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Bayesian Credible Intervals for Generalized Inverse Weibull Distribution

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Abstract

Bayesian credible intervals are obtained for Generalized Inverse Weibull distribution using different priors. Gibbs sampling procedure is used to draw Markov Chain Monte Carlo (MCMC) samples which are used to construct the Bayesian estimates and corresponding credible intervals. Simulation study is conducted by taking different configurations of parameter points and sample sizes to highlight the properties and comparison of the credible intervals. Illustrative example based on a real data set is also provided.

Key words: Generalized inverse Weibull distribution; Credible interval; MCMC algorithm; Posterior distribution.

1. Introduction

The three-parameter Generalized Inverse Weibull distribution (GIWD), introduced by Gusmao *et al.* (2011), is a positively skewed distribution used to model the income data and because of its ability of possessing decreasing and unimodal failure rate, is also useful in reliability and biological studies. Generalized inverse Weibull distribution is the generalization of various well-known and useful distributions, including inverse Weibull, inverse exponential, inverse Rayleigh and Fréchet distributions as special sub-models.

These distributions play an important role in many applications, including the dynamic components of diesel engines, several data sets such as the times to breakdown of an insulating fluid subject to the action of a constant tension, failure characteristics such as infant mortality, useful life and wear-out periods, analyzing the wind speed data (Drapella (1993), Jiang *et al.* (2001), Nelson (1982), Khan (2008), Zaharim *et al.* (2009)). Most of the sub-cases of generalized inverse Weibull distribution are families of inverse distributions, which can be easily fitted to income related data. These distributions have two parameters but in order to fit better at the tails, three parameters distribution (GIWD) is used in the present study.

The cdf of generalized inverse Weibull distribution is

$$F(x) = e^{\left(-\gamma\left(\frac{\alpha}{x}\right)^{\beta}\right)}, x > 0;\ \alpha, \beta, \gamma > 0,$$

where α is scale parameter and β, γ are shape parameters.

The pdf of generalized inverse Weibull distribution is

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$$f(x) = \gamma\beta\alpha^{\beta}x^{-(\beta+1)}e^{\left(-\gamma\left(\frac{\alpha}{x}\right)^{\beta}\right)}.$$

Sub-models:

- For $\alpha = 1$, it reduces to inverse Weibull (IW) distribution.
- For $\gamma = \alpha = 1$, it reduces to Fr$\acute{e}$chet (F) distribution.
- For $\beta = 2, \alpha = 1$, it reduces to inverse Rayleigh (IR) distribution.
- For $\gamma = \beta = 1$, it reduces to inverse exponential (IE) distribution.

In this paper, credible intervals for the parameters of the generalized inverse Weibull distributions are obtained. Some work using Generalized Pareto Distribution (Hosking (1987)), Weibull distribution (Kundu (2008)), Generalized Exponential Distribution (Kundu *et. al.* (2009)) and Generalized Inverted Exponential Distribution (Dey *et. al.* (2014) is already available in the literature in the case of credible interval, however, in the context of Bayesian and income inequality measure is already available in the literature for GIWD and some other distributions (Bhattacharya *et al*. (1999), Mahajan *et. al.* (2015), Arora *et. al.* (2017), Kaur *et. al.* (2018), Kaur *et. al.* (2021)). In the context of Credible interval, no work has been done for Generalized inverse Weibull distribution.

Credible interval is an interval in the domain of a posterior probability distribution or predictive distribution in Bayesian statistics. The Bayesian equivalent of the confidence interval in the classical inference is the credible interval. Bayesian interval estimators have a clearer and more direct interpretation than classical confidence intervals. Like classical confidence interval, the 95% Bayesian credible interval contains the true value with approximately 95% confidence. Bayesian intervals treat their bounds as fixed and the estimated parameter as a random variable, whereas frequentist confidence intervals treat their bounds as random variables and the parameter as a fixed value. 95% credible interval is any interval which contains a 95% percent of the posterior probability. Because the posterior density is a true probability density, we can compute quantiles and percentiles of the parameter. The simplest 95% credible interval is bounded by the 2.5th and 97.5th percentiles. This interval is called a symmetric credible interval because it removes equal probability (2.5%) from both tails of the distribution.

