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Reliability Analysis of a Phased Mission System under Degradation using Wiener Process and Copulas

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Abstract

In this paper, reliability of a Phased Mission System (PMS) under internal and external degradation has been analysed. The degradation path of a component in a phase is taken as a linear combination of the internal degradation and a proportion of common external degradation that influences each component and modelled using Wiener process. The PMS is modelled using cumulative exposure model with components' dependency and that of phases modelled using different copulas: Gumbel-Hougaard, Clayton, and Frank. A numerical illustration is presented using an aircraft flight PMS with comparative study amongst different copulas carried out. Sensitivity analyses are also undertaken to determine which parameters are sensitive to small deviations from their respective true value so that extra care may be taken by an engineer while setting the true value of each of such parameters.

Key words: Phased mission systems; Copulas; Degradation; Wiener Process; Sensitivity analyses.

AMS Subject Classifications: 62K05, 05B05

1. Introduction

Various environmental and operational conditions act on real-world systems leading to their degradation, and usually culminate into the system's failure that may be costly and sometimes catastrophic. Such systems are often required to complete their tasks in several non-overlapping phases or stages in which the structure of systems change phase-wise and are addressed in the literature as phased mission systems (PMSs).

Some examples of PMSs are:

- Aircraft flight PMS (Somani *et al.* (1992) and Xing (2007)), see also Srivastava and Rani (2024),
- a satellite launching PMS, (Huang *et al.* (2019)),

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- a boiling water reactor PMS, (Alam *et al.* (2006)),
- Mars orbiter mission system, (Xing (2007)).
- Spaceflight tracking, telemetry and command (TT&C) system, (Yang and Wu (2014)).
- Marine transportation mission of a ship, (Eruguz *et al.* (2017)).
- A distributed computer system, (Shrestha *et al.* (2010), Wang and Trivedi (2007), Levitin *et al.* (2018)).

The changes in system configuration from phase to phase, change in failure criteria and stresses due to differing environmental conditions acting on the PMS, and phase dependencies render reliability analyses of PMSs very challenging.

Two classes of approaches to the reliability evaluation of PMSs existing in the literature are Analytical Methods and Simulation Methods.

Analytical Methods include State-Space Based Approach, Combinatorial, Modular, and Recursive Methods.

State-space models based on Markov Chains (Alam and Al-Saggaf (1986), Kim and Park (1994)), Semi-Markov process (Wu and Hillston (2015)), Bayes Network (Bing and Xiao-yue (2012),Li *et al.* (2020)), and Petri-nets (Mural *et al.* (1999), Bondavalli *et al.* (2004)) have been used to describe the system behavior of the PMS. See also Li *et al.* (2021). However, they cannot be used in large-scale PMS due to the exponential growth of the systems' state numbers.

Combinatorial methods are based on the assumption that all the components are independent, meaning no dependency exists within one phase. Four different combinatorial methods used in the literature are the Mini-component Technique, Boolean Algebraic – based binary decision diagrams (BDD), multi-valued decision diagrams (MDD), and universal generating function-based method (Xing and Amari (2008)).

However, the assumption of components' independence limits the models' applicability because dependence within and across the phases occurs in some PMSs.

Phase modular techniques, introduced by Meshkat *et al.* (2003) and Ou and Dugan (2004), combine BDD-based solutions for static modules with Markov chain-based solutions for dynamic modules. These methods are specifically designed for analyzing non-repairable, binary-state PMSs.

While the phase modular approach offers greater modeling capability than purely combinatorial solutions and higher computational efficiency than state-space-based methods, it has notable limitations. It requires a costly path enumeration of the system BDD and the calculation of joint phase module probabilities for all system-level modules in the current path. The complicated state mapping between static and dynamic phase modules adds to the computational burden. As a result, the computational overhead of existing phase modular techniques remains significant in many practical applications See also Shrestha *et al.* (2010) and Shrestha *et al.* (2009).

A **Recursive method** based on conditional probability and the branch and bound has been proposed by Levitin *et al.* (2018) to evaluate the reliability of a PMS.

However, it may not be appropriate in large-scale PMS with complex dynamic behavior (Liu *et al.* (2020)).

Simulation methods are highly versatile and flexible for modeling a system and its components' behavior.

However, they only yield approximate reliability results, which may be inadequate or unsuitable for mission-critical systems See, for example, Yang and Wu (2014) and Hu *et al.* (2021).

More recently, Mura (2021) has presented a review of stochastic modeling approaches, emphasizing phase-dependent behavior and the quantitative analysis of PMSs' dependability attributes.

Copula-based approach for reliability analyses of PMSs wherein the dependency amongst components in a phase, and dependency across the phases are modelled using copulas have been studied by Srivastava *et al.* (2022),Srivastava and Rani (2024).

Dependency among degradation mechanisms or processes has received less attention in reliability modelling of a PMS, and assume Markovian state-space model wherein estimation of lifetime depends only upon the current state. In the literature, reliability of PMS has been estimated by Si *et al.* (2015) using condition-based monitoring and the degradation data. The numerical approach used by the authors is time consuming if the number of mission phases is substantially large.

The present paper deals with reliability analysis of PMS subject to internal and external degradation using Wiener degradation model with dependencies amongst components in a phase as well as across the phases modelled using copulas. Prior to the present work, Li *et al.* (2021) have used state –space model, *viz.*, semi-Markov model for PMSs under internal degradation and external shocks and, therefore, suffer from state-space explosion problem.

Section 2 presents the methodology for evaluating the reliability of a PMS under degradation, and in Section 3, the proposed method is explained using an aircraft flight PMS. The concluding remarks have been made in the last section.

2. Model formulation

2.1. Assumptions

- The components in a phase are dependent.
- The phases are dependent with structure of the PMS varying across the phases.
- Degradation of each component follows Wiener Process (WP) $\{W(t)\ ;\ t\ \geq\ 0\}$ with some positive drift ($\mu\ >\ 0$) and diffusion parameter $\sigma^2 > 0$.
- The degradation process is independent of the mission process.

A phased mission system (PMS) may consist of multiple degrading components.

Degradation of a component in a phase is internal as well as external; the former being unexplained and intrinsic to each component and the latter that is shared by all the components due to environmental factors such as temperature, humidity, or operational factors. Li *et al.* (2011) considered component degradation as a linear combination of internal and common external degradations for a single phase.

Suppose that the components of a phase of a PMS are subject to degradation due to different internal cause and common external cause. Let $\mathcal{Y}_1(t)$, $\mathcal{Y}_2(t)$ ... $\mathcal{Y}_m(t)$ be independent internal degradation paths and let degradation path due to external factor be denoted by $\mathcal{Z}(t)$. Also, suppose that the j^{th} component is exposed to the external degradation $\mathcal{Z}(t)$ with impact element α_j for $j = 1, 2, \ldots m$. The degradation path, $\mathcal{X}_j(t)$, of j^{th} component $(j = 1, \ldots, m)$ is taken as a linear combination of, $\mathcal{Y}_j(t)$, and, $\mathcal{Z}(t)$, which influences each component.

Wiener process and Gamma process are widely used degradation models in the literature. Wiener process (WP) is used for modelling degradation path in this paper.

It is assumed that internal degradation path as well as external degradation path of a component follows Wiener Process. Thus,

$$\mathcal{Y}_j(t) = \mu_j + \sigma_j W^1(t), \tag{1}$$

where $W^1(t)$ is a standard Wiener Process

$$\Longrightarrow \mathcal{Y}_j(t) \sim N\left(\mu_j, \sigma_j^2\right), \tag{2}$$

and

$$\alpha_j \mathcal{Z}(t) = \alpha_j \mu + \alpha_j \sigma W^1(t), \tag{3}$$

$$\Longrightarrow \alpha_j \mathcal{Z}(t) \sim N\left(\alpha_j \mu, \alpha_j^2 \sigma^2\right), \tag{4}$$

$\mathcal{Y}_j(t)$and $\mathcal{Z}(t)$ are both linear function of t.

