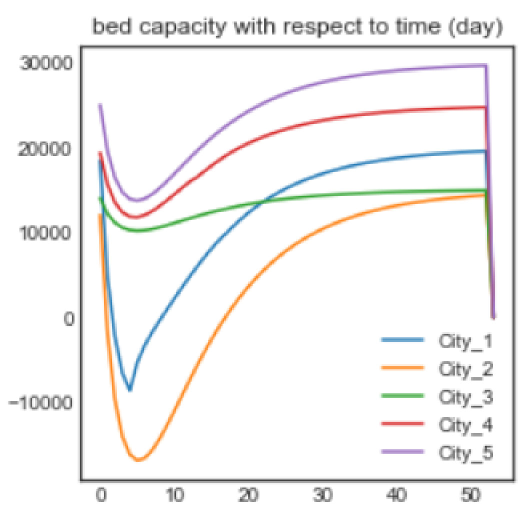

Figure 10: variation of bed capacity with number of days in lockdown

It is reflected in the result that for quantum-imposed lockdown the total number of deaths is 15062 after 60 days, where 192804 have been recovered and 7230041 have been vaccinated. Figure 11 depicts the comparison of infected who are incompletely vaccinated by classical knapsack-imposed lockdown and Quantum knapsack-imposed lockdown. Algorithm 2 describes the algorithm of quantum-imposed lockdown, see Annexure for Algorithm 2.

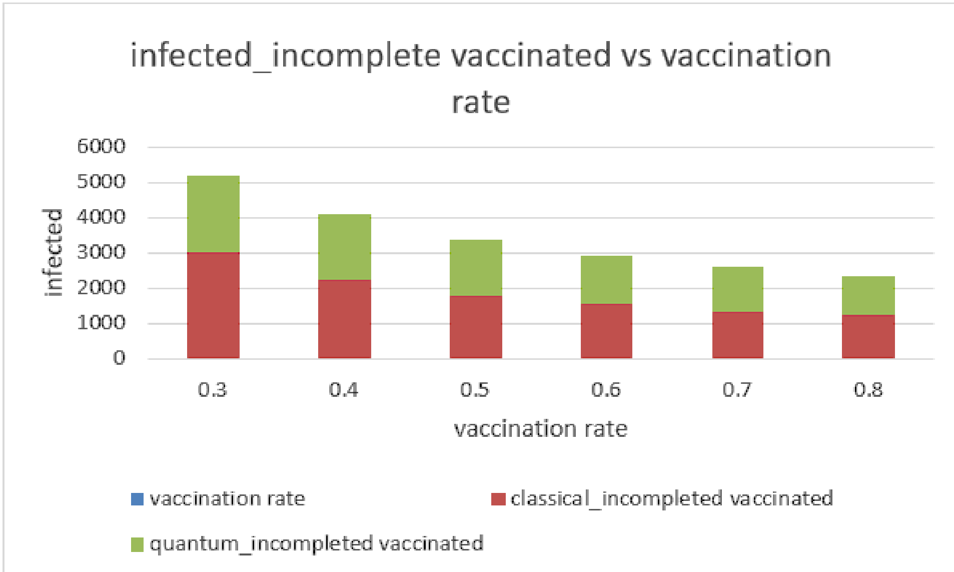

Figure 11: Comparison of infected who are incompletely vaccinated by classical knapsack-imposed lockdown and quantum knapsack-imposed lockdown