According to Eberly and Casella (2003) the $100\ (1-\alpha)\%$ equal tail credible interval for exact posterior distribution can be defined as

$$P(\theta < L) = \int_{-\infty}^{L} \pi(\theta|x)\, d\theta = \frac{\alpha}{2}\ ,\quad P(\theta > U) = \int_{U}^{\infty} \pi(\theta|x)\, d\theta = \frac{\alpha}{2} \tag{1}$$

where,

$\pi(\theta|x)$is posterior density of θ and
$(L\ , U)$are the lower and upper limits of the credible interval respectively for specified value of α (level of significance).

The posterior distribution is always available, although in realistically complex problems it cannot be represented analytically and becomes difficult in generation of random samples.

There are two types of algorithms used to draw samples from the true posterior. The first type is a direct method, when we draw a sample from an easily sampled density and reshape this sample by only accepting some of the values into the final sample in such a way that the accepted values constitute a random sample from the posterior. This method is inefficient as the number of parameters increases in the posterior distribution.

Secondly, the simulation method for sampling from posterior distribution is called the Markov Chain Monte Carlo (MCMC) method (Metropolis *et. al.* (1949)). The advantage of MCMC is that it gives not only a point estimator of the parameter, but also gives an interval estimation based on the final simulated empirical distribution. MCMC is essentially an iterative sampling algorithm, drawing values from the posterior distribution of the parameter in the model concerned. The simulation method for sampling from posterior distribution which computes posterior quantities of interest is called the Markov Chain Monte
Carlo (MCMC) method. A Markov chain is a well-known stochastic process model that can be used to characterize the probability of moving from one state to another. Numerous algorithms have been developed that will simulate samples from a discrete-time continuous-space Markov chain such that, after reaching a steady-state, the sequence of samples constitutes a sample from the desired joint posterior distribution. These simulated samples estimate the mean and especially the quantiles (used to compute credible intervals) of marginal posterior distributions for the parameters of interest. MCMC involves two methods, Metropolis–Hastings' algorithm and Gibbs sampling for generating samples from the posterior distribution (Metropolis *et al.* (1953), Hastings (1970)). For more details about MCMC and the related methodologies, one can refer to Gentle (1998), Chen *et al.* (2000) and Robert and Casella (2004). Gibbs sampling procedure and Metropolis-Hastings (M-H) method are used to generate samples from the posterior density function to compute the Bayesian point estimates and credible intervals. When using a Markov Chain Monte Carlo algorithm such as the Gibbs sampler to generate a sample from the posterior distribution (marginal) of interest, calculations are often easier.

1.1. Metropolis-Hastings (Bolstad, 2010) algorithm

The algorithm of Metropolis-Hastings (Bolstad, 2010) is as follows:

Let the proposed density using the Metropolis-Hastings algorithm is denoted by $q(\theta, \theta')$, which is close to target density $g(\theta|x)$,

where

$\theta$is starting value,
$\theta'$is the next generating value of θ and
$g(\theta|x)$is the posterior target density from which we need to generate θ.

1. Start at an initial value$\theta^{(0)}$.
2. Do for $n = 1,2,\ldots\ldots,n$ (n is the number of iterations)
   (a) Draw a sample from $q(\theta^{(n-1)}, \theta')$.
   (b) Calculate r = probability of acceptance = $\alpha(\theta^{(n-1)}, \theta')$.
   (c)Draw u from the uniform distribution $U(0,1)$.
   (d) If $u < r$, then let $\theta^{(n)} = \theta'$, else let $\theta^{(n)} = \theta^{(n-1)}$.

The density $q(\theta, \theta')$ close to the target density $g(\theta|x)$ leads to more points being accepted. In fact, proposed density has the same shape as the target density.

