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Wavelet-ARIMA-TDNN Model for Agricultural Commodity Price Forecasting

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Abstract

In every agricultural market, accurate agricultural commodity price forecasting is essential for farmers, traders, policymakers, and government sectors. Decomposition of the price series has sufficiently increased the forecast accuracy. In the past years, wavelet analysis has been widely used for the decomposition of price series, where it converted time series into high and low frequencies. Often, without accounting for the linearity of the frequencies in wavelet-hybrid models, those frequencies are modeled directly. A major problem arises when wavelet-hybrid models contain both linear and non-linear frequencies. Hence, a type of wavelet-hybrid model was developed to solve this problem. Tomato's monthly wholesale price in the Mumbai market was used in this study. First, linear, and non-linear frequencies are separated by the McLeod and Li test after the wavelet decomposition of the tomato price series. Autoregressive Integrated Moving Average (ARIMA) and Time Delay Neural Network (TDNN) were applied to linear and non-linear frequencies, respectively. Forecasts of ARIMA and TDNN were reconstructed to obtain forecasts of the tomato price series. Finally, our proposed wavelet-ARIMA-TDNN model was compared to ARIMA, TDNN, and Wavelet-ARIMA, Wavelet-TDNN. The result revealed that our proposed method outperformed other models.

Key words: McLeod and Li test; Non-linearity; Decomposition; Wavelet analysis; Wavelet-ARIMA-TDNN.

AMS Subject Classifications: 62K05, 05B05

1. Introduction

In a populous country like India, satisfying people's daily food demands is cumbersome. Thus, farmers have the responsibility to increase food production, especially for vegetables. It is due to the perishable nature of vegetables, which causes their prices to fluctuate and affects farmers' revenue. Therefore, predicting this price fluctuation is essential for farmers, traders, policymakers, government sectors, *etc.* Forecasting this highly volatile price is a very challenging task for forecasters. Understanding the nature of the price series

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is important for forecasting it. Generally, time series follow either a linear or non-linear pattern.

The ARIMA model is one of the most important and widely used models for linear time series. Due to its inherent statistical properties and use of the Box George *et al.* (1976) approach, the ARIMA model is popular. On the other hand, ANNs provide good self-learning and non-linear approximation skills when dealing with non-linear complex data sets. ANN has some success with predicting applications with a lag, particularly for non-linear time series. However, there is no specific model to handle all the circumstances. But we can get good results through the appropriate application of suitable models.

The ARIMA model performs well as a predictor for linear time series. Singla *et al.* (2021) found that ARIMA model outperformed wavelet-hybrid models for the onion price series. But, the ARIMA model's precision is insufficient to address complex non-linear situations. Jha and Sinha (2014) showed that ANN models provide better prediction accuracy for non-linear patterns than ARIMA models. Although ANNs are effective against non-linear time series, they might produce inconsistent results against linear models. Additionally, it shows that the sampling size and noise level affect the performance of linear regression model using ANN Markham and Rakes (1998).

Decomposition of time series is an essential process in modelling. Wavelet analysis Antoniadis (1997) is extensively used for decomposition and converts the time series into high and low frequencies. These frequencies are fitted using time series models, which are known as wavelet-hybrid models. Generally, without considering the linearity or non-linearity of the data's frequencies, time series models are used for modelling the frequencies in wavelet-hybrid models. Also, Paul *et al.* (2020) found that wavelet-ANN outperformed wavelet-ARIMA for modelling sub-divisional rainfall data. Nury *et al.* (2017) reported that wavelet-ARIMA was performed better than wavelet-ANN for temperature time series data. The above studies indicate that the linearity or non-linearity of the frequencies is a significant factor in wavelet-hybrid models. For instance, Anjoy *et al.* (2017) fitted the ANN model to all frequencies due to their non-linearity. Similarly,Ray *et al.* (2020) were fitted WNN to high frequencies and ANN to low frequency due to their non-linear pattern. Also, it is possible to get both linear and non-linear frequencies after the wavelet decomposition of a time series. In such a situation, it is not optimal to fit all frequencies using the same time series model.

