

Bayesian DCC-MGARCH Models for Understanding the Price Dynamics of Volatile Series: A Case Study on Pulses Price in India

Achal Lama¹, Girish K. Jha¹, Bishal Gurung² and Saurav Guha^{3,4}

¹ICAR – Indian Agricultural Statistics Research Institute, New Delhi, India

²North-Eastern Hill University, Shillong, India

³Bihar Agricultural University, Sabour, Bhagalpur, India

⁴Health Analytics Network, Pittsburgh, PA, USA

Received: 27 July 2024; Revised: 07 January 2025; Accepted: 10 January 2025

Abstract

This article explores the price movement and dependency among major pulses in India namely Moong, Masur, Urad and Arhar. To achieve our objective we have considered the well-known DCC-MGARCH model. A two-step estimation procedure, first, identifying the appropriate mean model using VAR and then the variance model using DCC-MGARCH has been performed. An attempt has also been made to document the comparative performance of the DCC-MGARCH model under both classical and Bayesian framework. The model parameters have been estimated using MLE and MCMC methods. The robustness of the Bayesian DCC-MGARCH model is evaluated under different prior settings and the number of MCMC samples drawn. Empirical studies indicate the superiority of Bayesian DCC-MGARCH model with errors satisfying multivariate t distribution for modelling the pulses series. The results suggest that the Bayesian DCC-MGARCH models outperforms the competing models in terms of parameters standard error, information criteria and its interpretability. This study also revealed the existence of co-movement of pulse prices to be dynamic, where prices of Moong and Urad exhibit the highest degree of dependency among them. The findings of this study can better assist the stakeholders in formulating policy decisions focusing on enhancing income of the pulse cultivators of India.

Key words: Pulses; VAR; MGARCH; DCC; Bayesian.

AMS Subject Classifications: 62M10

1. Introduction

India is one of the largest importer, producer, processor and consumer of pulses in the world. Thus pulses are an integral part of Indian agricultural system. Indian agriculture till date is largely dependent on monsoons. Pulses being no exception rely largely on monsoons. To aggravate the situation further, pulses are grown mainly on marginal lands. These factors

together contribute to the high level of fluctuations in pulse production and prices in India. Leading to the scenario that Indian farmers are not readily taking up cultivation of pulses despite of government ensuring high wholesale prices in recent years. Among the pulses, Moong, Masur, Urad and Arhar together are cultivated at around 50% of the total pulse growing area and account for more than 40% of the total production. In addition to it, these pulses face the inherent problem of fluctuation in production and prices. The movement of prices among the commodities can be short lived or long lived, depending upon the nature of the commodities. The fluctuation in prices leads to volatility and heteroscedasticity is an important characteristic of volatility. There are various models reported in literature to model the volatility of a series. In recent years due to liberalisation of the markets in domestic as well as international markets the prices of the commodities move together over markets and time. It is due to this very reason that the researchers were compelled to look for models that can model more than one series together, the multivariate modelling approach. Different multivariate time-series models are used for forecasting, but after the pioneering work of Sims (1977), Vector Autoregressive (VAR) model have become the most popular among them for correlated series. This phenomenon of correlation is also observed in agricultural data series. The VAR model is useful only for modelling the mean or the first order moment of the series. Thus to have a better understanding of the series, modelling and forecasting volatility has been a major area of time series research for years.