$q(\theta, \theta') = kg(\theta'|x)$

the acceptance probability

$$\begin{aligned}\alpha(\theta, \theta') &= min\left[1, \frac{g(\theta'|x)q(\theta', \theta)}{g(\theta|x)q(\theta, \theta')}\right] \\ &= min\left[1, \frac{g(\theta'|x)g(\theta|x)}{g(\theta|x)g(\theta'|x)}\right] \\ &= 1\end{aligned}$$

i.e in this case, all points will be accepted.

After generating the sample from the posterior distribution using MCMC simulation method, one important question is: how many samples are needed to accurately approximate the characteristics of the posterior distribution? This question is difficult to answer because samples generated on successive iterations are not independent of one another. Frequently, the values from one iteration and the next will be highly correlated, and a very large number of iterations will be necessary to make sure that the sample covers the entire range of the distribution. We would like our Markov chain to move about the space covered by the distribution freely. When outcome of one iteration has little effect on the next iteration, we say that the chain is mixing quickly. If the outcomes on successive iterations are highly linked, then we say that the chain is mixing slowly. If the chain is mixing slowly then it will have to be run for a long time until we can be sure that our sample properly represents the posterior distribution.

1.2. Trace plots

The simplest tool for visualizing the convergence of a Markov chain is the **trace plot**, the plot of the values generated from the Markov chain versus the iteration number. This plot shows that the chain is mixing well, moving back and forth over the space and suggests how much sample values are enough to produce accurate approximation of the posterior summaries.

It may be noted that if the chain does not converge to its stationary distribution, then there will be long burn-in period. One can observe from a trace plot that there is a relatively constant mean and variance in case of stationarity.

1.3. Burn-in period

To discard the initial portion of a Markov chain, so that the effect of initial values on the posterior inference is minimized, we use Burn-in procedure. The initial samples are not completely valid because the Markov Chain has not stabilized to the stationary distribution or at beginning of sequence, we need to run MCMC for a while to achieve convergence to target pdf. The burn in samples allows us to discard these initial samples that are not yet at the stationary distribution.

This study focuses on the generation of samples using MCMC algorithm from the posterior distribution. Then the generated samples using Metropolis–Hastings' algorithm and Gibbs sampling are used to compute the credible intervals for the parameters of interest using different prior and squared error loss function (SELF) in case of generalized inverse Weibull distribution.

The outline of the paper is: the Posterior distributions of generalized inverse Weibull distribution using different priors are given in Section 2. In Section 3, algorithms are given to compute credible intervals for the above said distributions. The convergence and mixing of Markov chain through graphical method are presented in Section 4. In this section, simulation study along with real life illustration is also carried out to compute credible intervals using different priors in case of generalized inverse Weibull distribution. Finally, Section 5 gives the conclusion of the study.

2. Posterior Distributions for Parameters of Generalized Inverse Weibull Distribution

The pdf of generalized inverse Weibull distribution is

$$f(x) = \gamma\beta\alpha^{\beta}x^{-(\beta+1)}e^{\left(-\gamma\left(\frac{\alpha}{x}\right)^{\beta}\right)}, \ \alpha,\beta,\gamma > 0.$$

The likelihood function of generalized inverse Weibull distribution is given by

$$L(x|\gamma,\alpha,\beta) = \gamma^{n}\beta^{n}\alpha^{n\beta}\left(\prod_{i=1}^{n} x_i^{-(\beta+1)}\right) exp\left(-\gamma\alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta}\right).$$

2.1. Posterior densities of parameters of GIWD using informative prior

Informative prior depends on the elicitation of prior distribution based on pre-existing scientific knowledge in the area of investigation. This information may be available from the previous investigation or from non-statistician experts. Assuming parameters α,β,γ have independent Gamma priors with the pdf's

$$g(\alpha; a_{2,}b_2) = \frac{{b_2}^{a_2}\alpha^{a_2-1}exp\,(-\alpha b_2)}{\Gamma(a_2)},$$

$$g(\beta; a_{3,}b_3)= \frac{{b_3}^{a_3}\beta^{a_1-1}exp\,(-\beta b_3)}{\Gamma(a_3)},$$

$$g(\gamma; a_{1,}b_1)= \frac{{b_1}^{a_1}\gamma^{a_1-1}exp\,(-\gamma b_1)}{\Gamma(a_1)},$$

where a_i, b_i for $i = 1,2,3$ are hyperparameters.

