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Partially Accelerated Reliability Demonstration Tests For A Parallel System With Weibull Distributed Components Under Periodic Inspection

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Abstract

A Reliability Demonstration Test (RDT) demonstrates whether a product has met a certain reliability requirement with a specific confidence. This paper deals with construction of RDTs for a two-component parallel system subject to constant-stress partially accelerated life testing (CSPALT) using periodic mode of inspection and Weibull life distribution. The data from periodic inspection consist of number of failures of systems due to each component in each inspection period. In CSPALT the test specimens are allocated to two test chambers with test items running at normal operating condition in one and at accelerated condition in the other till the termination of the experiment. The optimal test plan consists in obtaining optimal number of allocations in each test chamber and optimal inspections points. RDTs based on optimal test plan are carried out for mean lives of components as well as the system. A numerical example is presented to illustrate the method developed.

Key words: Reliability demonstration tests; Partially accelerated life tests; Periodic inspection; Two-component parallel system; Weibull life distribution; D-optimality criterion.

1. Introduction

Accelerated life tests (ALTs) facilitate bringing about early failures in highly reliable items lasting for several years and hence obtaining reliability information about them in timely manner. This in turn helps the manufacturer to sustain in competitive market where technology is constantly changing with change in consumers' tastes. The book by Nelson (2009) gives a detailed account of Accelerated Tests. The data from a periodic inspection referred to as "grouped data" or "interval data" comprises number of failures in each inspection period. In contrast to continuous inspection wherein exact failure times of the test units are observed, periodic inspection requires less testing effort and is administratively convenient as compared to continuous inspection. In the literature the periodic inspection has been used by many authors, for example, Kulldorff (1961); Ehrenfeld (1962); Nelson (1977); Archer (1982); Flygare *et al.* (1985); Meeker (1986); Yum and Choi (1989); Seo and Yum (1991); Ahmad *et al.* (1994); Islam and Ahmad (1994); Ahmad and Islam (1996); Ahmad *et al.* (2006). A PALT is modelled using an acceleration factor (AF) and a life distribution,

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where AF is defined as the ratio of a reliability measure, say mean life, at use condition to that at accelerated condition. AF= k (say) means that the unit under consideration runs k times longer at normal operating condition than at accelerated condition. In CSPALT the test specimens are allocated to two test chambers with test items running at normal operating condition in one and at accelerated condition in the other till the termination of the experiment. ALTs have been studied extensively in the literature see for example, Srivastava (2017) and Chen *et al.* (2018).

The theory of testing statistical hypotheses provides the tools for reliability demonstration. If either the life distribution or its parameters are unknown, then the problem of reliability demonstration is that of obtaining suitable data and using them to test the null hypothesis that $R(t_0) \geq R_0$ against the alternative that $R(t_0) < R_0$, where t_0 is the specified time point and R_0 is desired reliability. We wish to test whether the reliability of the device at age t_0, $R(t_0)$ satisfies the requirement that $R(t_0) \geq R_0$. Nelson (1977) has provided the optimal demonstration tests with grouped inspection data from an exponential distribution and has also explained how to use the results for a Weibull distribution with known shape parameter. Optimal demonstration tests with grouped inspection data for logistic, log-logistic, normal/Gaussian, and log-normal distributions have been obtained Wei and Bau (1987).

The present paper deals with formulation of reliability demonstration tests for a two-component parallel system subject to CSPALT using periodic mode of inspection and Weibull life distribution. The Weibull life distribution incorporates various failure rates-increasing, decreasing and constant and is therefore of importance in industries manufacturing electronic and mechanical components. It adequately fits the life of several types of capacitors and resistors, such as electrolytic aluminium and tantalum capacitors and carbon film resistors (Yang, 2007; Shaw, 1987).