Traditional econometric models assume a constant one-period forecast variances. To generalize this implausible assumption, Engle (1982) introduced a new class of stochastic process called Autoregressive Conditional Heteroscedasticity (ARCH) which is very useful where underlying forecast variances may change over time and is predicted by the past forecast errors. However, ARCH model has some drawbacks such as high number of unknown parameters and rapid decay of unconditional autocorrelation function of squared residuals *etc.* To overcome these difficulties, Bollerslev (1986) proposed the Generalized ARCH (GARCH) model in which conditional variance is also a linear function of its own lags. This model is also a weighted average of past squared residuals, but it has declining weights that never go completely to zero. It provides flexible lag structure and it permits parsimonious descriptions in most of the situations. The ability of GARCH model to capture volatility has been widely studied in literature (Lama *et al.*, 2015). A multivariate GARCH (MGARCH) model framework was proposed to model more than one series at a time. A class of MGARCH models have been developed over time. Engle and Kroner (1995) introduced a multivariate structure of GARCH model known as BEKK (Baba, Engle, Kraft and Kroner) model which is the direct generalization of univariate GARCH model and have huge flexibility. Bollerslev (1990) developed a relatively flexible approach known as Constant Conditional Correlation (CCC) model which allowed for combination of univariate GARCH model, with an assumption of constant correlation among the series over time. Engle (2002) proposed a new class of multivariate GARCH model known as Dynamic Conditional Correlation (DCC) model which has the flexibility of the univariate GARCH models coupled with parsimonious parametric model for the correlations. DCC model allows the conditional correlation to vary over time, adding dynamicity to the model. These models along with other variants of MGARCH models are explained lucidly by Bauwens *et al.* (2006). The use of these models for modelling the degree of interactions among various volatile commodities and markets can be widely seen in literature (Chevallier, 2012; Lean and Teng, 2013; Lin and Li, 2015; Gupta *et al.*, 2019). It is also interesting to note that most of the financial series exhibits leptokurtic behaviour

which has been documented well. There are number of ways to deal with this problem, such as the use of stable Paretian distribution with a characteristic exponent less than two to replace the normal error setting (Fama, 1965), discrete mixture of normal distributions can be used to explain the observed patterns of significant kurtosis and a positive skewness of data (Hull and White, 1998), but the use of Student- t distribution to model the error term is a promising alternative among them (Ku, 2008).

In recent times the use of Bayesian paradigm in time series literature has gained popularity. Researches have harnessed the benefits of Bayesian inference to interpret time series models explaining volatility process efficiently. Univariate as well as multivariate models have been estimated using Bayesian framework (Fioruci *et al.*, 2014). But, the use of Bayesian DCC-MGARCH model is very limited till date (Shiferaw, 2019). Furthermore, in majority of the cases the application is based on financial markets, very little efforts have been made to analyse volatile agricultural commodities. Hence, we undertake this study with two major objectives; first we model the multivariate agricultural price series in two step process using combination of VAR and DCC-MGARCH model. In the first step VAR has been used to capture the linear dependency among the series and then the DCC-MGARCH model has been used in the second step upon the residuals obtained from VAR model to capture the dynamic movement of non-linear volatile component among the series (Mohammadi and Tan, 2015; Lama *et al.*, 2016; Chen *et al.*, 2020). Secondly, this study also compares the performance of DCC-MGARCH model under Maximum Likelihood Estimation (MLE) and Bayesian framework using different error distributions for appropriate modelling of the inherent kurtosis nature of the series. Such comparative analyses are very few in multivariate time series literature (Shiferaw, 2019). The driving force behind implementation of Bayesian framework in the present study can be attributed largely to these three properties of Bayesian estimation:

1. Constraints on the model parameters to ensure positive variance are achieved in Bayesian framework by assigning priors to the model parameters.
2. The Bayesian framework also provides reliable results for finite samples and heavy tailed data sets (Hall and Yao, 2003).
3. The comparison of models is more consistent in Bayesian framework as it makes use of not only the estimates as in MLE, but also the marginal likelihood and Bayes factor or posterior model probabilities (Miazhynskaia and Dorffner, 2006).

The very basic assumption of a Bayesian framework is that the data is not exhaustive to explain all the underlying behavior of the series. Thus priors are to be assigned to the parameters of the model and then posterior is estimated under that prior information. Bayesian analysis also provides the density of the parameters of the model unlike the point or interval estimates provided by the classical approaches. Further, in Section 2 brief details of the VAR, MGARCH models and testing of MARCH effect have been described, followed by empirical results in Section 3. Section 4 of this paper deals with the discussion of the results. Finally, the paper is concluded in Section 5.