In this research, the problem of containing both linear and non-linear frequencies was addressed. According to this problem, the wavelet-ARIMA-TDNN model was developed to obtain reliable and accurate forecasting.

2. Materials and Methods

Monthly wholesale prices of Tomato for the Mumbai market (Jan-2011 to Dec-2021) was collected from AGMARKNET (https://agmarknet.gov.in/) website.

2.1. Wavelet Analysis

Wavelets are underlying building block functions like trigonometric sine and cosine functions. A wavelet function (Equation 1) oscillates about zero.

$$\psi_{\tau,s} = \frac{1}{\sqrt{|s|}}\,\psi\left(\frac{t-\tau}{s}\right) \qquad \tau, s \in R,\ s \neq 0 \tag{1}$$

Here, τ - Translation parameter s - Scaling parameter.

Wavelets are well-described in Daubechies *et al.* (1992), Ogden (1997), and Percival and Walden (2000). Based on scaling and translation parameters, two types of wavelet transforms (continuous and discrete) exist. The continuous wavelet transform (CWT) provides coefficients for the entire real axis, which are more than necessary for extracting frequencies. On the other hand, due to scaling proportional steps of translation parameters, the dyadic discrete wavelet transform (DWT) requires a sample size of multiples of two. If the sample size is $2^J, J$ is known as the maximum level of decomposition. Equation (2) is the dyadic DWT.

$$\psi_{m,n}\left(t\right) = \frac{1}{\sqrt{2}}\psi\left(\frac{t-n2^m}{2^m}\right) \tag{2}$$

where, $\frac{1}{\sqrt{2}}$ - variance preserving factor; m- scaling parameter; n- translation parameter (ranges from 1 to 2^{J-m}).

These reasons lead to the requirement of a modified wavelet transform, which is known as a maximal overlap discrete wavelet transform (MODWT).

2.2. Maximal Overlap Discrete Wavelet Transform

A MODWT (Equation 3) can be obtained from a slight modification of dyadic DWT. In MODWT, the translation parameter is not proportional to the scaling parameter where wavelets are convoluted in each time interval for all the dyadic scales, so there is no restriction on sample size. For N sample size,

$$\psi_{m,n}\left(t\right) = \frac{1}{\sqrt{2}}\psi\left(\frac{t-n}{2^m}\right) \tag{3}$$

where n ranges from 1 to N.

It produces an overlapping tile in the time-frequency plane, so the transform is not orthogonal Percival and Walden (2000). Because of the non-orthogonality, it demands an orthogonal filter for perfect reconstruction. Based on linear filter operation, MODWT gives high frequencies and low frequencies using synthesis filters. MODWT provides J high frequencies and one low frequency at the J^{th} decomposition level. The maximum level of decomposition for the N sample size is $J = \log_2(N)$. It ranges from 1 to J. This transform partitions variance across the scale. Frequencies are reconstructed by inverse maximal overlap discrete wavelet transform (IMODWT). The variance of reconstructed series at any J^{th} level and variance of actual time series are always equal, which explains that MODWT is the variance-preserving transform.