2. Notation

δ	Weibull shape parameter
μ_1	Weibull scale parameter for component 1
μ_2	Weibull scale parameter for component 2
λ_1	Exponential Scale parameter for component 1
λ_2	Exponential scale parameter for component 2
A	Acceleration Factor, $A > 1$
$R(t)$	Reliability function
n	Total number of two-component parallel systems
w_{1j}, w_{A1j}	The number of systems failing due to component 1 in $(t_{j-1}, t_j]$, $j = 1, 2, \ldots k+1$ in chamber 1 and chamber 2, respectively
w_{2j}, w_{A2j}	The number of systems failing due to component 2 in $(t_{j-1}, t_j]$, $j = 1, 2, \ldots k+1$ in chamber 1 and chamber 2, respectively
P_{1j}, P_{A1j}	The probability of failure of a system due to component 1 in $(t_{j-1}, t_j]$, $j = 1, 2, \ldots k+1$ in chamber 1 and chamber 2, respectively
P_{2j}, P_{A2j}	The probability of failure of a system due to component 2 in $(t_{j-1}, t_j]$, $j = 1, 2, \ldots k+1$ in chamber 1 and chamber 2, respectively
$N(0,1)$	Standard normal distribution

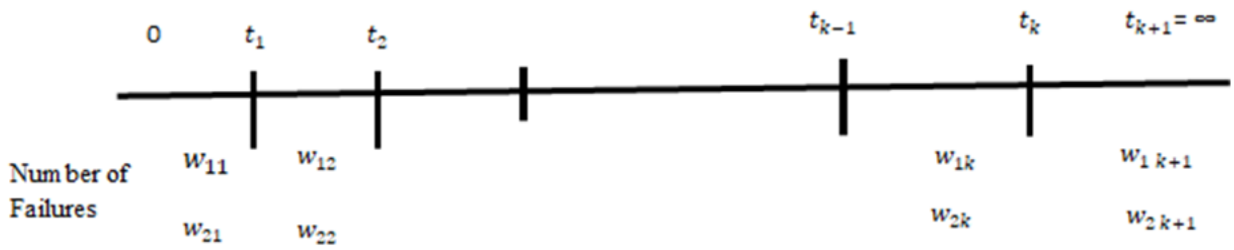

Figure 1: Structure of periodic inspection in chamber 1

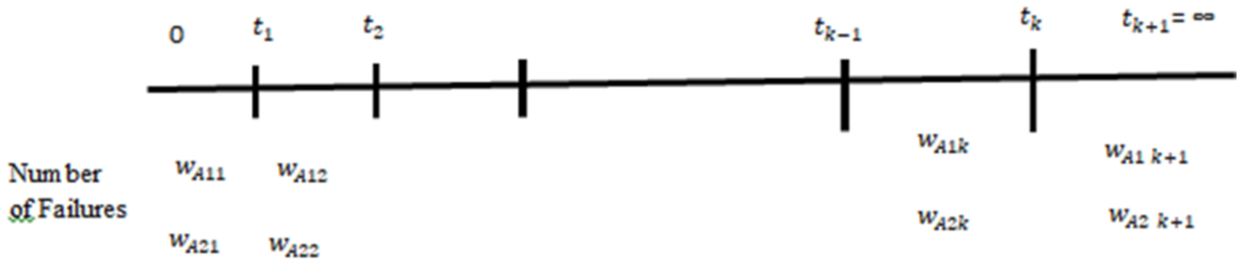

Figure 2: Structure of periodic inspection in Chamber 2

3. Formulation of likelihood function

n independent parallel systems each with two independent components are put to test under CSPALT. Out of n systems n_1 systems are put to test in test chamber 1 where they are run under normal operating condition, and n_2 systems are put to test in test chamber 2 where they are run at accelerated condition. The systems are examined for failures periodically at optimally spaced inspection $t_1,\ t_2,\ \ldots\ ,\ t_{k+1}$. Let, $t_0 = 0$ and $t_{k+1} = \infty$. Define ρ as the proportion of units that are allocated in chamber 1, and $1-\rho$ as the proportion of units that are allocated in chamber 2.

The structures of periodic inspection of systems at chamber 1 and chamber 2 are displayed in Figure 1 and Figure 2, respectively.