2. Theoretical background

2.1. VAR model

Let $Y_t = (y_{1t}, y_{2t}, \dots, y_{Nt})^T$ denote an $(N \times 1)$ vector of time series variables. The basic p -lag vector autoregressive VAR(p) model has the form:

$$Y_t = A + B_1 Y_{t-1} + B_2 Y_{t-2} + B_3 Y_{t-3} + \dots + B_p Y_{t-p} + \varepsilon_t \quad (1)$$

where, A is $N \times 1$ vector of intercepts, $B_i (i = 1, 2, \dots, p)$ is $N \times N$ matrices of parameters and $\varepsilon_t \stackrel{iid}{\sim} N(0, \Sigma)$. The number of parameters to be estimated in the VAR model is $N(1 + Np)$ which increases with the number of variables (N) and number of lags (p).

2.2. MGARCH models

For a multivariate time series $y_t = (y_{1t}, y_{2t}, \dots, y_{Nt})^T$ the MGARCH model is given by:

$$y_t = H_t^{1/2} \varepsilon_t \quad (2)$$

where, $H_t^{1/2}$ is $N \times N$ positive-definite matrix and of the conditional variance of y_t . N is the number of series and $t = 1, 2, \dots, n$ (number of observations). It is with the specification of conditional variance that the MGARCH model changes. The core issues in MGARCH model is to construct the conditional variance-covariance matrix H_t . A relatively easy estimation approach is the CCC model introduced by Bollerslev (1990). This model assumes the conditional correlations to be constant. This restriction strongly reduces the number of unknown parameter and thus simplified the estimation. In case of CCC model the H_t represented as follows:

$$H_t = D_t R D_t \quad (3)$$

where, $D_t = \text{diag}(h_{11,t}^{1/2}, \dots, h_{NN,t}^{1/2})$ and R is a symmetric positive-definite matrix whose elements are (constant) conditional correlations:

$$R(i, j) = \begin{cases} 1 & \text{for } i = j (i, j = 1, 2, \dots, N); \\ \rho_{ij} & \text{otherwise.} \end{cases}$$

Here $h_{11,t}^{1/2}, \dots, h_{NN,t}^{1/2}$ are conditional variances of each series estimated from univariate GARCH models. Thus each conditional covariance is given by (Bollerslev, 1990):

$$h_{ij,t} = \rho_{ij} \sqrt{h_{ii,t} h_{jj,t}}. \quad (4)$$

In case of DCC the R matrix is also time varying thus making it dynamic. The representation of the model is as follows:

$$H_t = D_t R_t D_t \quad (5)$$

where, $R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2}$ and $Q_t = (1 - \alpha - \beta) R + \alpha u_{t-1} u_{t-1}^T + \beta Q_{t-1}$ and $u_t = D_t^{-1} y_t$. R is the unconditional covariance matrix of u_t . And the conditional covariances are given by:

$$h_{ij,t} = q_{ij,t} \sqrt{h_{ii,t} h_{jj,t}} / \sqrt{q_{ii,t} q_{jj,t}}. \quad (6)$$

Q_t is written as GARCH(1, 1) type equation and then transformed to get R_t .

2.3. Parameter estimation

2.3.1. Maximum likelihood estimation

The log likelihood estimator (Engle, 2002) is as follows:

$$y_{t|t-1} \sim N(0, H_t)$$

$$\begin{aligned} L &= -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + \log |H_t| + y_t^T H_t^{-1} y_t) \\ &= -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + \log |D_t^T R_t D_t| + y_t^T D_t^{-1} R_t^{-1} D_t y_t) \\ &= -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + 2 \log |D_t| + \log |R_t| + \varepsilon_t^T R_t^{-1} \varepsilon_t) \\ &= -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + 2 \log |D_t| + y_t^T D_t^{-1} D_t y_t - \varepsilon_t^T \varepsilon_t + \log |R_t| + \varepsilon_t^T R_t^{-1} \varepsilon_t) \end{aligned} \quad (7)$$