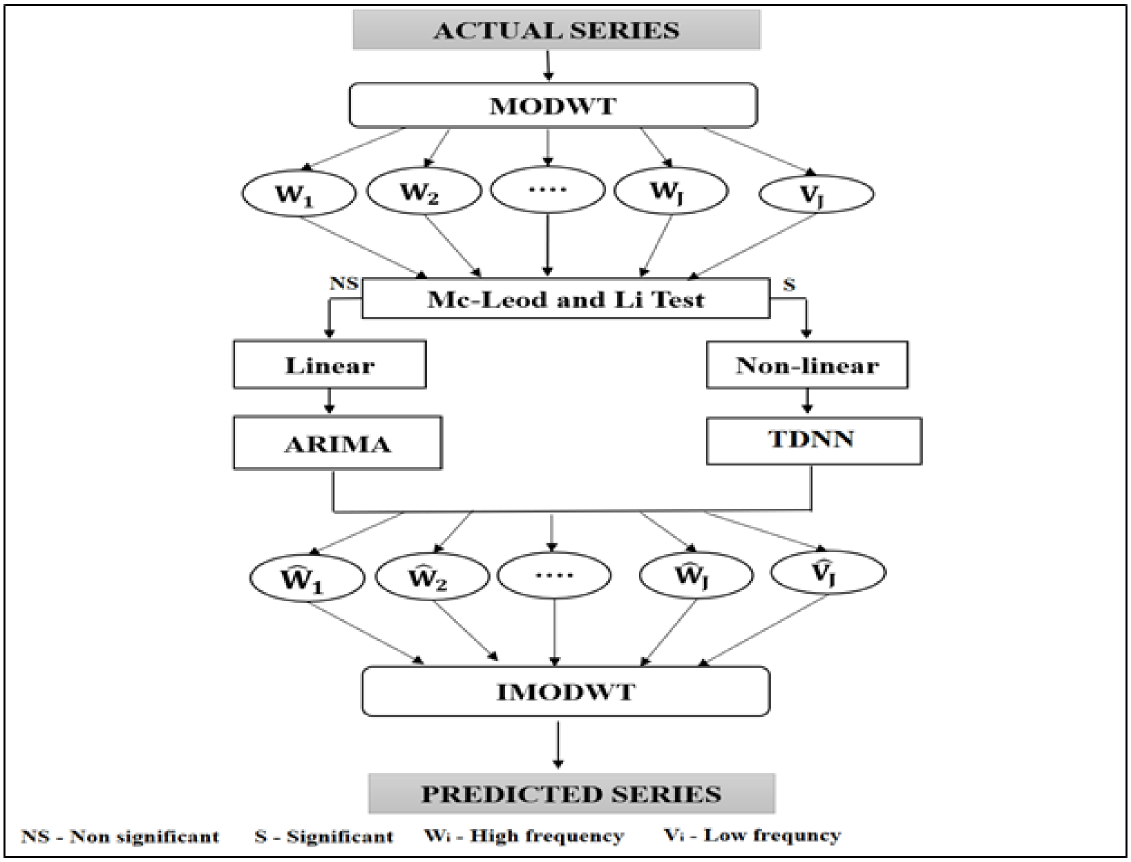

Figure 1: Schematic representation of the proposed wavelet-ARIMA-TDNN model

2.3. Wavelet-ARIMA-TDNN

Wavelet-hybrid models are the combination of wavelet analysis and time series analysis, in which time series are converted into high and low frequencies using wavelet analysis and then fitted by any time series model to increase the forecast accuracy. In this research, we developed a hybrid time series forecasting method that combines features of wavelet transformation, ARIMA, and TDNN based on the non-linearity test.

Since some of the time series data contain both linear and non-linear frequencies, the following method is developed:

Step 1: MODWT divides time series into high and low-frequency components.

Step 2: Test the non-linearity for each frequency using the McLeod and Li test.

Step 3: Identify the linear and non-linear frequencies which are fitted by ARIMA and TDNN, respectively.

Step 4: Reconstruct the forecast value of frequencies obtained from fitted models by IMODWT.

Figure 1 shows the schematic representation of the wavelet-ARIMA-TDNN model. The Haar filter was used in this study. Among all types of filters, only the Haar filter has the property of discontinuity. So, it can capture sudden changes in the signal.

2.4. Non-linearity test

McLeod and Li (1983) is the Ljung-Box test for squared time series data.

$$Q\,(m) = n(n+2)\sum_{j=1}^{m}\frac{r_j^2}{n-j} \tag{4}$$

where, r_j- autocorrelation at j^{th} lag; m-number of lags. Under the null hypothesis of linearity, the statistic (Q) is asymptotically distributed as a Chi-square distribution with m degrees of freedom.