Using (1) and (3)

$$\mathcal{X}_j(t) = \mathcal{Y}_j(t) + \alpha_j \mathcal{Z}(t); \qquad j = 1, 2, \ldots, m, \tag{5}$$

where

$$\mathcal{X}_j(t) \sim N\left(\mu_j + \alpha_j \mu, \sigma_j^2 + \alpha_j^2 \sigma^2\right). \tag{6}$$

Let

$$\eta_j = \mu_j + \alpha_j \mu, \text{and } \delta_j^2 = \sigma_j^2 + \alpha_j^2 \sigma^2, \tag{7}$$

then

$$\mathcal{X}_j(t) \sim N\left(\eta_j, \delta_j^2\right). \tag{8}$$

2.2. First passage time distribution of WP

Let $y(t)$ be performance degradation measure at time t defined as:

$$y\left(t\right) = \ \gamma t \ + \ \lambda W^1\left(t\right), \ \left(\gamma \ , \ \lambda\right) > \ 0. \tag{9}$$

Then, $y(t)$ is a WP with drift γ and diffusion constant λ^2.

When the degradation path follows WP, then the time when the degradation level first reaches a fixed failure critical level, w, has an inverse Gaussian (IG) distribution with reliability function (Chhikara and Folks (1989)):

$$R\left(t\right) = \phi\left[\frac{w - \gamma.t - w_0}{\lambda\sqrt{t}}\right] - exp\left(\frac{2.\gamma.\left(w - w_0\right)}{\lambda^2}\right)\phi\left[-\frac{w + \lambda.t - w_0}{\lambda\sqrt{t}}\right]; \left(\gamma \ , \lambda\right) \ > \ 0, \tag{10}$$

where w_0 is the initial degradation level at time zero.

2.3. Copulas

In this paper, copula-based approach is used to model dependency amongst the components in a phase, and also across the phases. The dependence structure relates the first passage time distribution of the Wiener degradation path depicted by the components within the phase to their multivariate distribution. The choice of copula determines the underlying dependence structure (Nelsen (2006)).

Some types of copula functions existing in the literature are **Clayton copula, Frank copula, Gaussian copula, Gumbel-Hougaard (GH) copula, and Student's t-copula**.

In this work ***Gumbel-Hougaard copula, Clayton copula, and Frank copula*** have been used.

Let $X_1, \ X_2, \ldots, \ X_n$ be random variables and let $\overline{\mathcal{G}}_1\left(x_1\right), \ \overline{\mathcal{G}}_2\left(x_2\right), \ldots, \ \overline{\mathcal{G}}_n\left(x_n\right)$ be their respective marginal reliability functions. Further, let $\overline{\mathcal{G}}\left(x_1, x_2, \ldots, x_n\right)$ denote their corresponding joint reliability function. Then, Sklar's Theorem (Nelsen (2006)) states that, $\exists$ a copula reliability function $C(\cdot, \cdot, \cdot)$ s.t $\forall$ $(X_1, \ X_2, \ldots, \ X_n)$ in the defined range:

$$\overline{\mathcal{G}}\left(x_1, x_2, \ldots, x_n\right) = C\left(\overline{\mathcal{G}}_1\left(x_1\right), \overline{\mathcal{G}}_2\left(x_2\right), \ldots, \overline{\mathcal{G}}_n\left(x_n\right)\right), \tag{11}$$

The copulas used in this work are:

(i) Gumbel-Hougaard (GH) copula

$$C\left(\overline{\mathcal{G}}_1\left(x_1\right), \overline{\mathcal{G}}_2\left(x_2\right), \ldots, \overline{\mathcal{G}}_n\left(x_n\right)\right) =$$
$$exp\left[-\left(\left(-log\left(\overline{\mathcal{G}}_1\left(x_1\right)\right)\right)^\theta + \left(-log\left(\overline{\mathcal{G}}_2\left(x_2\right)\right)\right)^\theta + \cdots + \left(-log\left(\overline{\mathcal{G}}_n\left(x_n\right)\right)\right)^\theta\right)^{1/\theta}\right], \tag{12}$$

where $\theta \epsilon \left[1, \infty\right)$

(ii) Clayton copula,

$$C\left(\overline{\mathcal{G}}_1(x_1), \overline{\mathcal{G}}_2(x_2), \ldots, \overline{\mathcal{G}}_n(x_n)\right) = \left(\left(\overline{\mathcal{G}}_1(x_1)\right)^{-\theta} + \left(\overline{\mathcal{G}}_2(x_2)\right)^{-\theta} + \cdots + \left(\overline{\mathcal{G}}_n(x_n)\right)^{-\theta} - n + 1\right)^{-\frac{1}{\theta}}, \quad (13)$$

where $\theta \epsilon (0, \infty)$

(iii) Frank copula

$$C\left(\overline{\mathcal{G}}_1(x_1), \overline{\mathcal{G}}_2(x_2), \ldots, \overline{\mathcal{G}}_n(x_n)\right) = -\frac{1}{\theta} ln\left[1 + \frac{\left(e^{-\theta.\overline{\mathcal{G}}_1(x_1)} - 1\right)\left(e^{-\theta.\overline{\mathcal{G}}_2(x_2)} - 1\right)\ldots\left(e^{-\theta.\overline{\mathcal{G}}_n(x_n)} - 1\right)}{\left(e^{-\theta} - 1\right)^{n-1}}\right], \quad (14)$$

where $\theta \epsilon \mathrm{R}\backslash\{0\}$.

(12)-(14) belong to the Archimedean class of copulas. The Archimedean copula class is easy to work with from a mathematical point of view (Wilson and Y. (2005)), Chapter 8, p.- 114). These copulas find a wide range of applications for several reasons, such as the ease with which they can be constructed and the variety of families of copulas which belong to the class (see Nelsen (2006), Chapter 4). n-dimensional GH copula with each marginal as Weibull is n-variate Weibull distribution (see Lee and Wen (2006)) which is widely used in manufacturing industries.

2.4. Computation of reliability of a phased-mission system

Let T_j be the lifetime of component j, $j = 1, 2, \ldots, m$ of phase i, $i = 1, 2, \ldots, n$ following inverse Gaussian (IG) distribution with parameters γ (> 0) and λ (> 0), and

$$\overline{\mathcal{G}}_{ji}(t) = R_{ji}(t), \quad (15)$$

be its reliability function, where $R_{ji}(t)$ can be obtain from (10) on replacing the failure threshold w by w_j, γ by η_j, and λ by δ_j^2.

Let $\overline{F}_{p1}(t), \overline{F}_{p2}(t), \ldots$ and $\overline{F}_{pn}(t)$ be the respective reliability of phases 1, 2, …, n, respectively. Then, reliability of PMS is:

$$\overline{F}_{PMS}(t) = \begin{cases} \overline{F}_{p1}(t), & 0 \le t \le t_1 \\ \overline{F}_{p2}(t), & t_1 \le t \le t_2 \\ \vdots & \\ \overline{F}_{pn}(t), & t_{n-1} \le t \le t_n \end{cases}, \quad (16)$$

where $[t_{i-1}, t_i]$ represents time-duration of functioning of phase i of the phased mission system $i = 1, 2, 3, 4, \ldots, n, \quad t_0 = 0$.

Let i^{th} phase be composed of m dependent components and reliability of i^{th} phase be denoted by $\overline{F}_{pi}(t)$ then dependency is modelled using Gumbel-Haugaard, Clayton, and Frank copula giving:

$$\overline{F}_{pi}(t) = C\left(\overline{\mathcal{G}}_{1i}(t), \overline{\mathcal{G}}_{2i}(t), \ldots, \overline{\mathcal{G}}_{mi}(t)\right), \tag{17}$$

and, reliability of PMS:

$$\overline{F}_{PMS}(t) = C\left(\overline{F}_{p1}(t_1), \overline{F}_{p2}(t_2), \overline{F}_{p3}(t_3), \ldots, \overline{F}_{pn}(t_n)\right). \tag{18}$$

The cumulative exposure model Nelson (2009) is used in equation (17), to obtain the reliability of i^{th} phase at t_i. Thus,

$$\overline{F}_{pi}(t_i) = C\left(\overline{\mathcal{G}}_{1i}(t_i - t_{i-1} + l_{1i}), \overline{\mathcal{G}}_{2i}(t_i - t_{i-1} + l_{2i}), \ldots, \overline{\mathcal{G}}_{mi}(t_i - t_{i-1} + l_{mi})\right). \tag{19}$$

where for the component j in the phase i of the PMS, $i = 1, .., n,$ & $j = 1, .., m,$ l_{ji} is determined in such a way that (Srivastava *et al.* (2022))

$\overline{\mathcal{G}}_{ji}(l_{ji}) = \overline{\mathcal{G}}_{ji-1}(t_{i-1} - t_{i-2} + l_{ji-1})$, and $l_{j1-1} = 0$.

3. Numerical illustration

An Aircraft Flight PMS has been used to explain the model proposed.