Now this equation is to be maximized over the parameters of the model. The main advantage of this estimation procedure is that it allows the model to be estimated even when the covariance matrix is very large. The essence of estimation of DCC model lies in the fact that it is a two-step estimation procedure. The first one being the estimation of the volatility part and the next one is the correlation part. Thus the likelihood can be written as the sum of the volatility and correlation components. To do so we denote the D by φ and R by ϕ , now the likelihood can be written as :

$$L(\varphi, \phi) = L_v(\varphi) + L_c(\varphi, \phi).$$

The volatility component is represented as:

$$L_v(\varphi) = -\frac{1}{2} \sum_t \left[n \log(2\pi) + \log |D_t|^2 + y_t^T D_t^{-2} y_t \right]$$

and the correlation part as:

$$L_c(\varphi, \phi) = -\frac{1}{2} \sum_t \left[(\log |R_t| + \varepsilon_t^T R_t^{-1} \varepsilon_t - \varepsilon_t^T \varepsilon_t) \right].$$

The two-step maximization is done and the estimates are obtained as follows:

$$\hat{\varphi} = \arg \max \{L_v(\varphi)\}$$

and then take this value as given in the second stage as $\max_{\phi} \{L_c(\hat{\varphi}, \phi)\}$.

In this estimation procedure maximum at second step is a function of the first step parameter estimates. Thus to ensure the consistency, first step estimates' consistency will ensure the consistency of the second step as long as the function is continuous in the neighborhood of the true parameters. The error distribution can be multivariate normal distribution or the multivariate Student-t distribution depending on the kurtosis of the data. The density function of the multivariate t distribution is given by:

$$p(\varepsilon_t) = \frac{\Gamma((v+k)/2)}{\Gamma(v/2) [\pi(v-2)]^{k/2}} \left[1 + \frac{\varepsilon_t^T \varepsilon_t}{v-2} \right]^{-(v+k)/2} \quad (8)$$

where, v is the degree of freedoms of t distribution and $\Gamma(\cdot)$ is the Gamma function.

2.3.2. Bayesian estimation

The MLE method is the classical approach for parameter estimation of the GARCH models. The Bayesian estimation technique is different from MLE and is growing rapidly. This technique is based on the generation of the posterior distribution based on the prior distribution through various techniques such as Markov Chain Monte Carlo (MCMC). In our study we have adopted Metropolis-Hastings (MH) algorithm where the GARCH parameters are updated by blocks (one block for α and one block for β) while the degrees of freedom parameter is sampled using an optimized rejection technique from a translated exponential source density. The basis of Bayesian estimation is the Bayes' Theorem. Let us consider the parametric space θ which is the vector of the parameters of the model with a prior density function $\pi(\theta)$ and Y is the data vector. According, to Bayes' rule the posterior density

$$\pi(\theta|Y) \propto L(Y|\theta)\pi(\theta) \quad (9)$$

where, $L(Y|\theta)$ is the likelihood function same as in 7.

At first model is fitted using MLE technique and the obtained log likelihood value is noted. Then the priors are defined for the parameters. Incorporating both log likelihood and prior, facilitating the algorithm to draw samples from the posterior distribution following the Bayes' rule. It generates Markov chain for each parameters based on the initial value. The ML estimates of parameters are taken as initial value for sampling in the present study. Based on the extensive review of literature Gaussian proposal density is taken for the current study. A new value of each parameter δ^{t+1} , where $\delta \in \theta^* = (\omega, \alpha, \beta, DCC_\alpha, DCC_\beta)$ is obtained with the acceptance probability $\alpha(\delta^{t+1}|\delta^t) = \min(1, r(\delta^{t+1}, \delta^t))$ for each $\delta \in \theta^* = (\omega, \alpha, \beta, DCC_\alpha, DCC_\beta)$ where, $r(\delta^{t+1}, \delta^t) = \pi(\delta^{t+1}|Y)/\pi(\delta^t|Y)$. The new value is either accepted or rejected and this will be continued until a sufficient number of iterations are obtained.