Assuming that the parameters are mutually independent, the posterior distribution is proportional to the product of the prior and the likelihood function given by

$$g^{*}(\alpha,\beta,\gamma|x) \propto \gamma^{n}\beta^{n}\alpha^{n\beta}\prod_{i=1}^{n} x_i^{-(\beta+1)}\, exp\left(-\gamma\alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta}\right)\gamma^{a_1-1}\, exp(-\gamma b_1)\,\alpha^{a_2-1}$$

$$exp\,(-\alpha b_2)\beta^{a_3-1}exp(-\beta b_3)$$

The full conditional posterior density of α is

$$g^{*}(\alpha|\beta,\gamma,x) \propto \alpha^{n\beta+a_2-1}exp\left(-\gamma\alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta} - b_2\alpha\right).$$

The full conditional posterior density of β is

$$g^*(\beta|\alpha,\gamma,x) \propto \beta^{n+a_3-1}\alpha^{n\beta+a_3-1}\prod_{i=1}^{n} x_i^{-(\beta+1)} exp\left(-\gamma\alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta} - b_3\beta\right).$$

The full conditional posterior density of γ is

$$g^*(\gamma|\alpha,\beta,x) \propto \gamma^{n+a_1-1} exp\left(-\left(\alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta} - b_1\right)\gamma\right) \sim Gamma(n + a_1,\ \alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta} - b_1).$$

2.2. Posterior densities of parameters of GIWD using Jeffreys' prior

Jeffreys' (1946) prior based on the Fisher's information, is defined as

$$\pi(\theta) \propto \sqrt{I(\theta)},$$

where $I(\theta) = -E\left[\frac{\partial^2}{\partial\theta^2} lnL(\theta|x)\right]$ is Fisher's information based on likelihood function $L(\theta|x)$.

The expected value of double derivatives is not in the closed form, hence the explicit experrsion for the Jeffreys' prior is not obtained. For simplicity it is assumed that all the three parameters are independent, therefore joint prior in case of Jeffreys' prior (Guure, 2012), Singh (2011) is written as

$$g(\alpha,\beta,\gamma) \propto \frac{1}{\gamma\alpha\beta}.$$

The full conditional posterior density of α is

$$g^*(\alpha|\beta,\gamma,x) \propto \alpha^{n\beta-1} exp\left(-\gamma\alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta}\right).$$

The full conditional posterior density of β is

$$g^*(\beta|\alpha,\gamma,x) \propto \beta^{n-1}\alpha^{n\beta-1}\prod_{i=1}^{n} x_i^{-(\beta+1)} exp\left(-\gamma\alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta}\right).$$

The full conditional posterior density of γ is

$$g^*(\gamma|\alpha,\beta,x) \propto \gamma^{n-1} exp\left(-\left(\alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta}\right)\gamma\right) \sim Gamma(n, \alpha^{\beta}\sum_{i=1}^{n} x_i^{-\beta}).$$

Note: The full conditional posterior densities of α, β and γ using Jeffreys' prior are obtained by taking hyperparametres as zero ($a_1 = b_1 = a_2 = b_2 = a_3 = b_3 = 0$).

3. Algorithms to Compute Credible Intervals for Generalized Inverse Weibull Distribution

The posterior densities using different priors cannot be solved directly to compute lower limit (L) and upper limit (U) of credible interval as stated in equation 1. MCMC simulation techniques allow us to generate a sample from these posterior densities using Metropolis-Hastings (M-H) method and Gibbs sampling method.

The conditional posterior distributions of α and β cannot be reduced analytically to well-known distributions and therefore it is not possible to simplify it directly by standard methods, but their graphs indicate that they are like the Gamma and Weibull distributions, respectively.

So, to generate random numbers from these distributions, use the Metropolis-Hastings (M-H) method with Gamma and Weibull as the proposed distributions. To generate γ from the posterior density, Gibbs sampling method is used.

The following algorithm is given to generate α, β and γ from their posterior density functions and in turn to obtain the Bayes estimates and the corresponding credible intervals.