Consider an aircraft flight from departure A to destination B and vice versa. The four-phase aircraft flight PMS are depicted in Figure 1 (see also (Srivastava and Rani (2024)).

The taxiing phase takes around 10-20 minutes, depending on airport traffic and distance from the gate to the runaway. The take-off rolls and initial climb to cruising altitude can take about 10-20 minutes. The cruising phase, where the plane is at its highest altitude and flying in a straight line, might last for about 2 hours and 20 minutes. The conditions during landing would be similar to those at the arrival airport B. The decent and landing process can take around 20-30 minutes.

Thus, the following data set has been used:

The duration of

- Taxiing and Take-Off phases has been taken to be 20 minutes,
- cruising phase is 140 minutes, and
- The landing phase is 20 minutes.

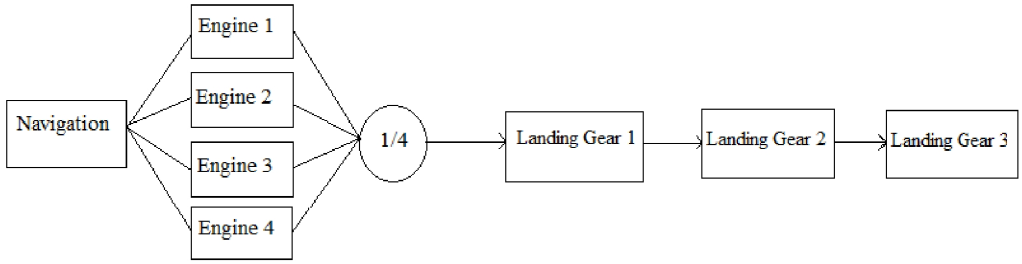

Figure 1(a): *Taxiing Phase*

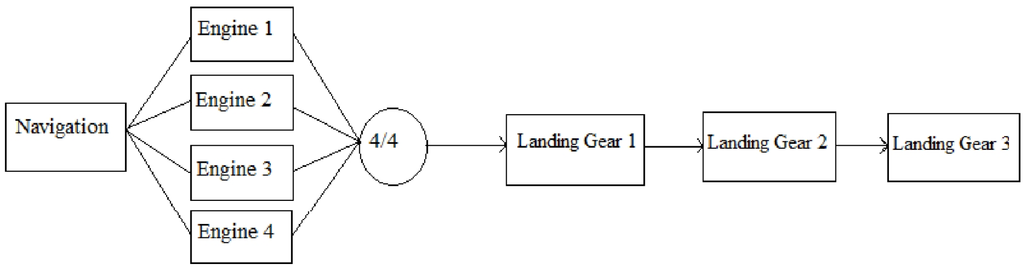

Figure 1(b): *Take-Off Phase*

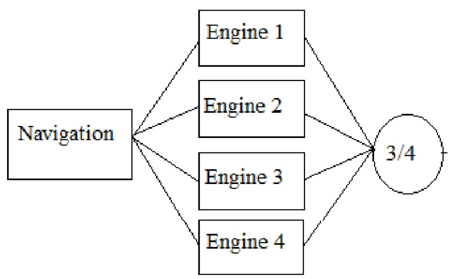

Figure 1(c): *Cruising Phase*

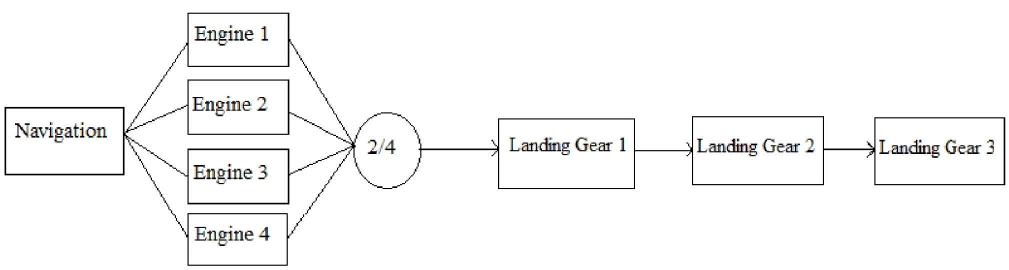

Figure 1(d): *Landing Phase*

Figure 1: 1(a)-(d) Reliability block diagrams for the four- phase aircraft flight

The hypothetical values of the parameters for the eight components, namely, Navigation, Engine 1, 2, 3, and 4; landing gear 1, 2, and 3 are taken as:

$$\mu_1 = 0.6, \quad \mu_2 = \mu_3 = \quad \mu_4 = \quad \mu_5 = 0.5, \; \mu_6 = \mu_7 = \mu_8 = 0.7, \; \mu = 1.4,$$

$\alpha_1 = 0.4, \; \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5 \;, \; \alpha_6 = \alpha_7 = \alpha_8 = 0.7,$ and

$\sigma_1 = 1.4, \quad \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 1.3 \;, \quad \sigma_6 = \sigma_7 = \sigma_8 = 1.2, \quad \sigma = 1.1$, respectively in (7).

Degradation levels for the three components: Navigation, Engine and landing gear have been taken as

$w_1 = 19,\ w_2 = 18,\ \text{and } w_3 = 17,\ \ w_0 = 1$, copula parameter $\theta = 2.17$.

Table 1 gives reliability of Aircraft Flight PMS for Gumble-Hougaard, Clayton, and Frank copulas. Figure 2 obtained using Table 1 depicts reliability of Aircraft Flight PMS for these three copulas. It is quite evident from the Table and its corresponding Figure that although there is no copula that performs the best in all the phases, the reliability of the PMS is highest for Gumble-Hougaard copula for the hypothetical data set used.

Table 1: Reliability of aircraft flight PMS using Gumbel-Hougaard, Clayton, and Frank copulas

Copula	Phase 1	Phase 2	Phase 3	Phase 4	PMS
Gumble-Hougaard	1	1	0.9997517	0.9859937	0.9859927
Clayton	1	1	0.999883	0.9765497	0.9765496
Frank	1	1	0.9998831	0.9764228	0.9763125

3.1. Sensitivity analysis

Sensitivity analysis is carried out on reliability of PMS using percentage deviation (PD) of the reliability of PMS given as:

$$\left(\frac{|R - R^*|}{R}\right) \times 100,$$

where R is Reliability of PMS obtained with the given design parameters, and R^* is the value of reliability obtained using mis-specified value. $|.|$ denote the absolute value.

Sensitivity analysis is carried out on reliability of PMS with

$(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%,\ \pm 30\%,\ \pm 40\%,\ \pm 50\%)$ deviations in:

(i) copula parameter θ keeping other parameters fixed,

(ii) degradation level w_1, w_2, and w_3 of Nevigation, Engines and Landing gears, respectively, with other parameters fixed,

(iii) Parameters, $\mu,\ \sigma$, associated with external degradation,

(iv) impact element $\alpha_j,\ j = 1, 2, \ldots, 8$ of external degradation,

using Gumbel-Hougaard, Clayton, and Frank copulas.

For Gumble-Hougaard copula, Table 2 and Figure 3 show the results for (i), Table 3 and Figure 4 show the results for (ii), Table 4 and Figure 5 show the results for (iii), Table 5 and Figure 6 show the results for (iv). For Clayton copula, Table 6 and Figure 7 show the results for (i), Table 7 and Figure 8 show the results for (ii), Table 8 and Figure 9 show the results for (iii), Table 9 and Figure 10 show the results for (iv). For Frank copula, Table 10