2.5. ARIMA

The combination of Autoregressive and Moving Average processes and the integration is more efficient for achieving higher adaptability of actual time series data. It is denoted as ARIMA (p, d, q). It is one of the linear nonstationary time series models, defined in equation 5. For seasonal time series, ARIMA expanded into SARIMA $(p, d, q)(P, D, Q)$, which stands for Seasonal Autoregressive Integrated Moving Average. It is stated in equation 6.

$$\left(1-\sum_{i=1}^{p}\ \emptyset_i L^i\right)(1-L)^d y_t = c + \left(1+\sum_{k=1}^{q}\theta_k L^k\right)\varepsilon_t \tag{5}$$

$$\left(1-\sum_{i=1}^{p}\ \emptyset_i L^i\right)\left(1-\sum_{j=1}^{P}\mathit{\Phi}_j L^j\right)(1-L)^d\,(1-L^D)y_t = c + \left(1+\sum_{k=1}^{q}\theta_k L^k\right)\left(1+\sum_{r=1}^{Q}\mathit{\Theta}_r L^r\right)\ \varepsilon_t \tag{6}$$

where, L- lag operator; y_t- time series; p- Autoregressive order; P- Seasonal autoregressive order; d- No. of. Differences; D- No. of. Seasonal differences; q- Moving average order; Q- Seasonal moving average order; ε_t- white noise.

2.6. Time-Delay Neural Network

An Artificial Neural Network is for modelling non-linear data sets Ogden (1997), especially unknown relations between input and output datasets, through a data-driven and self-adaptive approach. Over the last few decades, neural modelling systems have been used to deal with a variety of prediction difficulties. The primary theoretical guideline for resolving problematic situations with ANNs is based on the learning principle Valiant (1984). ANN is inspired by human neurological science.

A network of basic processing nodes or neurons that are connected in a certain order to carry out basic arithmetic manipulations is known as a neural network and can be used to forecast future values of potentially noisy time series based on historical data Adamowski and Chan (2011). A Time-delay neural network is an illustration of such a design (TDNN). The number of layers and the total number of nodes in each layer must be selected to create the neural network structure that is appropriate for a given application in time-series prediction. A feed-forward neural network with a single hidden layer and an output node has been employed in the present investigation. In the hidden layer, the sigmoid function has been used as an activation function with form for the y time series,

$$f\left(y\right)=\frac{1}{1+e^{-y}} \tag{7}$$

For g input lag, h hidden nodes in the hidden layer, and one output node, the total number of parameters in a three-layer feed-forward neural network is $h(g+2)+1$.

3. Evaluation criteria

It is necessary to verify the model's accuracy to choose the most suitable model for forecasting. Root Mean Square Error (RMSE) is the standard deviation of the residuals of the model; Mean Absolute Error (MAE) is the average difference of residuals of the model; and Mean Absolute Percentage Error (MAPE) is the percentage of average absolute error which give a way to compare the performance of the different models.

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(Y_t-\widehat{Y}_t\right)^2} \tag{8}$$

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n}\left|Y_t-\widehat{Y}_t\right| \tag{9}$$

$$\text{MAPE} = \frac{1}{n}\left(\sum_{t=1}^{n}\left|\frac{Y_t-\widehat{Y_t}}{Y_t}\right|\right)*100 \tag{10}$$

Finally, the Tomato price series forecast accuracy of the developed model (Wavelet-ARIMA-TDNN) was compared to that of Wavelet-ARIMA and Wavelet-TDNN, and the single ARIMA, TDNN in this investigation.

4. Results and Discussion

The tomato price series of the Mumbai market was used to apply the developed hybrid methodology. Descriptive statistics of tomato price series of Mumbai market is given in Table 1. The data set was separated into training data (Jan-2011 to Dec-2020) and validation data groups (Jan-2021 to Dec-2021). The validation data set is used to determine the predictive accuracy after model fitting. The predicting outcomes of various methods, including ARIMA, TDNN, Wavelet-ARIMA, and Wavelet-TDNN, were examined to compare the performance of the suggested methodology with other related techniques in the field. The Ljung-Box (LB) test Ljung and Box (1978) was used to test residual series.