Assume that the lifetimes of test units are iid as Weibull with shape parameter δ known and scale parameter μ unknown. That is, the pdf, cdf, and reliability function of lifetime T at normal operating condition are:

$$f(t) = \mu\delta(\mu t)^{\delta-1}e^{-(\mu t)^{\delta}},\ t \geq 0\ , \tag{1}$$

$$F(t) = 1 - e^{-(\mu t)^{\delta}},\ t \geq 0\ , \tag{2}$$

and

$$\overline{F}(t) = e^{-(\mu t)^{\delta}},\ t \geq 0\ , \tag{3}$$

respectively, and the pdf, cdf, and reliability function of lifetime T at accelerated operating condition are given as

$$f(t) = A\mu\delta(\mu t)^{\delta-1}e^{-A(\mu t)^{\delta}},\ t \geq 0\ , \tag{4}$$

$$F(t) = 1 - e^{-A(\mu t)^{\delta}},\ t \geq 0\ , \tag{5}$$

and

$$\overline{F}(t) = e^{-A(\mu t)^{\delta}},\ t \geq 0\ , \tag{6}$$

Assuming transformation times $y_j = t_j^{\delta}$, $j = 1, \ldots, k+1$, are iid from an exponential distribution with failure rate $\lambda = \mu^{\delta}$.

So the Weibull distribution reduces to the exponential distribution with pdf, cdf, and reliability function under accelerated condition as:

$$f(y) = A\lambda e^{-A\lambda y},\ y > 0\ , \tag{7}$$

$$F(y) = 1 - e^{-A\lambda y},\ y > 0\ , \tag{8}$$

and

$$\overline{F}(y) = e^{-A\lambda y},\ y > 0\ , \tag{9}$$

respectively. At normal operating condition, $A = 1$ in the above equations. The probability of failure of a system due to component 1 under normal operating condition in $(y_{j-1}, y_j]\,, j = 1, 2, \ldots k+1$,

$$P_{1j} = \int_{y_{j-1}}^{y_j} F_2(y)\, f_1(y)\, dy\ ,$$

giving

$$P_{1j} = e^{-\lambda_1 y_{j-1}} - e^{-\lambda_1 y_j} + \frac{\lambda_1\left(e^{-(\lambda_1+\lambda_2)y_j} - e^{-(\lambda_1+\lambda_2)y_{j-1}}\right)}{\lambda_1+\lambda_2},\ for\ j =\ 1, 2, \ldots,\ k+1. \tag{10}$$

The probability of failure of a system due to component 2 under normal operating condition in $(y_{j-1}, y_j]\,, j = 1, 2, \ldots k+1$,

$$P_{2j} = \int_{y_{j-1}}^{y_j} F_1(y)\, f_2(y)\, dy\ ,$$

giving

$$P_{2j} = e^{-\lambda_2 y_{j-1}} - e^{-\lambda_2 y_j} + \frac{\lambda_2\left(e^{-(\lambda_1+\lambda_2)y_j} - e^{-(\lambda_1+\lambda_2)y_{j-1}}\right)}{\lambda_1+\lambda_2},\ for\ j =\ 1, 2, \ldots,\ k+1. \tag{11}$$

The probability of failure of a system due to component 1 under accelerated condition in $(y_{j-1}, y_j]\,, j = 1, 2, \ldots k+1$,

$$P_{A1j} = \int_{y_{j-1}}^{y_j} F_2(y)\, f_1(y)\, dy\ ,$$

giving

$$P_{A1j} = e^{-A\lambda_1 y_{j-1}} - e^{-A\lambda_1 y_j} + \frac{\lambda_1\left(e^{-A(\lambda_1+\lambda_2)y_j} - e^{-A(\lambda_1+\lambda_2)y_{j-1}}\right)}{\lambda_1+\lambda_2}, for\ \ j =\ 1, 2, \ldots,\ k+1. \tag{12}$$

The probability of failure of a system due to component 2 under accelerated condition in $(y_{j-1}, y_j]\,, j = 1, 2, \ldots k+1$,

$$P_{A2j} = \int_{y_{j-1}}^{y_j} F_1(y)\, f_2(y)\, dy\ ,$$

giving

$$P_{A2j} = e^{-A\lambda_2 y_{j-1}} - e^{-A\lambda_2 y_j} + \frac{\lambda_2 \left(e^{-A(\lambda_1+\lambda_2)y_j} - e^{-A(\lambda_1+\lambda_2)y_{j-1}} \right)}{\lambda_1 + \lambda_2}, \; for\; j = 1, 2, \ldots, k+1. \quad (13)$$