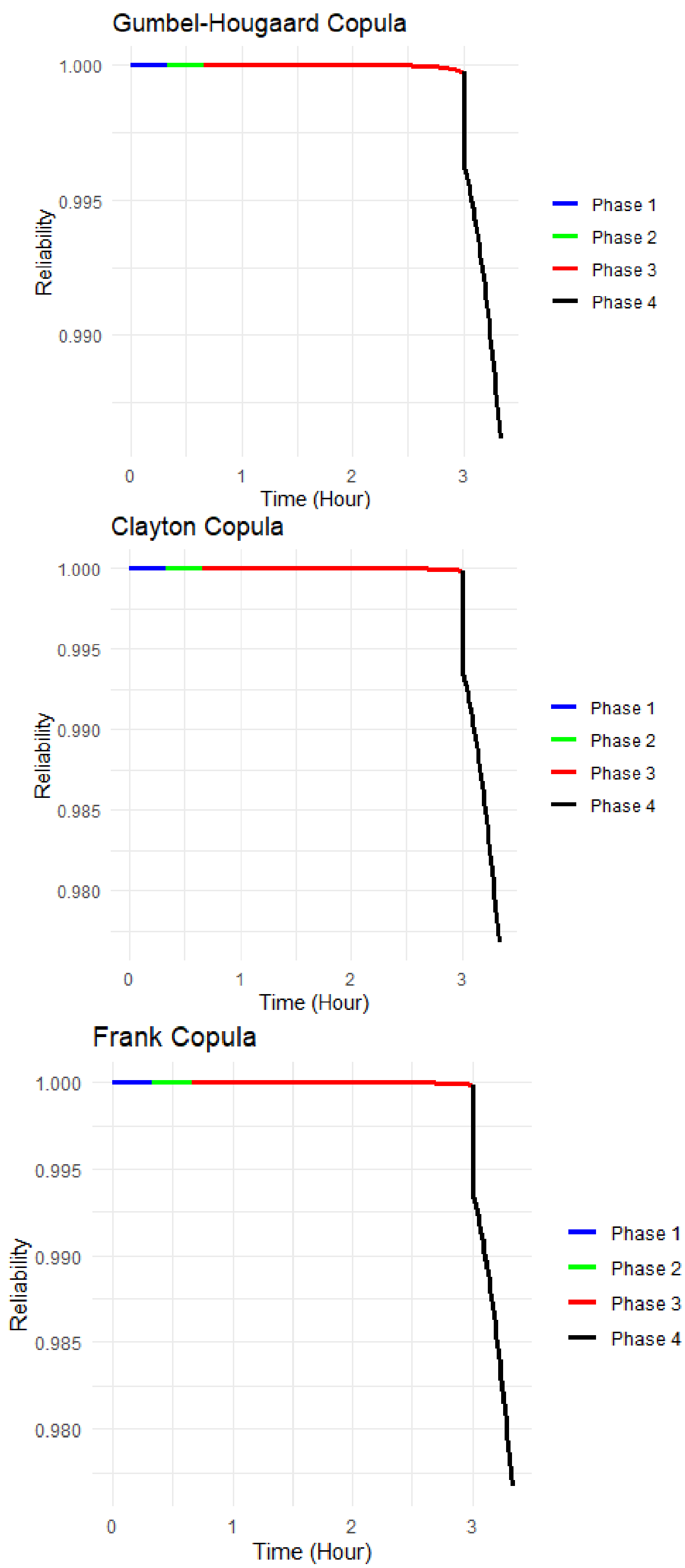

Figure 2: Reliability of aircraft flight PMS using Gumbel-Hougaard, Clayton, and Frank copulas

and Figure 11 show the results for (i), Table 11 and Figure 12 show the results for (ii), Table 12 and Figure 13 show the results for (iii), Table 13 and Figure 14 show the results for (iv).

Table 2: Sensitivity analysis on reliability of aircraft flight PMS w.r.t ($\pm 1\%$, $\pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%$, $\pm 30\%$, $\pm 40\%$, $\pm 50\%$) **deviations in** $\theta = 2.17$ **using Gumbel-Hougaard copula** $w_1 = 19, w_2 = 18, w_3 = 17, \mu = 1.4, \sigma = 1.1, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Initial values of copula parameter θ =2.17	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9859927	0.9860537	0.006179741
-1%	0.9859927	0.9859302	0.006336702
+ 5 %	0.9859927	0.986283	0.02943944
-5 %	0.9859927	0.9856636	0.03337802
+10 %	0.9859927	0.9865409	0.05559442
- 10 %	0.9859927	0.9852874	0.07153325
+15 %	0.9859927	0.9867715	0.07898237
-15%	0.9859927	0.9848533	0.1155581
+20%	0.9859927	0.9869789	0.1000179
-20%	0.9859927	0.9843471	0.166901
+30%	0.9859927	0.9873368	0.1363148
-30%	0.9859927	0.9830335	0.3001304
+40%	0.9859927	0.9876346	0.1665184
-40%	0.9859927	0.9810778	0.4984735
+50%	0.9859927	0.9878862	0.1920378
-50%	0.9859927	0.9778959	0.8211879

For the Gumbel-Hougaard copula, Tables 2, 3, 4, and 5 indicate that the reliability function is not sensitive to small deviations from the true parameter values.

For the Clayton copula, Tables 6, 7, 8, and 9 indicate that the reliability function is not sensitive to small deviations from the true parameter values.

For Frank copula Tables 10, 11, 12, and 13 indicate that the reliability function is not sensitive to small deviations from the true parameter values.

Further, it can be observed from Table 2 and Figure 3, Table 6 and Figure 7, and Table 10 and Figure 11 that for the Gumbel-Hougaard, Clayton, and Frank copulas, respectively, as the value of copula parameter, θ, increases the reliability of aircraft flight PMS increase, and decreasing the value of parameter, θ, results in decrease in the reliability of aircraft flight PMS, when other parameters are held constant.

Table 3 and Figure 4, Table 7 and Figure 8, and Table 11 and Figure 12, corresponding to the Gumbel-Hougaard, Clayton, and Frank copulas, respectively, show that an increase in the degradation level of components, w_i, results in an increase in the reliability of the aircraft flight PMS. Conversely, a decrease in, w_i, leads to a decrease in reliability.

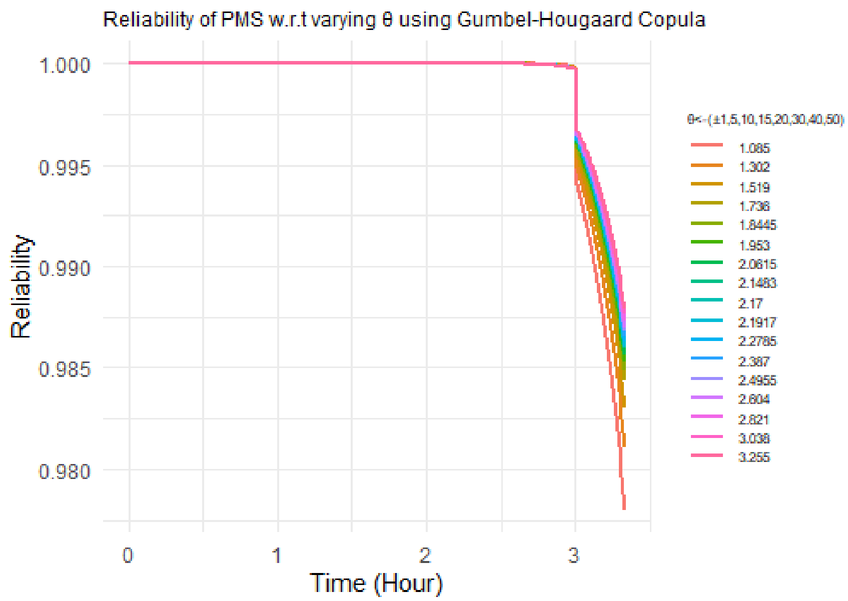

Figure 3: Reliability of aircraft flight PMS w.r.t $(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ deviations in $\theta = 2.17$ using Gumbel-Hougaard copula

Table 3: Sensitivity analysis on reliability of aircraft flight PMS w.r.t $(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$, deviations in $w_1 = 19,\ w_2 = 18,\ w_3 = 17$ using Gumbel-Hougaard copula $\theta = 2.17, \mu = 1.4, \sigma = 1.1, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Degradation level initial values $w_1 = 19, w_2 = 18, w_3 = 17$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9859927	0.9878902	0.1924424
-1%	0.9859927	0.9838344	0.218895
+ 5 %	0.9859927	0.9933866	0.7498937
-5 %	0.9859927	0.9719673	1.422466
+10 %	0.9859927	0.99705	1.121439
- 10 %	0.9859927	0.9470123	3.953421
+15 %	0.9859927	0.9987573	1.294592
- 15 %	0.9859927	0.9055028	8.163342
+20%	0.9859927	0.9995059	1.370514
-20%	0.9859927	0.8413032	14.6745
+30%	0.9859927	0.9999345	1.413982
-30%	0.9859927	0.4639339	52.94753
+40%	0.9859927	0.9999932	1.419932
-40%	0.9859927	0.1378241	86.02179
+50%	0.9859927	0.9999994	1.42057
-50%	0.9859927	0.02954872	97.00315