For ARIMA fitting, ACF and PACF plots of stationary series were used to get the possible orders for model fitting. Among all possible models, ARIMA (1,0,1) $(2,1,1)_{[12]}$ gave low AIC and BIC values. Parameter estimates are given in Table 2. The performance of the ARIMA model and its residual test is shown in Table 3.

Table 1: Descriptive statistics of tomato price series in Mumbai market

Mean	Standard deviation	Minimum	Maximum	Range	Skewness	Kurtosis	CV (%)
1439.09	835.67	459.17	4159.38	3700.21	1.18	0.57	58.07

Table 2: Fitted ARIMA model parameter estimates

Parameters	C	AR (1)	MA (1)	SAR (1)	SAR (2)	SMA (1)
Coefficient (S.E)	7.54** (2.31)	0.32* (0.14)	0.41** (0.15)	0.46** (0.16)	0.22 (0.14)	-0.76 (0.22)
AIC	1698.66					
BIC	1717.43					

Table 3: Results of fitted ARIMA and TDNN models

Models	Training set			Validation set			Ljung-Box test	
	RMSE	MAE	MAPE	RMSE	MAE	MAPE	Statistic	P value
ARIMA	502.44	339.56	24.92	973.02	808.02	47.04	7.12	0.85
TDNN	594.04	419.45	26.51	855.03	783.06	46.63	20.96	0.06

This study applied an TDNN with a hidden layer, along with sigmoid and identity as activation functions at the hidden and output layers, respectively as per prior studies Jha and Sinha (2014). The backpropagation algorithm can be used to train feed-forward networks in several different ways. In this study, the second-tier training speed was obtained using the Levenberg-Marquardt algorithm Hagen and Menhaj (1994). Rapid convergence into the modestly sized feed forward neural network is provided by this algorithm. Thus, functional approximation issues were addressed by this technique Demuth and Beale (2002). For model fitting, several combinations of input lags and hidden node sizes were tested. Input delays ranged from 1 to 8, whereas hidden neurons ranged from 1 to 10. In the validation data set for the tomato price series, three tapped delay and two hidden nodes (3:2s:1l) provided the lowest RMSE, MAE and MAPE values. Table 3 shows the performance of the TDNN model.

Table 4: Results of Wavelet-ARIMA model for all the decomposition levels

Decomposition Level	Training set			Validation set			Ljung-Box test	
	RMSE	MAE	MAPE	RMSE	MAE	MAPE	Statistic	P value
1	649.52	449.16	42.84	824.55	674.95	34.27	13.64	0.32
2	625.69	431.87	39.04	822.93	650.61	31.80	9.53	0.66
3	628.95	433.14	40.19	844.59	810.80	45.87	8.77	0.72
4	630.34	433.98	44.07	884.13	863.94	56.37	8.04	0.78
5	629.78	434.24	43.97	883.40	862.96	56.25	8.29	0.76
6	629.02	433.50	43.38	884.64	865.01	56.50	8.27	0.76
7	628.46	433.37	42.65	880.66	851.98	54.49	8.30	0.76

Actual tomato price series were decomposed through MODWT from one to seven ($\log_2[120] = 6.9$) decomposition levels. In each J^{th} level of decomposition, tomato price series were separated into J high frequencies (W_1, W_2, ... ,W_J) and one low frequency (V_J) by the Haar mother wavelet, which is a frequently used wavelet, especially for the price series. In the Wavelet-ARIMA model, all the high and low frequencies were fitted by ARIMA for all the decomposition levels without conducting the non-linearity test, and the results of Wavelet-ARIMA are reported in Table 4. Similarly, TDNN was used to fit all high and low frequencies at every decomposition level for the Wavelet-TDNN model without taking linearity into account. The results of Wavelet-TDNN are given in Table 5. Results of the non-linearity test for all the high and low frequencies are shown in Table 6. In our developed model, ARIMA was used to predict linear frequencies (W_1 and W_2) and TDNN was used for modelling non-linear frequencies (W_3, W_4, W_5, W_6, W_7, V_1, V_2, V_3, V_4, V_5, V_6, V_7). Table 7 gives that the models were used to forecast the frequencies at each level of decomposition in the developed hybrid model. Finally, the predicted values of different frequencies from fitted ARIMA and TDNN were used to reconstruct the data series at each level of decomposition.