- Start with $\alpha_0 = \hat{\alpha}$ and $\beta_0 = \hat{\beta}$ as their initial approximation.
- Set $j = 1$, using Metropolis – Hasting generate α_j from conditional posterior density of α with the Gamma $(\alpha_{j-1}, 2)$ as the proposal distribution and also generate β_j from conditional posterior density of β with the Weibull $(\beta_{j-1}, 2)$ as the proposal distribution. Generate γ_j from Gamma $(n + a_1, (\alpha^{\beta} \sum_{i=1}^{n} x_i^{-\beta} + b_1))$ using Gibbs sampling.
- Set $j = j + 1$
- Repeat step 2, N times.
- Obtain the Bayes estimates of α, βandγ using SELF as
  $\hat{\alpha}= \frac{\sum_{i=M+1}^{N} \alpha_i}{N-M}$, where$M$ is the burn-in period.
  $\hat{\beta}= \frac{\sum_{i=M+1}^{N} \beta_i}{N-M}$, where M is the burn-in period.
  $\hat{\gamma}= \frac{\sum_{i=M+1}^{N} \gamma_i}{N-M}$, where M is the burn-in period.
- To compute the credible intervals of α, β and γ, order $\alpha_{M+1}, \alpha_{M+2} \dots \dots, \alpha_N$; $\beta_{M+1}, \beta_{M+2} \dots \dots, \beta_N$ and $\gamma_{M+1}, \gamma_{M+2} \dots \dots, \gamma_N$ in ascending order as $\alpha_{(1)}, \alpha_{(2)} \dots \dots, \alpha_{(N-M)}$; $\beta_{(1)}, \beta_{(2)} \dots \dots, \beta_{(N-M)}$; $\gamma_{(1)}, \gamma_{(2)} \dots \dots, \gamma_{(N-M)}$. Then the $100(1-\eta)\%$ credible intervals for α, βandγ are
- $\left(\alpha_{\left(\frac{(N-M)\eta}{2}\right)}, \alpha_{\left(1-\frac{\eta}{2}\right)(N-M)}\right)$, $\left(\beta_{\left(\frac{(N-M)\eta}{2}\right)}, \beta_{\left(1-\frac{\eta}{2}\right)(N-M)}\right)$ and $\left(\gamma_{\left(\frac{(N-M)\eta}{2}\right)}, \gamma_{\left(1-\frac{\eta}{2}\right)(N-M)}\right)$,

where η is the level of significance.

In the next section, credible intervals are computed using R-software by the above algorithm.

4. Simulation Study

Statistical simulation study is carried out to compute the 95% and 99% credible intervals using different priors for generalized inverse Weibull distribution. The comparisons of priors are also done based on the width of the credible intervals; smaller the width better is the interval. According to distributions, combinations of parameters, hyper parameters and sample size should be chosen, and these are discussed below for all the three parameters. The credible intervals are computed based on 10,000 MCMC samples and first 500 values are discarded as burn-in. We plot the trace plots of the chains to determine whether the chain is exploring the parametric space well for all the parameters of GIWD. The monitoring MCMC convergence and mixing is also checked using trace and autocorrelation plots. The autocorrelation shows the mixing rate, and it is measured by autocorrelations of different lags.

The credible interval are computed for Jeffreys' and informative prior for the parametres of GIWD obtained using squared error loss function. These intervals are computed for different sample sizes $n = 30, 50$ with parameters combinations $\alpha = \beta = \gamma = 2$. The combinations of hyperparameters are taken as $a_1 = b_1 = 6, a_2 = b_2 = a_3 = b_3 = 4$ (Kaur *et. al*. (2018)) according to misfit measure. The trace, posterior density and autocorrelation plots of α, β and γ are plotted in case of informative prior.