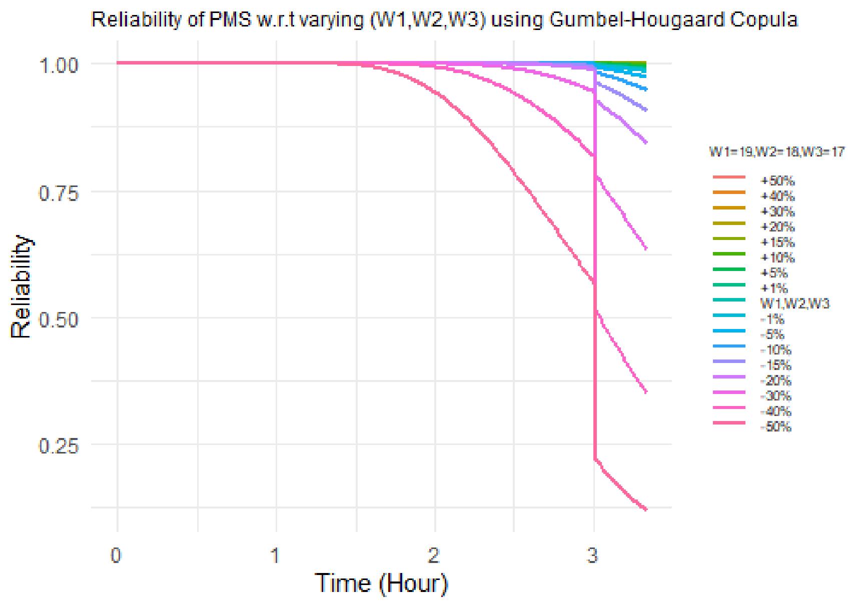

Figure 4: Reliability of aircraft flight PMS w.r.t $(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ **deviations in** $w_1 = 19,\ w_2 = 18,\ w_3 = 17$ **using Gumbel-Hougaard copula**

Table 4: Sensitivity analysis on reliability of aircraft flight PMS w.r.t $(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ **deviations in external degradation parameters** $\mu = 1.4,\ \sigma = 1.1$ **using Gumbel-Hougaard copula** $\theta = 2.17, w_1 = 19, w_2 = 18, w_3 = 17, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Initial values of external degradation parameters $\mu = 1.4,\ \sigma = 1.1$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9859927	0.9849241	0.1083774
-1%	0.9859927	0.9869985	0.1020064
+ 5 %	0.9859927	0.9799717	0.6106535
-5 %	0.9859927	0.990443	0.4513527
+10 %	0.9859927	0.972077	1.411342
- 10 %	0.9859927	0.9936299	0.7745633
+15 %	0.9859927	0.9620341	2.429903
- 15 %	0.9859927	0.9958411	0.9988327
+20	0.9859927	0.9496229	3.688652
-20	0.9859927	0.9973298	1.149814
+30	0.9859927	0.9172041	6.976582
-30	0.9859927	0.9989263	1.311736
+40	0.9859927	0.7950699	19.36351
-40	0.9859927	0.9995567	1.375664
+50	0.9859927	0.7333875	25.61938
-50	0.9859927	0.9997959	1.399928

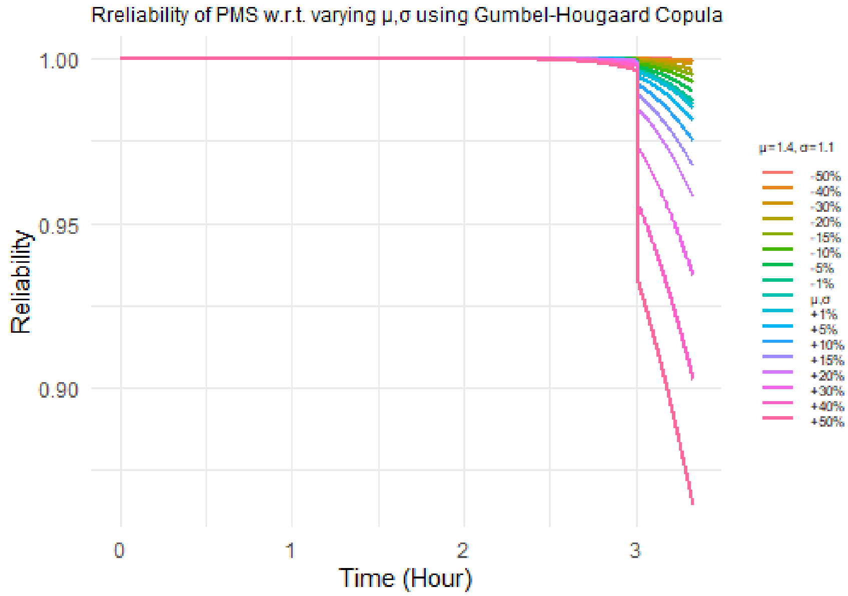

Figure 5: Reliability of aircraft flight PMS w.r.t $(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ **deviations in** $\mu = 1.4,\ \sigma = 1.1$, **using Gumbel-Hougaard copula**

Table 5: Sensitivity analysis on reliability of aircraft flight PMS w.r.t $(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ **deviations in impact element of external degradation** $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$ **using Gumbel-Hougaard copula** $\theta = 2.17, w_1 = 19, w_2 = 18, w_3 = 17, \mu = 1.4, \sigma = 1.1$

Initial values of common factors $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9859927	0.9850911	0.0914448
-1%	0.9859927	0.9868494	0.08688919
+ 5 %	0.9859927	0.9810052	0.5058418
-5 %	0.9859927	0.9898571	0.3919228
+10 %	0.9859927	0.97471	1.144305
- 10 %	0.9859927	0.9927842	0.6887922
+15 %	0.9859927	0.9669341	1.932937
- 15 %	0.9859927	0.9949515	0.9086028
+20	0.9859927	0.9575276	2.886954
-20	0.9859927	0.9965205	1.06773
+30	0.9859927	0.9333932	5.334674
-30	0.9859927	0.9984033	1.258685
+40	0.9859927	0.9018792	8.530849
-40	0.9859927	0.9992809	1.347693
+50	0.9859927	0.7776503	21.13022
-50	0.9859927	0.9996672	1.386876

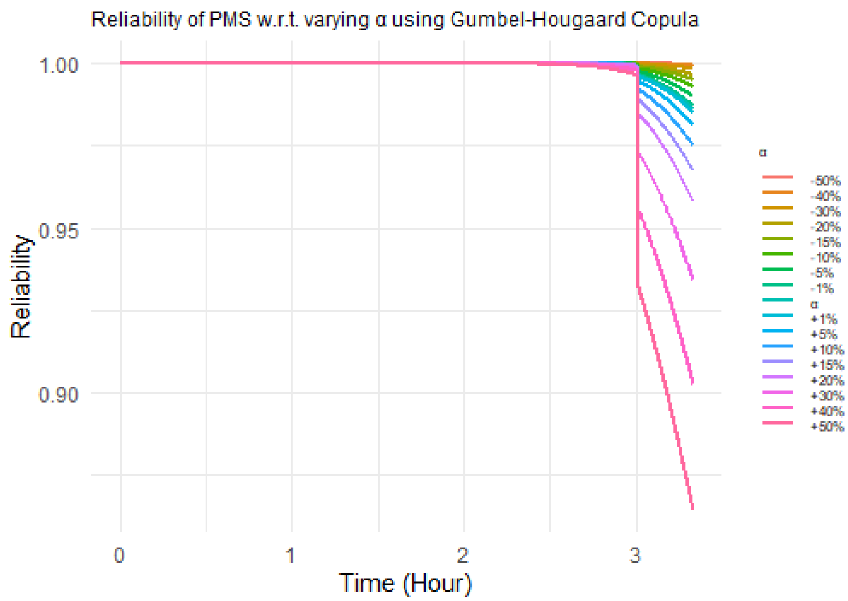

Figure 6: Reliability of aircraft flight PMS w.r.t $(\pm1\%,\ \pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%)$ **deviations in** $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$ **using Gumbel-Hougaard copula**

Table 6: Sensitivity analysis on reliability of aircraft flight PMS w.r.t $(\pm1\%,\ \pm5\%, \pm10\%, \pm15\%, \pm20\%,\ \pm30\%,\ \pm40\%,\ \pm50\%)$ **deviations in** $\theta = 2.17$ **using Clayton copula.** $w_1 = 19, w_2 = 18, w_3 = 17, \mu = 1.4, \sigma = 1.1, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Initial values of copula parameter θ =2.17	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9765496	0.9765533	0.0003794482
-1%	0.9765496	0.9765459	0.0003798237
+ 5 %	0.9765496	0.9765681	0.00189386
-5 %	0.9765496	0.976531	0.001903387
+10 %	0.9765496	0.9765865	0.003780273
- 10 %	0.9765496	0.9765123	0.00382023
+15 %	0.9765496	0.9766049	0.005660352
-15%	0.9765496	0.9764934	0.005757995
+20%	0.9765496	0.9766232	0.007534697
-20%	0.9765496	0.9764741	0.00773097
+30%	0.9765496	0.9766597	0.01126736
-30%	0.9765496	0.9764332	0.01191837
+40%	0.9765496	0.9766959	0.01497956
-40%	0.9765496	0.9763828	0.01708787
+50%	0.9765496	0.976732	0.01867179
-50%	0.9765496	0.9762956	0.02601556