Table 5: Results of Wavelet-TDNN model for all the decomposition levels

Decomposition Level	**Training set**			**Validation set**			**Ljung-Box test**	
	RMSE	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**	**Statistic**	**P value**
1	342.76	246.59	19.22	595.83	462.56	30.49	11.84	0.46
2	422.48	306.10	23.37	487.13	393.36	26.16	35.19	<0.01
3	440.05	319.89	24.57	651.53	554.93	37.66	36.65	<0.01
4	454.30	325.86	24.96	788.35	735.23	48.05	34.93	<0.01
5	458.38	330.33	25.32	877.61	820.73	52.22	47.10	<0.01
6	462.07	333.82	25.61	770.39	698.75	44.37	36.38	<0.01
7	459.32	331.50	25.42	769.76	694.30	44.04	46.13	<0.01

McLeod and Li's test for the actual series shows (Table 6) that the tomato price series is non-linear. Both ARIMA and TDNN models were fitted for the tomato price series. But TDNN gave better results than the ARIMA model. Therefore, TDNN performed well for non-linear time series.

Table 6: Results of McLeod and Li test

Actual Series		**Statistic**		**P value**	
Y		80.46		<0.01	
Decomposed series					
High frequency	**Statistic**	**P value**	**Low frequency**	**Statistic**	**P value**
W_1	11.95	0.98	V_1	125.15	<0.01
W_2	28.67	0.23	V_2	216.08	<0.01
W_3	97.03	<0.01	V_3	324.69	<0.01
W_4	127.27	<0.01	V_4	806.21	<0.01
W_5	297.19	<0.01	V_5	667.16	<0.01
W_6	589.54	<0.01	V_6	924.31	<0.01
W_7	697.13	<0.01	V_7	226.14	<0.01

Next, without considering the non-linearity, the Wavelet-ARIMA model was used for model fitting, which was fitted at all the levels of decomposition. The 2^{nd} decomposition level gave better results than other decomposition levels. Although Wavelet-ARIMA was fitted, Table 9 shows that it gave less RMSE, MAE and MAPE than the ARIMA model, which confirm that wavelet analysis improves the performance of the ARIMA. Similarly, Wavelet-TDNN was also tried for model fitting, which was fitted only at the first level of decomposition. But Table 6 shows that the tomato price series consists of both significant linear and non-linear frequencies. Due to modelling the linear high frequencies (W_1 and W_2) using TDNN in Wavelet-TDNN, the Ljung-Box test shows that Wavelet-TDNN was not fitted for other levels of decomposition. But Wavelet-TDNN enhanced the performance of TDNN at single level decomposition. To overcome these contrasted applications of linear and non-linear models, the developed hybrid model was applied to all the levels of decomposition. Models used for Wavelet-ARIMA-TDNN at each decomposition level are given in Table 7. The developed hybrid model (Wavelet-ARIMA-TDNN) was given better forecasts than the Wavelet-ARIMA and Wavelet-TDNN models at every decomposition level. Finally, Wavelet-ARIMA-TDNN gave a better forecast at the 2^{nd} level of decomposition than at any other level of decomposition (Table 8). In, W_1, W_2, and V_2 are the outcome frequencies where high frequencies are linear and a low frequency is non-linear. Two level decomposition of tomato price series is given in Figure 2.