2.3.3. Model evaluation criteria

Standard model evaluation criteria, such as Akaike information criterion (AIC) and Bayesian information criterion (BIC), are used to compare the performance of different models. These criteria penalize the decrease in the degrees of freedom when more variables are added. The AIC and BIC values for GARCH model with Gaussian distributed errors are computed by:

$$AIC = 2\log(\text{likelihood}) + 2T$$

$$BIC = 2\log(\text{likelihood}) + \log(Tk)$$

where, k is model degrees of freedom. In case of models estimated using Bayesian technique these AIC and BIC evaluation criteria are not applicable as it involves samples estimated through MCMC from posterior distribution. Hence, Deviance Information criterion (DIC) is used as we have assumed posterior distributions to be normally distributed. The formula is as follows:

$$DIC = 2E[D(\theta M)] - D(E[\theta M])$$

where θM is the set of parameters in model M and $D(\cdot)$ is the deviance function defined as minus twice the log-likelihood function. So, given a sample from the posterior distribution of θM , it is straightforward to approximate the DIC. The implementation of the models to the dataset was done using R software and a schematic representation is given in Figure 1.

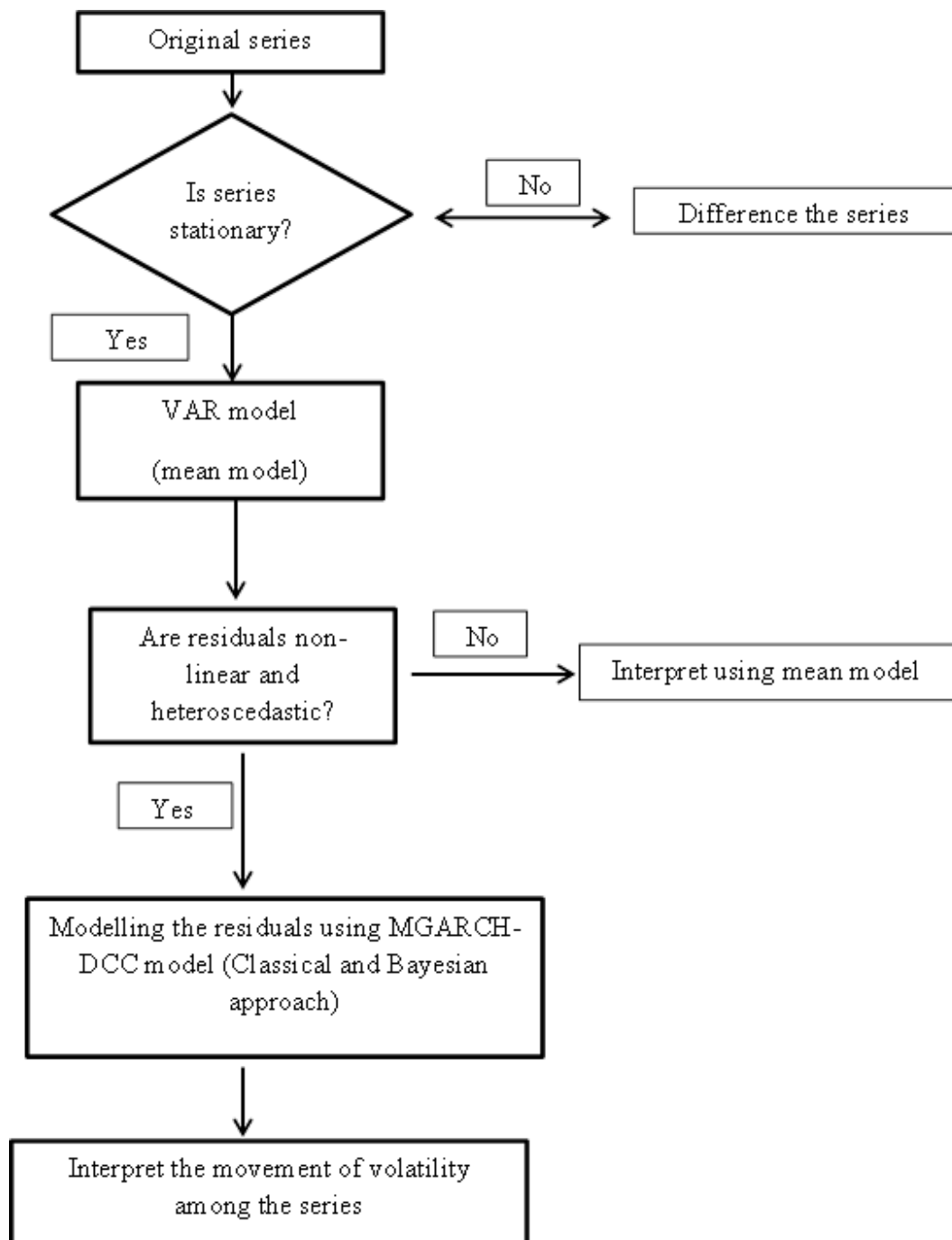


Figure 1: Flowchart of the employed methodology

3. Empirical results

3.1. Data description

The four major pulses series namely price index of Moong, Masur, Urad and Arhar are used in this study. The source of the data being Office of the Economic Adviser, Ministry of Commerce and Industry, Government of India (<https://eaindustry.nic.in/>). These series very well depict the properties of typical