At stress level, the grouped data $w_{ij}, j = 1, 2, \ldots, k+1$ are multinomially distributed with parameters n_i and $P_{ij}, j = 1, 2, \ldots, k+1$. The likelihood function of parallel system for independent components is then given by

$$L(\lambda_1,\ \lambda_2,\ A) = L_1 L_2, \quad (14)$$

where L_1 is the likelihood corresponding to systems' failures in chamber 1 (normal operating condition) is:

$$\mathrm{L}_1 = n_1! \left[\prod_{j=1}^{k+1} (w_{1j} + w_{2j})! \right]^{-1} \left[\prod_{j=1}^{k+1} P_{1j}^{w_{1j}} P_{2j}^{w_{2j}} \right],$$

L_2 is the likelihood systems' failures in chamber 2 (accelerated operating condition) is:

$$L_2 = n_2! \left[\prod_{j=1}^{k+1} (\ w_{A1j} + w_{A2j})! \right]^{-1} \left[\prod_{j=1}^{k+1} P_{A1j}^{w_{A1j}} P_{A2j}^{w_{A2j}} \right],$$

$$L = L_1 L_2\ ,$$

From properties of Multinomial Distribution,

$w_{1j} + w_{2j} = n_{1j}$, and, $w_{A1j} + w_{A2j} = n_{2j}$,

$\sum_{j=1}^{k+1} (P_{1j} + (P_{2j}) = 1$, and, $\sum_{j=1}^{k+1} (P_{A1j} + P_{A2j}) = 1$,

Thus, the log-likelihood function is a function of unknown parameters $\lambda_1,\ \lambda_2,\ and\ A$ given as:

$$lnL(\lambda_1,\ \lambda_2,\ A) = lnL_1 + lnL_2.$$

$$lnL(\lambda_1,\ \lambda_2,\ A)$$

$$= \ln \left\{ \left[n_1! \left(\prod_{j=1}^{k+1} (w_{1j} + w_{2j})! \right)^{-1} \left(\prod_{j=1}^{k+1} P_{1j}^{w_{1j}} P_{2j}^{w_{2j}} \right) \right] \cdot \left[n_2! \left(\prod_{j=1}^{k+1} (\ w_{A1j} + w_{A2j})! \right)^{-1} \left(\prod_{j=1}^{k+1} P_{A1j}^{w_{A1j}} P_{A2j}^{w_{A2j}} \right) \right] \right\}$$

$$= ln\,(n_1!)\ + ln\,(n_2!)\ - \sum_{j=1}^{k+1} \ln(w_{1j} + w_{2j})! - \sum_{j=1}^{k+1} \ln\,(w_{A1j} + w_{A2j})\ !$$

$$+ \sum_{j=1}^{k+1} w_{1j} \ln(P_{1j}) + \sum_{j=1}^{k+1} w_{2j} \ln\,(P_{2j})\ + \sum_{j=1}^{k+1} w_{A1j} \ln(P_{A1j}) + \sum_{j=1}^{k+1} w_{A2j} \ln(P_{A2j})$$

$$= \sum_{j=1}^{k+1} w_{1j} \ln(P_{1j}) + \sum_{j=1}^{k+1} w_{2j} \ln(P_{2j}) + \sum_{j=1}^{k+1} w_{A1j} \ln(P_{A1j}) + \sum_{j=1}^{k+1} w_{A2j} \ln(P_{A2j}) + C, \tag{15}$$

where C is a constant independent of parameters. Maximum Likelihood (ML Estimates of $\lambda_1,\ \lambda_2,\ and\ A$ are obtained by maximizing $lnL\,(\lambda_1,\ \lambda_2, A)$ using NMaximize option of Mathematica 10 software package.

If $\widehat{\lambda}$ is the ML estimate for $\lambda = \mu^\delta$ in the transformed problem, then $\widehat{\mu} = \widehat{\lambda}^{1/\delta}$ is the ML estimate for μ.