Table 7: Models used for Wavelet-ARIMA-TDNN at each decomposition level

Frequencies	Decomposition level						
	1	**2**	**3**	**4**	**5**	**6**	**7**
W1	ARIMA	ARIMA	ARIMA	ARIMA	ARIMA	ARIMA	ARIMA
W2	-	ARIMA	ARIMA	ARIMA	ARIMA	ARIMA	ARIMA
W3	-	-	TDNN	TDNN	TDNN	TDNN	TDNN
W4	-	-	-	TDNN	TDNN	TDNN	TDNN
W5	-	-	-	-	TDNN	TDNN	TDNN
W6	-	-	-	-	-	TDNN	TDNN
W7	-	-	-	-	-	-	TDNN
V1	TDNN	-	-	-	-	-	-
V2	-	TDNN	-	-	-	-	-
V3	-	-	TDNN	-	-	-	-
V4	-	-	-	TDNN	-	-	-
V5	-	-	-	-	TDNN	-	-
V6	-	-	-	-	-	TDNN	-
V7	-	-	-	-	-	-	TDNN

When some time series have both linear and non-linear frequencies, it is very difficult to detect the relationship between such a series using either ARIMA or TDNN. The developed hybrid method has captured this complicated relationship significantly. It is important to note that the authors attempted to model this complicated relationship using Wavelet-ARIMA and Wavelet-TDNN, which proved to be less effective than the developed model. Hence, the model fitting and forecast accuracy became worse because ARIMA was unable to model non-linear frequencies and TDNN was unable to capture the linear relationship of the linear frequencies.

Model performance for the validation set (Table 9) shows that a combination of ARIMA and TDNN models based on the non-linearity test along with the MODWT can improve the overall accuracy. Finally, our developed hybrid method can give more appropriate results than the other methods such as ARIMA, TDNN, Wavelet-ARIMA, and Wavelet-TDNN models, especially for the series that contain linear and non-linear frequencies. Forecasts of the tomato price series from the developed model are given in Figure 3.

Table 8: Results of Wavelet-ARIMA-TDNN model for all the decomposition levels

Decomposition Level	Training set			Validation set			Ljung-Box test	
	RMSE	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**	**Statistic**	**P value**
1	463.48	331.44	25.05	524.19	415.35	23.74	6.00	0.92
2	453.52	314.16	23.45	458.34	353.15	18.93	11.54	0.48
3	481.20	341.22	26.83	587.90	474.78	24.38	12.52	0.41
4	505.52	351.16	27.38	751.53	624.70	32.02	9.84	0.63
5	510.13	356.81	28.49	861.84	726.78	37.73	12.68	0.39
6	511.64	359.27	28.57	754.63	611.60	30.34	10.21	0.60
7	526.08	367.91	27.51	814.63	637.90	31.83	15.28	0.23