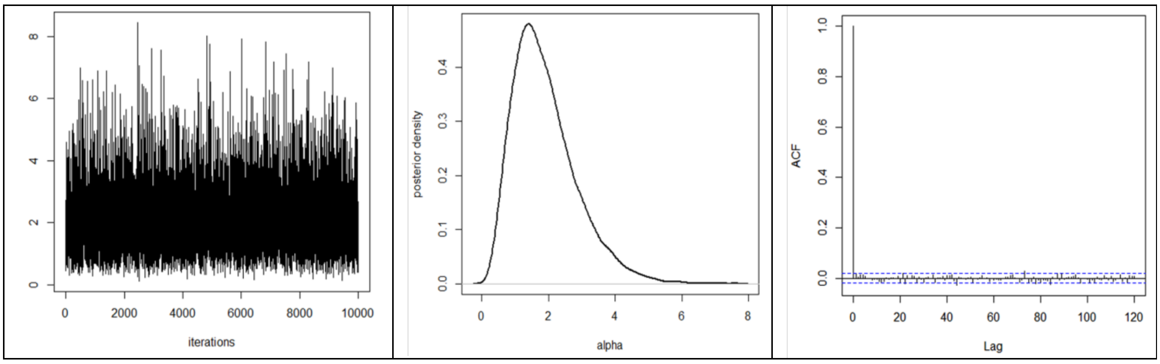

Figure 1: Trace, posterior density and autocorrelation plots of α

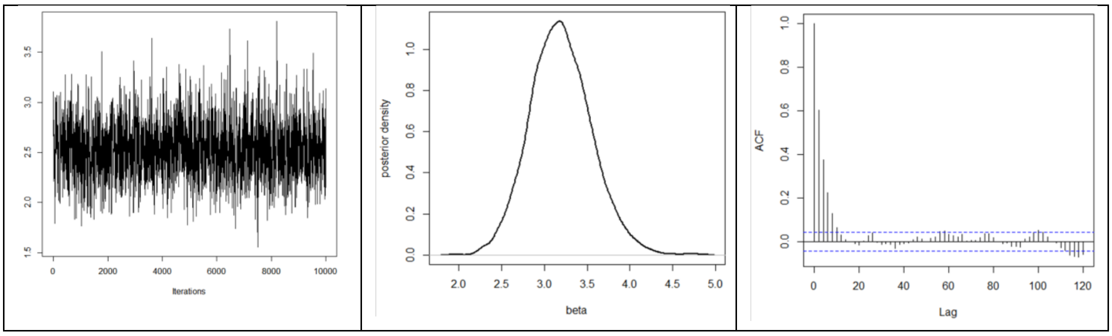

Figure 2: Trace, posterior density and autocorrelation plots of β

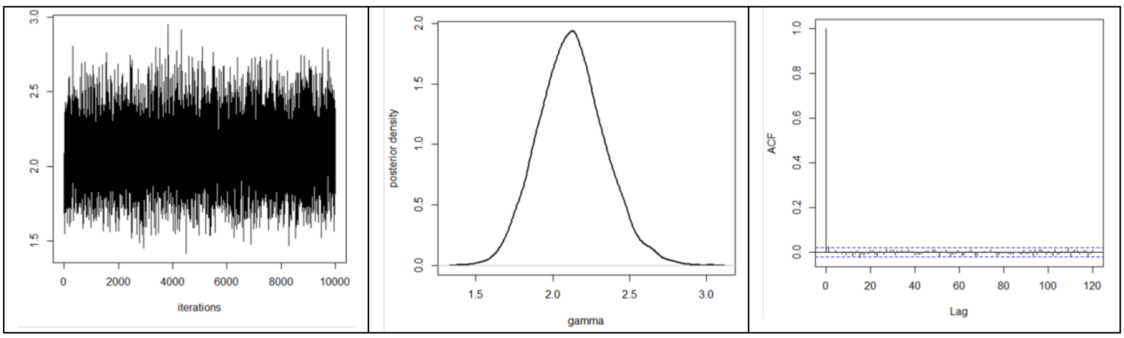

Figure 3: Trace, posterior density and autocorrelation plots of γ

Based on trace, autocorrelation and posterior plots (Figure 1-3), we conclude that

- Markov Chain (MC) has reached convergence,
- trace plot is perfect and the centre of the chain having small fluctuations indicates that the MC has reached the right distribution,
- all autocorrelations are close to zero for α and γ *i.e.*, MCMC sampling is done in independent manner and stationarity is reached. The autocorrelation plots for β shows low mixing at the starting lags and good mixing after 10^{th} lag.