4. Fisher information matrix

The Fisher Information Matrix (Nelson, 1977) is given by,

$$F = n\rho F_1 + n\,(1-\rho)\,F_2,$$

where,

$$F_i = \begin{bmatrix} E\left[-\frac{\partial^2 lnL_i}{\partial {\lambda_1}^2}\right] & E\left[-\frac{\partial^2 lnL_i}{\partial\lambda_1\partial\lambda_2}\right] & E\left[-\frac{\partial^2 lnL_i}{\partial\lambda_1\partial A}\right] \\ E\left[-\frac{\partial^2 lnL_i}{\partial\lambda_1\partial\lambda_2}\right] & E\left[-\frac{\partial^2 lnL_i}{\partial {\lambda_2}^2}\right] & E\left[-\frac{\partial^2 lnL_i}{\partial\lambda_2\partial A}\right] \\ E\left[-\frac{\partial^2 lnL_i}{\partial\lambda_1\partial A}\right] & E\left[-\frac{\partial^2 lnL_i}{\partial\lambda_2\partial A}\right] & E\left[-\frac{\partial^2 lnL_i}{\partial A^2}\right] \end{bmatrix},\ i = 1, 2. \tag{16}$$

5. Optimization problem

The optimum plan consists in determining optimum allocation ρ and optimal inspection times using D-optimality which consists in maximizing the determinant of Fisher information matrix which is the same as the reciprocal of the asymptotic variance-covariance matrix. The volume of the asymptotic joint confidence region of parameters, say, (μ, δ) is proportional to the square root of the determinant of the inverse of the Fisher information matrix, $|F^{-1}|^{1/2}$, at a fixed confidence level. In other words, it is inversely proportional to $|F|^{1/2}$. Consequently, a smaller value of the determinant would correspond to a higher (joint) precision of the estimators of μ, δ. The D-optimality criterion is therefore preferred to other optimality criteria existing in the literature such as A-optimality criterion, C-optimality or variance-optimality criterion. Thus, the optimization problem for determining optimal allocation and two inspection points $y1$ and $y2$ with $y3$ specified is:

Maximize $|F|$

$$s.t. 0 < \rho < 1, 0 < y_1 < y_2 < y_3. \tag{17}$$

Using transformed problem $t_j = {y_j}^\delta$, j =1, 2, 3, we get inspection points t_1 and t_2 with t_3 specified.

6. Reliability demonstration testing

In the present section, reliability demonstration testing for the mean life of the components and the system comprising these components has been presented. The acceptance of the null hypothesis in Section 6.1 and Section 6.2 corresponds to a demonstration of mean life of at least the specified value with confidence $100((1-\alpha_1)\%)$, where α_1 is the probability of committing Type-I error.

6.1. Reliability demonstration for components

For component $i, i = 1, 2$, the objective is to test:

$$H_{0i} : \frac{1}{\mu_i} \geq \frac{1}{\mu_{i0}}, versus H_{1i} : \frac{1}{\mu_i} < \frac{1}{\mu_{i0}}, \tag{18}$$

where $\mu_i = {\lambda_i}^{1/\delta}$ and $\mu_{i0} = {\lambda_{i0}}^{1/\delta}$ (18) is equivalent to testing

$H_{0i} : \frac{1}{\mu_i^\delta} \geq \frac{1}{\mu_{i0}^\delta}$, versus $H_{1i} : \frac{1}{\mu_i^\delta} < \frac{1}{\mu_{i0}^\delta}$,

or,

$H_{0i} : \frac{1}{\lambda_i} \geq \frac{1}{\lambda_{i0}}$, versus $H_{1i} : \frac{1}{\lambda_i} < \frac{1}{\lambda_{i0}}$,

where $\frac{1}{\lambda_{i0}}$ is a specified mean life of component i, and $\frac{1}{\widehat{\lambda}_i}$ is estimated value of $\frac{1}{\lambda_i}$. Under H_{0i}, the test statistic ((Nelson, 1977))

$$T_i = \frac{\frac{1}{\widehat{\lambda}_i} - \frac{1}{\lambda_{i0}}}{\sqrt{est.var\left(1/\ \widehat{\lambda}_i\right)}} \sim N(0,1) \ as\ n \ \to \infty\ . \tag{19}$$

6.2. Reliability demonstration test for system

The time to failure of a two-component parallel structure is not Weibull distributed, even if both components have Weibull distributed times to failure.