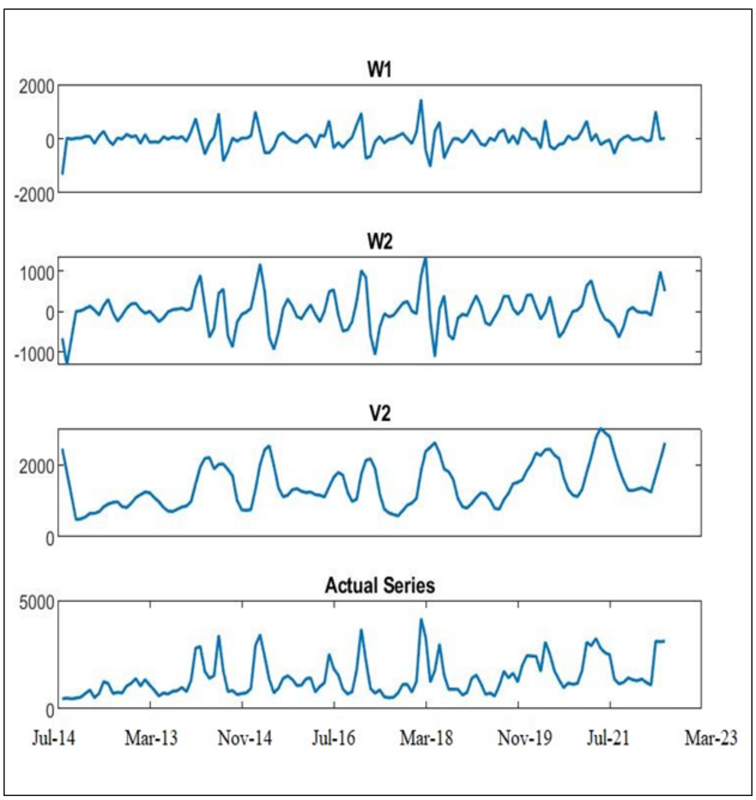

Figure 2: MODWT of tomato price series at level 2

Table 9: Forecasting ability of five different models in the validation set

Models	RMSE	MAE	MAPE
ARIMA	973.02	808.02	47.04
Wavelet-ARIMA	822.93	650.61	31.80
TDNN	855.03	783.06	46.63
Wavelet-TDNN	595.83	462.56	30.49
Wavelet-ARIMA-TDNN	458.34	353.15	18.93

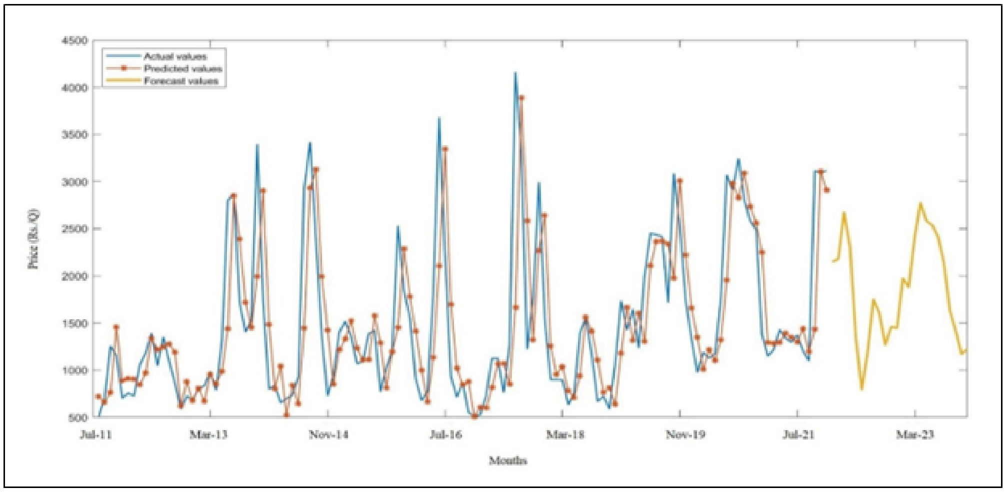

Figure 3: Actual and predicted tomato price series with its forecasts

5. Conclusion

This paper has developed a wavelet-ARIMA-TDNN model for forecasting the tomato price series in the Mumbai market. In the developed model, the McLeod and Li test was used to separate the frequencies into linear and non-linear frequencies, whereas ARIMA and TDNN were applied to model the linear and non-linear frequencies, respectively. Additionally, the developed model is confirmed to be superior to Wavelet-ARIMA, Wavelet-TDNN, ARIMA, and TDNN for modelling the series consisting of linear and non-linear frequencies. The choice of the best model was determined by forecast accuracy in the validation data set.

Finally, this study supports the following statements: (1) the linear time series model (non-linear time series model) is not appropriate for modelling the non-linear time series (linear time series); (2) wavelet decomposition (MODWT) improves the performance of the both time series models; and (3) whenever some time series contain both linear and non-linear frequencies, logical application of linear and non-linear models to the respective frequencies helps to enhance the wavelet-hybrid model fitting and forecasting.

Future research on other important non-linear models (LSTM, SVR, WNN, and so on) for modelling non-linear frequencies, as well as the use of different mother wavelets, is expected to improve our hybrid model.

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