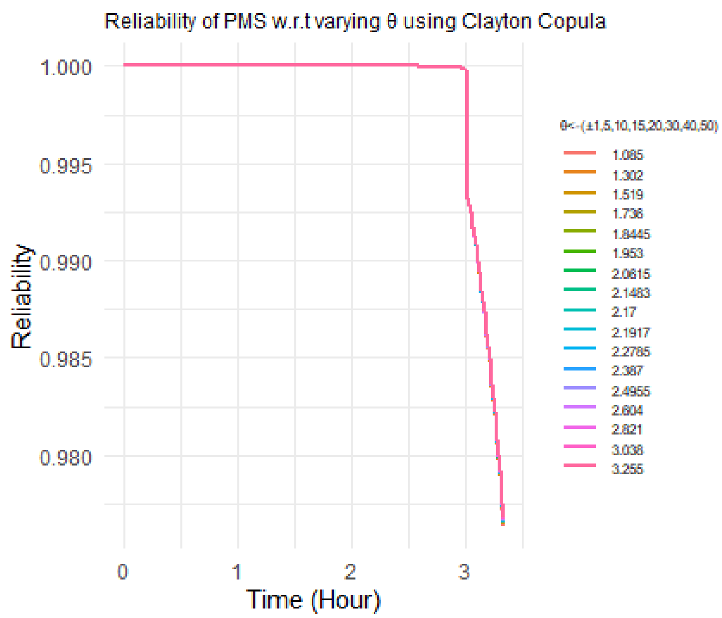

Figure 7: Reliability of aircraft flight PMS w.r.t ($\pm1\%$, $\pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%$) deviations in $\theta = 2.17$ using Gumbel-Hougaard copula

Table 7: Sensitivity analysis on reliability of aircraft flight PMS w.r.t ($\pm1\%$, $\pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%$) deviations in $w_1 = 19, w_2 = 18, w_3 = 17$ **using Clayton copula** $\theta = 2.17, \mu = 1.4, \sigma = 1.1, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Degradation level initial values $w_1 = 19$, $w_2 = 18$, $w_3 = 17$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9765496	0.9796191	0.3143206
-1%	0.9765496	0.9730903	0.3542398
+ 5 %	0.9765496	0.9886741	1.241559
-5 %	0.9765496	0.9546094	2.246707
+10 %	0.9765496	0.9948695	1.875979
- 10 %	0.9765496	0.9181977	5.975312
+15 %	0.9765496	0.9978156	2.177667
- 15 %	0.9765496	0.8632117	11.60595
+20%	0.9765496	0.9991249	2.311739
-20%	0.9765496	0.7878829	19.31972
+30%	0.9765496	0.9998828	2.389354
-30%	0.9765496	0.4396652	54.97769
+40%	0.9765496	0.9999877	2.40009
-40%	0.9765496	0.1508246	84.55536
+50%	0.9765496	0.999999	2.401247
-50%	0.9765496	0.04002497	95.90139

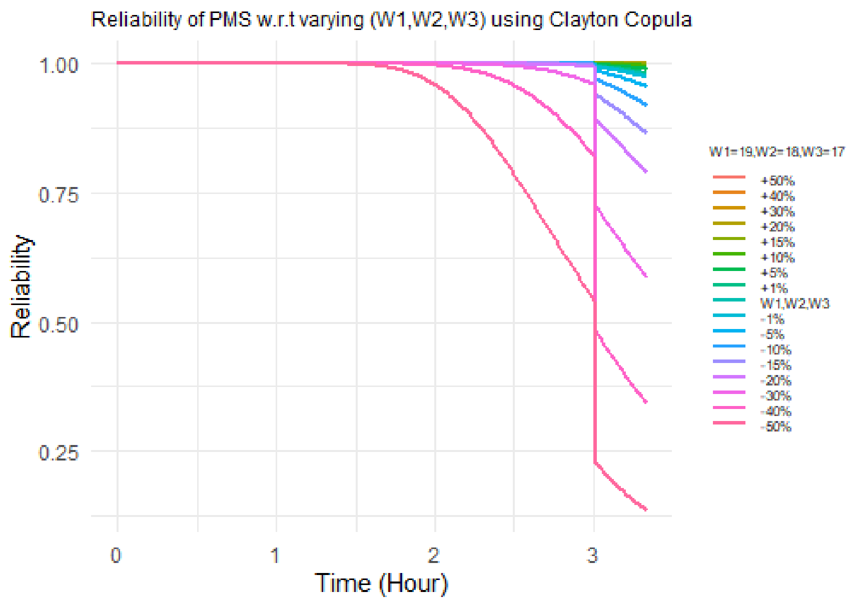

Figure 8: Reliability of aircraft flight PMS w.r.t $(\pm1\%,\ \pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%)$ **deviations in** $w_1 = 19, w_2 = 18, w_3 = 17$ **using Clayton copula**

Table 8: Sensitivity analysis on reliability of aircraft flight PMS w.r.t $(\pm1\%,\ \pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%)$ **deviations in external degradation parameters** $\mu = 1.4, \sigma = 1.1$**using Clayton copula** $\theta = 2.17, w_1 = 19, w_2 = 18, w_3 = 17, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Initial values of external degradation parameters $\mu = 1.4, \sigma = 1.1$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9765496	0.9746644	0.1819646
-1%	0.9765496	0.9781185	0.1717792
+ 5 %	0.9765496	0.9665029	1.017805
-5 %	0.9765496	0.9838988	0.7637562
+10 %	0.9765496	0.9537403	2.32486
- 10 %	0.9765496	0.9892922	1.31611
+15 %	0.9765496	0.9379365	3.943372
- 15 %	0.9765496	0.9930545	1.701409
+20	0.9765496	0.9190432	5.878283
-20	0.9765496	0.9955899	1.961072
+30	0.9765496	0.8727034	10.62407
-30	0.9765496	0.9982945	2.238055
+40	0.9765496	0.7229708	25.95859
-40	0.9765496	0.9993404	2.345168
+50	0.9765496	0.6612145	32.28323
-50	0.9765496	0.9997218	2.384225

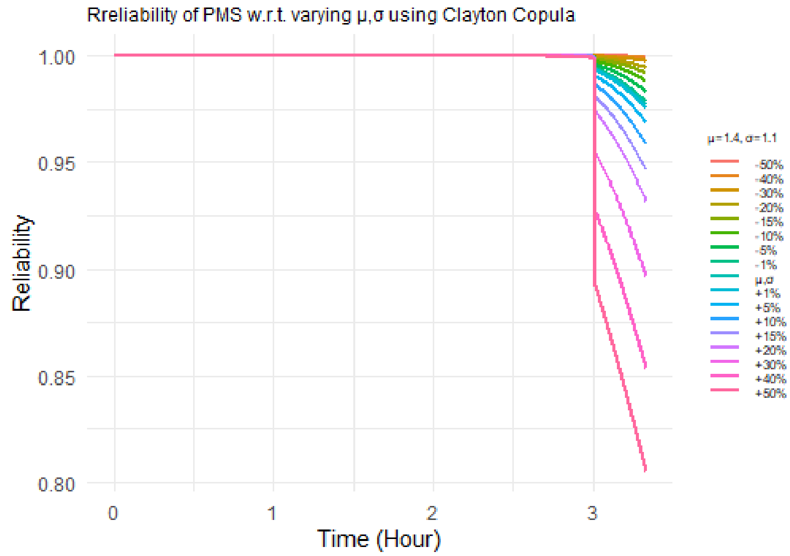

Figure 9: Reliability of aircraft flight PMS w.r.t ($\pm1\%, \pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%$) deviations in $\mu = 1.4, \sigma = 1.1$ using Clayton copula

Table 9: Sensitivity analysis on reliability of aircraft flight PMS w.r.t ($\pm1\%, \pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%$) deviations in impact element of external degradation $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$ using Clayton copula $\theta = 2.17, w_1 = 19, w_2 = 18, w_3 = 17, \mu = 1.4, \sigma = 1.1$