The MTTF (mean time to failure) of the 2-component parallel system is,

$$MTTF = \int_0^\infty R(t)\, dt = \frac{\Gamma\left(\frac{1}{\delta}\right)}{\delta}\left(\frac{1}{{\lambda_1}^{1/\delta}} + \frac{1}{{\lambda_2}^{1/\delta}} - \frac{1}{(\lambda_1+\lambda_2)^{1/\delta}}\right).$$

Under H_{03}: MTTF$\geq MTTF_0$ versus H_{13} : MTTF $< MTTF_0$ the test statistic (Nelson, 1977),

$$T_s = \frac{(Est.MTTF - MTTF_0)}{\sqrt{Est.variance\ of\ Est.MTTF}} \sim N(0,1)\ as\ n \to \infty, \tag{20}$$

where

Est.$MTTF = \frac{\Gamma\left(\frac{1}{\delta}\right)}{\delta}\left(\frac{1}{\widehat{\lambda}_1^{1/\delta}} + \frac{1}{\widehat{\lambda}_2^{1/\delta}} - \frac{1}{\left(\widehat{\lambda}_1+\widehat{\lambda}_2\right)^{1/\delta}}\right) = h(say).$

$$h_1 = \frac{dh}{d\lambda_1},\ h_2 = \frac{dh}{d\lambda_2},\ h_3 = \frac{dh}{dA}\ ,$$

$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}, h^T = [h_1\ h_2\ h_3]$

Est.$variance\ of\ Est.MTTF = h^T F^{-1} h.$

7. Numerical example

Let the total number of inspection k = 5, $\lambda_1 = 0.1$ and $\lambda_2 = 0.5$, inspection times $y_0 = 0,\ y_5 = 30,\ y_6 = \infty$, acceleration factor $A = 1.7$, for $n = 35$ items put to test. The optimal allocation $\rho = 0.521892$, and times inspection times $y_1 = 2, y_2 = 4, y_3 = 8,$ and $y_4 = 16$. The simulated data is depicted in Table 1.1.

Table 1: Simulated data

Intervals	Chamber 1		Chamber 2	
	Component 1	Component 2	Component 1	Component 2
(0, 2]	3	1	0	3
(2, 4]	3	0	2	5
(4, 8]	2	3	0	2
(8,16]	3	0	0	4
(16,30]	2	0	0	1
(30, ∞)	1	0	0	0

7.1. Hypothesis testing problem for component 1

$H_{01} : \frac{1}{\lambda_1} \geq 10$ versus $H_{11} : \frac{1}{\lambda_1} < 10$

Under H_{01}, the test statistic:

$$T_1 = -0.786602$$

Thus accept H_{01} at 5% level of significance.

7.2. Hypothesis testing Problem for component 2

$H_{02} : \frac{1}{\lambda_2} \geq 2$ versus $H_{12} : \frac{1}{\lambda_2} < 2$

Under H_{02}, the test statistic:

$$T_2 = 2.71621$$

Thus, accept H_{02} at 5% level of significance.

7.3. Hypothesis testing problem for the 2-component parallel system

Under H_{03}: MTTF$\geq$ 5 versus H_{13} : MTTF $<$ 5 the test statistic,

$$T_s = 1.04019$$

Thus, accept H_{03} at 5% level of significance. Thus, components as well as the system meet the specified reliability requirements.

8. Conclusion

The paper deals with reliability demonstration tests for a two-component parallel system with subject to CSPALT under periodic inspection using Weibull life distribution. The optimal plan consists in determining optimal allocation and optimal inspection times using D-optimality criterion. The method proposed is illustrated using a numerical example.

The future scope of RDTs under normal operating or accelerated conditions is vast and still unexploited.

These tests can be also constructed for two-component parallel systems with dependent components. RDTs can also be formulated for other reliability systems such as series-parallel, parallel-series, and k-out-of-n system, etc. Conducting RDTs for small sample size for various reliability systems is still an open problem. Parametric approach has been used in the present paper. The tests can also be formulated using Non-parametric and Bayesian approaches.

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