agricultural price data with inherent fluctuations as shown in Figure 2. Each series (monthly) contained 312 data points (April, 1994 to March, 2020) and the entire series was used for model development purpose. The statistical properties of the dataset are presented in the Table 1. We also explore the stationarity of the series by implementing the augmented Dickey–Fuller (ADF) test and we find in Table 1 that the series are stationary after first order differencing. The visual inspection from Figure 3 of the stationary series clearly indicated the presence of volatility at several time-epochs. The data set was tested for the presence of seasonality using the correlogram. The ACF (auto correlation function) in Figure 4 for all the series was devoid of any oscillatory pattern, indicating absence of any seasonality. In addition, skewness and kurtosis of the stationary series were computed along with the correlation coefficients. Table 2 shows a fair degree of linear dependency was observed among the series ranging from 0.364 to 0.593. It is evident from Table 3 that the stationary series tends to be non-skewed apart from Urad showing moderate skewness and leptokurtosis being a common feature to all.

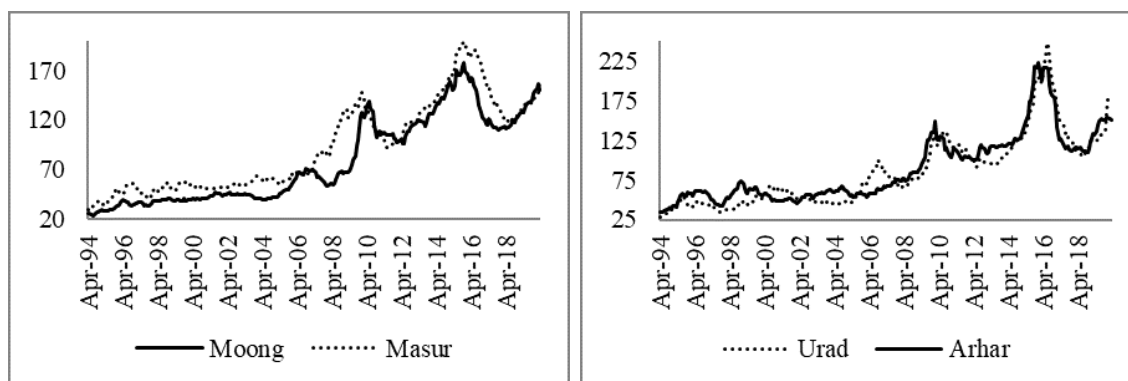


Figure 2: Time plot of the series

Table 1: Summary statistics of different pulses

Properties	Moong	Masur	Urad	Arhar
Mean	78.49	92.25	86.88	89.66
Median	61.90	74.95	76.00	70.10
Standard Deviation	43.32	44.97	45.69	42.91
Kurtosis	-1.18	-0.86	0.84	0.64
Skewness	0.52	0.57	1.07	1.06
Minimum	23.66	28.84	28.60	34.60
Maximum	178.10	198.80	247.60	222.80
C.V(%)	55.20	48.75	52.58	47.86
ADF (Level)	-3.32	-2.93	-2.89	-3.20
ADF (1st Differenced)	-4.90*	-4.51*	-4.71*	-4.72*