Initial values of common factors $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9765496	0.9749421	0.1535289
-1%	0.9765496	0.9778692	0.1462483
+ 5 %	0.9765496	0.9681982	0.8441897
-5 %	0.9765496	0.9829097	0.6624599
+10 %	0.9765496	0.9579671	1.891978
- 10 %	0.9765496	0.9878536	1.168775
+15 %	0.9765496	0.9455936	3.159183
- 15 %	0.9765496	0.9915347	1.545763
+20 %	0.9765496	0.9310032	4.653427
-20 %	0.9765496	0.9942061	1.819356
+30 %	0.9765496	0.8306779	8.306779
-30 %	0.9765496	0.9974064	2.147105
+40 %	0.9765496	0.8521386	12.73017
-40 %	0.9765496	0.998881	2.298122
+50 %	0.9765496	0.7047044	27.8293
-50 %	0.9765496	0.9995144	2.362989

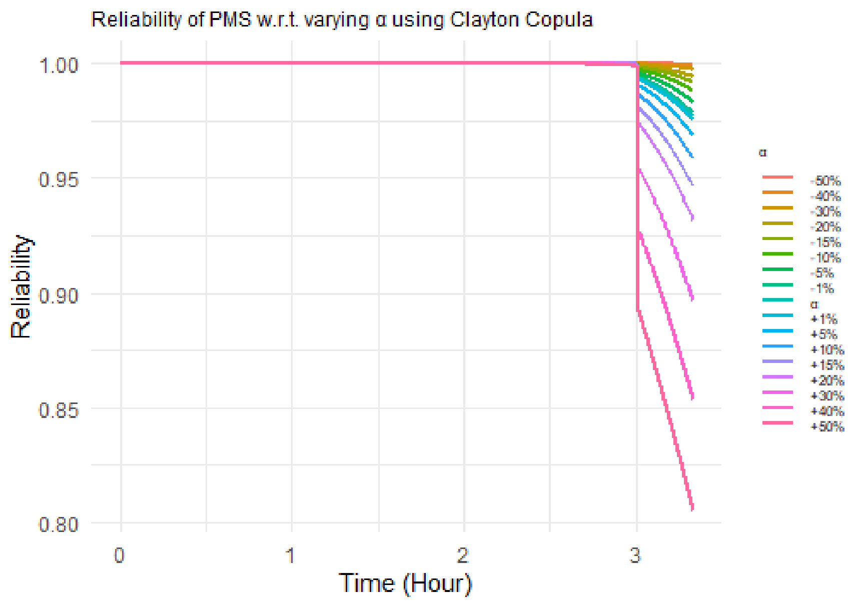

Figure 10: Reliability of aircraft flight PMS w.r.t ($\pm1\%$, $\pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%$) deviations in $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$ using Clayton copula

Table 10: Sensitivity analysis on reliability of aircraft flight PMS w.r.t ($\pm1\%$, $\pm5\%, \pm10\%, \pm15\%, \pm20\%$, $\pm30\%$, $\pm40\%$, $\pm50\%$) deviations in $\theta = 2.17$ using Frank copula $w_1 = 19, w_2 = 18, w_3 = 17, \mu = 1.4, \sigma = 1.1, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Initial values of copula parameter $\theta = 2.17$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9763125	0.9763156	0.0003141117
-1%	0.9763125	0.9763094	0.0003133606
+ 5 %	0.9763125	0.9763279	0.001577897
-5 %	0.9763125	0.9762973	0.001559119
+10 %	0.9763125	0.9763435	0.003173496
- 10 %	0.9763125	0.9762822	0.003098377
+15 %	0.9763125	0.9763592	0.004785734
-15%	0.9763125	0.9762674	0.004616689
+20 %	0.9763125	0.9763751	0.006413563
-20 %	0.9763125	0.9762528	0.006112972
+30 %	0.9763125	0.9764073	0.009711903
-30 %	0.9763125	0.9762243	0.009035165
+40 %	0.9763125	0.97644	0.01306057
-40 %	0.9763125	0.9761967	0.01185655
+50 %	0.9763125	0.9764731	0.01645208
-50 %	0.9763125	0.9761703	0.01456915

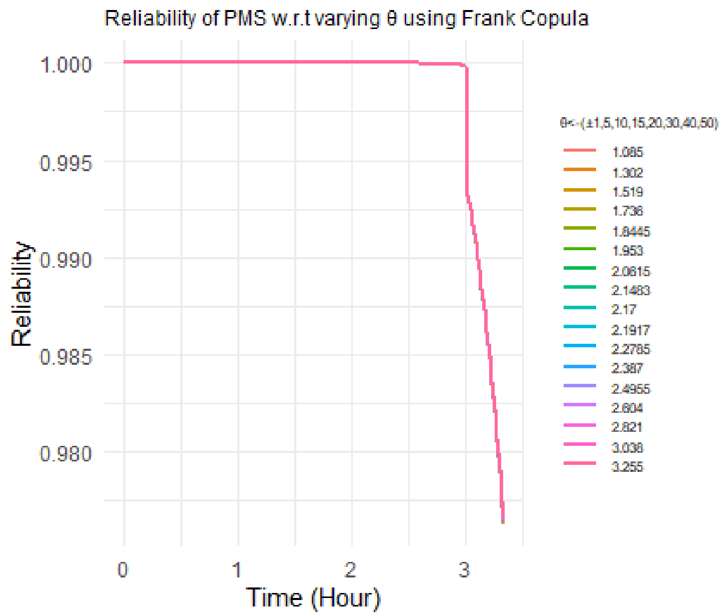

Figure 11: Reliability of aircraft flight PMS w.r.t $(\pm 1\%, \ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ **deviations in** $\theta = 2.17$ **using Clayton copula**

Table 11: Sensitivity analysis on reliability of aircraft flight PMS w.r.t $(\pm 1\%, \ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ **deviations in** $w_1 = 19, w_2 = 18, w_3 = 17$ **using Frank copula** $\theta = 2.17, \mu = 1.4, \sigma = 1.1, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Degradation level initial values $w_1 = 19, w_2 = 18, w_3 = 17$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9763125	0.9794347	0.3197983
-1%	0.9763125	0.9727865	0.3611527
+ 5 %	0.9763125	0.988609	1.259486
-5 %	0.9763125	0.9538239	2.303418
+10 %	0.9763125	0.994853	1.899037
- 10 %	0.9763125	0.9158703	6.190866
+15 %	0.9763125	0.9978117	2.202086
- 15 %	0.9763125	0.8571313	12.20728
+20 %	0.9763125	0.999124	2.3365
-20 %	0.9763125	0.7740416	20.71784
+30 %	0.9763125	0.9998828	2.414218
-30 %	0.9763125	0.3703219	62.06933
+40 %	0.9763125	0.9999877	2.424961
-40 %	0.9763125	0.08096258	91.70731
+50 %	0.9763125	0.999999	2.426118
-50 %	0.9763125	0.009363906	99.04089

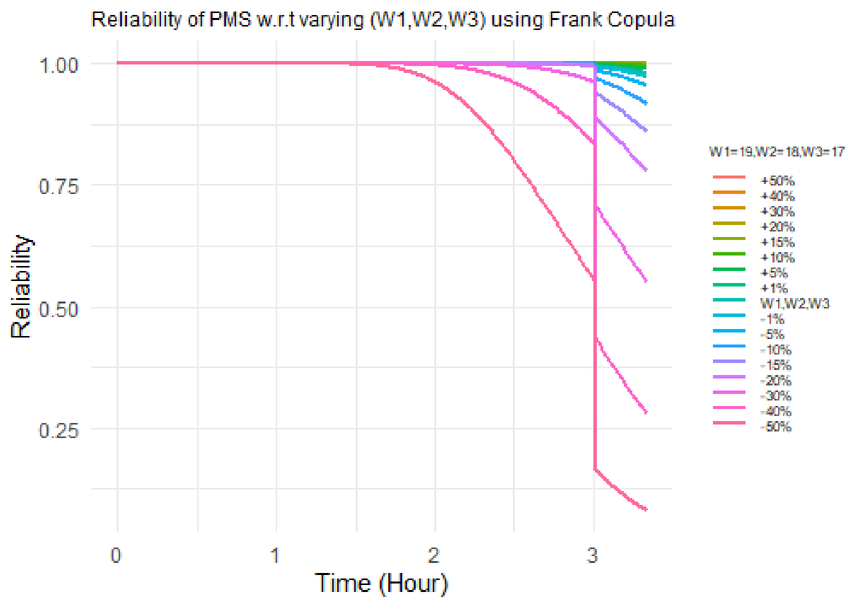

Figure 12: Reliability of aircraft flight PMS w.r.t $(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ deviations in $w_1 = 19, w_2 = 18, w_3 = 17$ using Frank copula