The credible intervals are reported in the following Tables 1-3. From the Tables, it may be seen that

(i) Credible intervals using informative priors lead to smaller width of the interval as compared to non-informative prior for all the three parameters both for 95% and 99% C.I.

(ii) As the sample size increases, the width of the credible intervals decreases

Table 1: Credible intervals for α

n	Prior	Estimate	95% C.I. (width)	99% C.I. (width)
30	Jeffrey	1.93205	(0.56643,4.87645) 4.31002	(0.35410,6.02160) 5.6675
	Informative	2.17695	(0.51799,4.46347) 3.94548	(0.36721,5.68138) 5.31417
50	Jeffrey	2.01670	(0.57516,4.48867) 3.91351	(0.34895, 5.91940) 5.57045
	Informative	2.05906	(0.54841,4.37218) 3.82377	(0.33537,5.60383) 5.26846

Table 2: Credible intervals for β

N	Prior	Estimate	95% C.I (width)	99% C.I (width)
30	Jeffrey	2.17912	(1.73348, 2.67003) 0.93655	(1.60958, 2.86279) 1.25321
	Informative	2.05582	(1.62910, 2.53639) 0.90729	(1.49765, 2.71638) 1.21873
50	Jeffrey	1.82986	(1.55504, 2.13338) 0.57834	(1.48468, 2.25778) 0.7731
	Informative	1.90897	(1.59993, 2.15669) 0.55676	(1.51976, 2.27387) 0.75411

Table 3: Credible intervals for γ

n	Prior	Estimate	95% C.I. (width)	99% C.I. (width)
30	Jeffrey	2.01406	(1.49211, 2.60510) 1.11299	(1.46391, 2.92407) 1.46016
	Informative	2.13966	(1.61030, 2.71203) 1.10173	(1.36377, 2.81584) 1.45207
50	Jeffrey	2.15621	(1.75397, 2.59709) 0.84312	(1.65186, 2.76850) 1.11664
	Informative	2.00077	(1.64707, 2.39050) 0.74343	(1.55070, 2.51419) 0.96349

Table 4: Credible intervals for α, β and γ

Parameters	Prior	estimate	95% C.I (width)	99% C.I. (width)
α	Jeffrey	1.01693	(0.29426, 2.72271) 2.42845	(0.18097, 3.62492) 3.44395
	Informative	1.14379	(0.26616, 2.34805) 2.08189	(0.17551, 2.97161) 2.7961
β	Jeffrey	2.20413	(1.81032, 2.65222) 0.8419	(1.69353, 2.83398) 1.14045
	Informative	2.04210	(1.67144, 2.46173) 0.79029	(1.56256, 2.63407) 1.07151
γ	Jeffrey	5.12292	(3.53113, 6.96405) 3.43292	(3.12454, 7.53388) 4.40934
	Informative	4.83125	(3.36143, 6.57182) 3.21039	(2.95502, 7.17773) 4.22271

Real Life Example

The real-life data of percentage of GDP of different countries is taken from Dataset: Central Government Dept of 2009. The GIWD is used to fit this data set. To check the validity of the model, we compute of Kolmogorov-Smirnov test and p-value for this test is 0.1859, suggesting thereby the appropriateness of the GIWD. The credible intervals are computed based on 10,000 MCMC samples and first 500 values are discarded as burn-in. The trace plots are also plotted to determine whether the chain is exploring the parametric space well for all the parameters of three distributions in case of real-life example.

It is seen from the above tables, for all three parameters of GIWD the informative prior performs better as compared with non-informative prior (Jeffreys' prior).

5. Conclusion

The informative prior performs better as compared to non-informative prior and findings from the analysis of real life example are in accordance with those of simulation study in case of generalized inverse Weibull distribution. One can further infer that as the sample sizes increases, the width of the credible interval decreases for both 95% and 99% credible intervals in case of Generalized inverse Weibull distribution.

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