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Optimal group divisible designs from affine resolvable designs

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Abstract

A method of construction of optimal group divisible designs from affine resolvable BIB designs is described. We also obtain an optimal group divisible design by dualization.

Key words: Affine resolvable designs; Optimal group divisible designs.

1 Introduction

Group divisible designs form the most important class of 2-associate partially balanced incomplete block designs and have been studied extensively. Good sources of references are Clatworthy(1973),Dey(1986) and Raghavarao(1971).Some recent methods of construction have been given by Sinha (2006).Optimality of 2-associate PBIB designs have been investigated by Cheng and Bailey(1991).Here, A method of construction of optimal group divisible designs from affine resolvable BIB designs is described.

2 The Constructions

For preliminaries and terminologies one may refer to Raghavarao (1971), Dey (1986). Let us recall the terminologies for affine α-resolvable designs. For affine α-resolvable block designs q_1 and q_2 denote within and between resolutions common block intersections numbers. For affine resolvable designs $\alpha = 1$ and $q_1 = 0$.

It was shown by Shrikhande and Bhagwandas (1965) op. cit. Raghavarao (1971) that duals of affine α-resolvable BIB and PBIB designs are semi-regular group divisible designs(SRGDs).In fact this results may easily be extended to hold as follows:

Theorem 2.1 *The dual of an affine-resolvable binary incomplete block design with parameters :*$v^*, b^*, r^*, k^*, q_1, q_2, v^*/k^* = b^*/r^* = n$ *is a semi-regular group divisible design* $v = b^*, r = k^*, k = r^*, b = v^*, \lambda_1 = q_1, \lambda_2 = q_2, m(= r^*), n(= b^*/r^*)$.

Proof 2.1 *The mere property of affine resolvability implies that the dual design is semi- regular group divisible design; the pairwise concurrency of treatments has no role to play. In particular, when* $\alpha = 1$, *the semi-regular group divisible design is having* $\lambda_1 = 0$.

Example 2.1 Let us consider the affine -resolvable binary block design :
[(1248)(3567)][(2358)(1467)][(3468)(2571)][(4578)(3612)] having parameters :
$v = 8 = b, r = 4 = k, q_1 = 0, q_2 = 2$.

Its dual is a semi-regular group divisible design given as :
(1468)(1368)(2358)(1457)(2367)(2458)(2467)(1357) having parameters:
$v = 8 = b, r = 4 = k, \lambda_1 = 0, \lambda_2 = 2, m = 4, n = 2$. □

By applying the above theorem to well known Latin square type PBIB design (see Dey, 1986) with parameters:
$v = s^2, r = s - 1, k = s, b = s(s - 1), \lambda_1 = 0, \lambda_2 = 1$ (s is a prime or prime power) and satisfying affine resolvability, we get,

Corollary 2.1 *There exists a semi-regular group divisible design with parameters:*
$v = s(s-1), r = s, k = s-1, b = s^2, \lambda_1 = 0, \lambda_2 = 1, m = s-1, n = s$, *where* s *is a prime or prime power.*

This SRGD design is optimal, see, Cheng and Bailey (1991) among the class of designs with same (v, b, k).

Theorem 2.2 *The existence of an affine resolvable BIBD with parameters:*
$v', b', r', k', \lambda', q_s(= v/k = b/r)$ *implies the existence of a SRGD design with parameters:*
$v = v'r' = b, k = r'k' = r, \lambda_1 = \lambda, \lambda_2 = \lambda + 1, m = r'k', n = v'/k'$.

Proof 2.2 *Let us consider an affine-resolvable BIBD:* v', b', r', k', λ', $q_2(= v/k = b/r)$. *It is well known that this affine resolvable BIBD corresponds to an* $OA(v', m = r', q = v/k, 2)$.

Let the set of ordered tuplets: $(i, j), i = 1, 2, ..., v'$; $j = 1, 2, ...r'$, *denote the* $v = v'r'$ *treatments of the required SRGD.*

The treatments in blocks of each resolution class are indexed by the symbols $0, 1, ..., q(= v/k)$ and without loss of generality the symbol 0 will denote the blocks containing treatments 1.

Let us consider a column of the OA. Let its entries be $(x_1, x_2, \ldots, x_n)$ in that order. Let B_j denote the x_jth block in a resolution of BIBD. Let $M = 1, 2, ..., r'$ be the set of rows of the OA. Then we form the blocks of required SRGD as: $\{(j, x) : j\varepsilon M, x\varepsilon b_j\}$.

This process is repeated for every column of the OA and for every resolution class of the BIBD. Thus we get the desired SRGD. The derivation of the parameters is similar to Bagchi (2004) .

A series of affine -resolvable BIBD: $v = s^2, b = s(s + 1), r = s + 1, k = s, \lambda = 1$, is well known. By the application of the above theorem to this affine-resolvable BIBD, we get the following:

Corollary 2.2 *There exists a new series of SRGD with parameters:* $v = s^2(s+1) = b, r = s(s+1) = k, \lambda_1 = s, \lambda_2 = s+1, m = s+1, n = s^2$, *where* s *is a prime or prime power.*

The SRGD design obtained in Theorem 3 and corollary 4 are also optimal see Cheng and Bailey (1991) among the class of designs with same (v, b, k).

For $s = 2$, the design is given in Bagchi (2004). For $s = 3$, we get an optimal SRGD with parameters: $v = 36 = b, r = 12 = k, \lambda_1 = 3, \lambda_2 = 4, m = 4, n = 9$.

Remark 2.1 *The existence of another semi-regular group divisible design with parameters:* $v = s(s + 1), b = s^2, r = s, k = s + 1, \lambda_1 = 0, \lambda_2 = 1, m = s + 1, n = s$ *obtained as dual of the above affine-resolvable BIBD is already well known, see Dey (1986).*

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