Table 12: Sensitivity analysis on reliability of aircraft flight PMS w.r.t $(\pm 1\%,\ \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%)$ deviations in external degradation parameters $\mu = 1.4, \sigma = 1.1$using Frank copula $\theta = 2.17, w_1 = 19, w_2 = 18, w_3 = 17, \alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$

Initial values of external degradation parameters $\mu = 1.4, \sigma = 1.1$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9763125	0.9745152	0.1840875
-1%	0.9763125	0.9780078	0.1736391
+ 5 %	0.9763125	0.9662398	1.031709
-5 %	0.9763125	0.9838395	0.7709588
+10 %	0.9763125	0.953233	2.363941
- 10 %	0.9763125	0.9892664	1.326817
+15 %	0.9763125	0.9370136	4.025242
- 15 %	0.9763125	0.9930438	1.713723
+20 %	0.9763125	0.9174555	6.028502
-20 %	0.9763125	0.9955857	1.974083
+30 %	0.9763125	0.8686875	11.02362
-30 %	0.9763125	0.9982939	2.251474
+40 %	0.9763125	0.7026077	28.03455
-40 %	0.9763125	0.9993403	2.358652
+50 %	0.9763125	0.6303879	35.43175
-50 %	0.9763125	0.9997217	2.397721

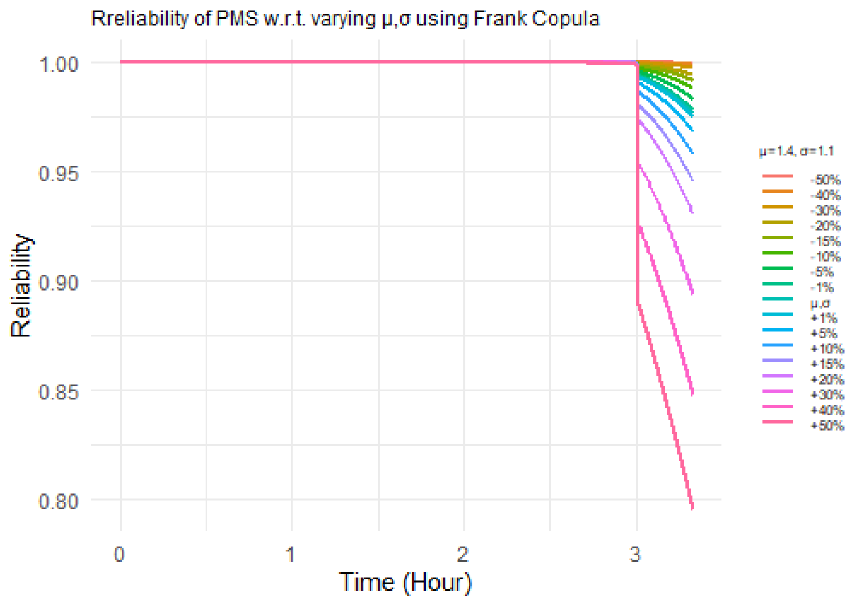

Figure 13: Reliability of aircraft flight PMS w.r.t ($\pm 1\%, \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%$) deviations in $\mu = 1.4, \sigma = 1.1$ using Frank copula

Table 13: Sensitivity analysis on reliability of aircraft flight PMS w.r.t ($\pm 1\%, \pm 5\%, \pm 10\%, \pm 15\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%$) deviations in impact element of external degradation $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$ using Frank copula $\theta = 2.17, w_1 = 19, w_2 = 18, w_3 = 17, \mu = 1.4, \sigma = 1.1$

Initial values of common factors $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$	True reliability (R)	Obtained reliability (R*)	Percentage deviation (PD) %
+1%	0.9763125	0.9747962	0.1553092
-1%	0.9763125	0.9777559	0.1478403
+ 5 %	0.9763125	0.9679614	0.855368
-5 %	0.9763125	0.9828427	0.668864
+10 %	0.9763125	0.9575498	1.921793
- 10 %	0.9763125	0.9878201	1.178685
+15 %	0.9763125	0.9448881	3.218686
- 15 %	0.9763125	0.9915186	1.557509
+20 %	0.9763125	0.929858	4.758156
-20 %	0.9763125	0.9941988	1.832024
+30 %	0.9763125	0.8926443	8.569819
-30 %	0.9763125	0.997405	2.160427
+40 %	0.9763125	0.8466733	13.27845
-40 %	0.9763125	0.9988808	2.311584
+50 %	0.9763125	0.6814569	30.20094
-50 %	0.9763125	0.9995144	2.376479

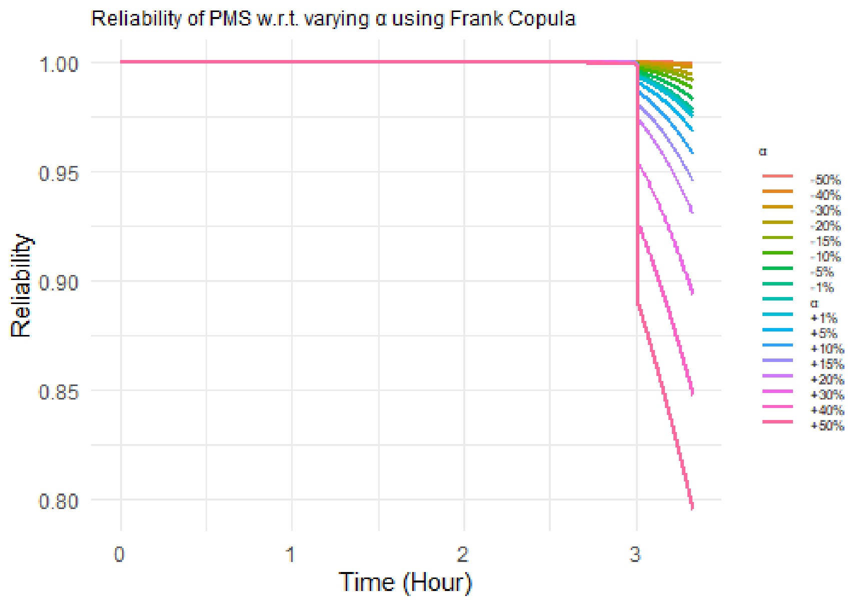

Figure 14: Reliability of aircraft flight PMS w.r.t $(\pm1\%,\ \pm5\%, \pm10\%, \pm15\%, \pm20\%, \pm30\%, \pm40\%, \pm50\%)$ **deviations in** $\alpha_1 = 0.4, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5, \alpha_6 = \alpha_7 = \alpha_8 = 0.7$ **using Frank copula**

Table 4 and Figure 5, Table 8 and Figure 9, and Table 12 and Figure 13 corresponding to the Gumbel-Hougaard, Clayton, and Frank copulas, respectively, show that increasing the parameters associated with external degradation of components, μ, σ, result in decrease in the reliability of aircraft flight PMS, and decrease in their values result in increase in the reliability of aircraft flight PMS.

Finally, Table 5 and Figure 6, Table 9 and Figure 10, and Table 13 and Figure 14 corresponding to the Gumbel-Hougaard, Clayton, and Frank copulas, respectively, show that increasing impact element, α_j, of external degradation of components results in decrease in the reliability of aircraft flight PMS, and decrease in their values result in increase in the reliability of aircraft flight PMS.

4. Concluding remarks

In the present paper reliability analysis of a PMS with each component in a phase subject to internal and external degradation is studied. A linear combination of the internal degradation and a proportional common external degradation constitute a degradation path of a component in a phase. Wiener process is used to model both internal and common external degradation. The cumulative exposure model has been used to model a PMS, with the dependency amongst the components in a phase and that across the phases modelled using different copulas: Gumbel-Hougaard, Clayton, and Frank. An aircraft flight PMS is used to demonstrate the methodology proposed and comparative study amongst PMS models with different copulas carried out. The results of the sensitivity analyses show that the model proposed is not sensitive to small deviations in the values of the selected parameters. The method developed can be generalized to missions with any number of phases.

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Conflict of interest

The authors do not have any financial or non-financial conflict of interest to declare for the research work included in this article.

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