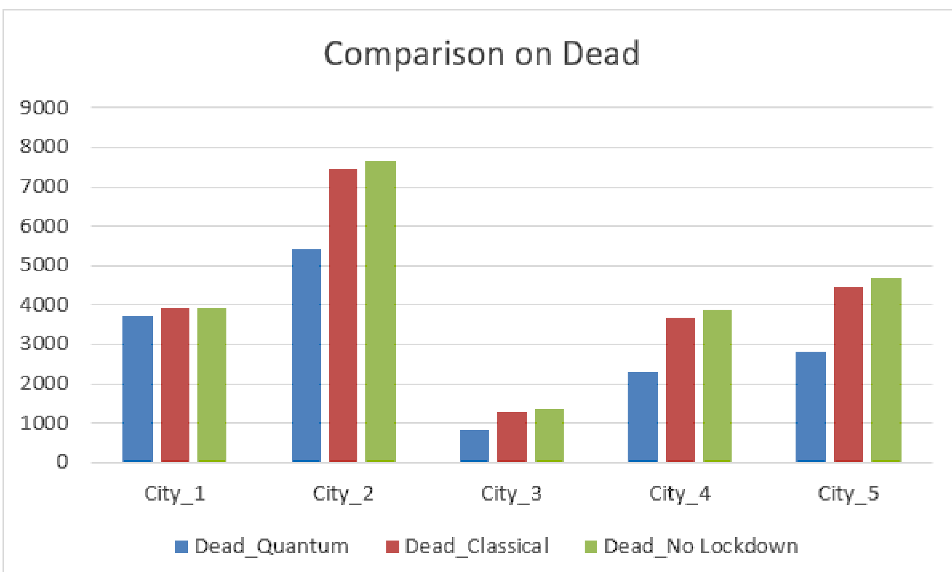

Figure 12: Comparison of dead by no lockdown, classical knapsack-imposed lockdown and quantum knapsack-imposed lockdown

In Figure 12, a comparison of the death count is done between no lockdown, classical knapsack-assisted lockdown, and quantum knapsack-assisted lockdown algorithm. It is reflected from the result that quantum causes 15062 total deaths whereas in classical knapsack death count is 20123.

4. Discussion

This paper fulfills the objective by validating the proposed model with real data. The model fits better and the root mean square error concerning actual data is lesser compared to the classical SEIR model. The model shows the effectiveness of vaccination by showing the variation of infected and death with vaccination rate. It is observed that number of infected is much lesser in complete vaccination compared to incomplete vaccination. An optimal lockdown schedule is derived by applying a quantum knapsack algorithm and it is found that compared with classical knapsack-based lockdown quantum assisted lockdown results in lesser death. For 60 days, quantum-assisted lockdown yields a death toll of 15062 compared to 20123 in classical knapsack-induced lockdown. Compared to classical, quantum knapsack implements a lockdown schedule more efficiently so that the number of infections decreases resulting increase in available bed capacity and thus number of deaths. Because of this, the death toll of quantum-assisted method is much smaller compared to classical Knapsack algorithm. However, the paper has the limitations that the exact date of obtaining predictor values is not known. Despite this limitation, the $\mathrm{SEIRDVI_m}$ model can predict the possible infected and death as well as help to decide on lockdown.

5. Conclusion

In this paper, our objective is to propose an effective compartmental model $\mathrm{SEIRDVI_m}$ considering complete and partially vaccinated populations with immunized as a separate compartment. The model yields better results compared to the classical SEIR model in terms of R^2 and RMSE values. This model yields an optimal lockdown schedule using classical and quantum knapsack algorithms. It is reflected in the result that for 60 days, quantum-based lockdown resulted death toll of 15062 compared to 20123 in classical knapsack-induced lockdown.

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ANNEXURE

Algorithm 1: An algorithm to obtain Binary Quadratic Model for Quantum Knapsack

Require: $city_{index}$, $city_{GDP}$, $city_{infected}$, $city_{bedCapacity}$
bqm : Binary Quadratic Model
$lagrange \leftarrow \max(city_{value})$
$x_{size} \leftarrow length(city_{infected})$
$y_{index_{max}}$: maximum index in y
for $k \leftarrow 1,\ x_{size}$ **do**
    bqm.setLinear($city_{index_k}, lagrange * (city_{infected_k})^2 - city_{GDP_k})$)
end for

for $i \leftarrow 1,\ x_{size}$ **do**
    for $j \leftarrow i+1,\ x_{size}$ **do**
        bqm.setQuadratic$[city_{index_i}, city_{index_j}] \leftarrow 2(lagrange * city_{infected_i} * city_{infected_j})$
    end for
end for

for $k \leftarrow 1,\ y_{index_{max}}$ **do**
    bqm.setLinear('y' + string(k), $lagrange * (y_k)^2$)
end for

for $i \leftarrow 1,\ y_{index_{max}}$ **do**
    for $j \leftarrow i+1,\ y_{index_{max}}$ **do**
        bqm.setQuadratic$[y_i, y_j] \leftarrow 2 * lagrange * y_i * y_j$
    end for
end for

for $i \leftarrow 1,\ x_{size}$ **do**
    for $j \leftarrow i+1,\ y_{index_{max}}$ **do**
        bqm.setQuadratic$[city_{index_i}, y_j] \leftarrow -2 * lagrange * city_{infected_i} * y_j$
    end for
end for

Algorithm 2: An algorithm to lockdown city based on Binary Quadratic Model from Quantum Knapsack

Require: bqm : Binary Quadratic Model for Quantum Knapsack
  Get OpenCity & ClosedCity sampleset from bqm based on bed Capacity threshold.
  for each city **do**
    if $city_{index}$ is in OpenCities **then** LockdownList $\leftarrow 1$
    else LockdownList $\leftarrow 0$
    end if
  end for
    return LockdownList