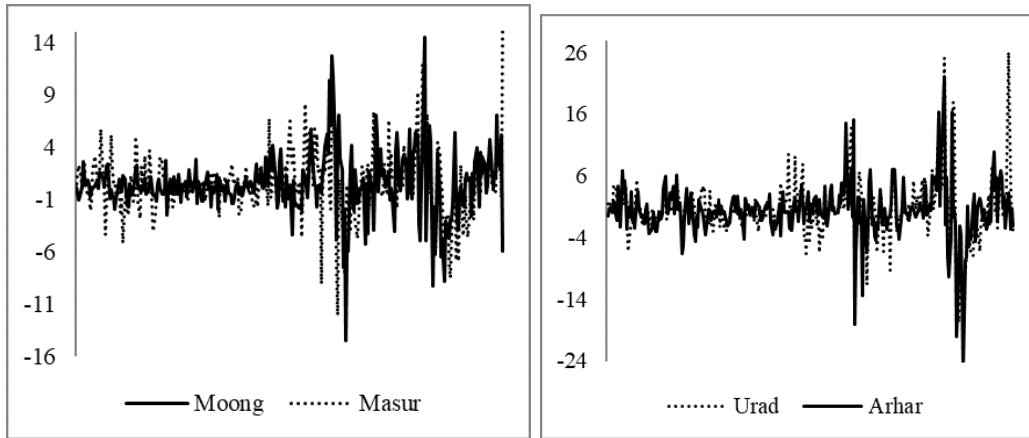


Figure 3: Plot of the first differenced series

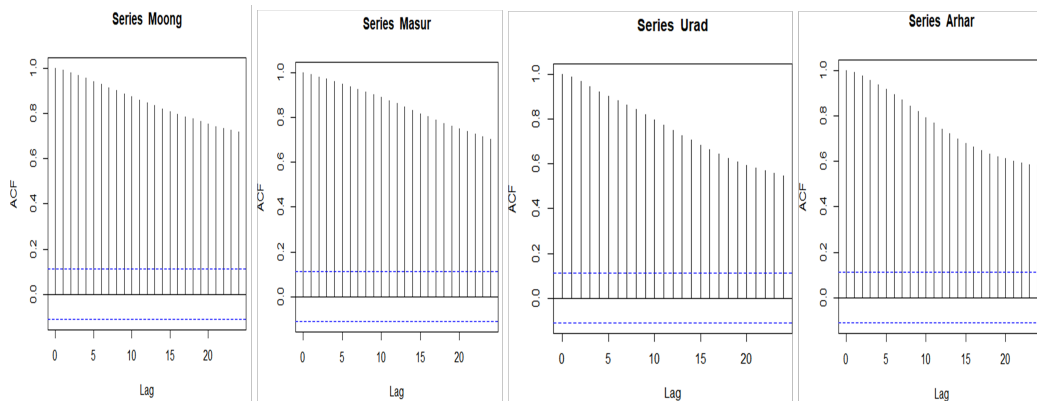


Figure 4: ACF plots of the original series

3.2. Fitting VAR model

The VAR model as discussed earlier was fitted to the stationary dataset. The appropriate order of the VAR model was selected based on Bayesian information criterion (BIC) and Hannan and Quinn (HQ) information criterion. We focused on these two criteria as they penalize depending on the sample size, unlike AIC which penalizes each parameter by a factor of 2. The lag order with lowest values for both the criteria was selected (BIC: 8.893; HQ: 8.777). We obtained VAR (1) model for the present investigation. The estimates of the model given in Table 4 reveal linear dependency within and between the series at first lag. Within series dependency ranged between 0.219 to 0.428 and 0.003 to 0.293 for between the series. The strongest linear dependency within series was found for Urad (0.428) and Arhar and Masur (0.293) exhibited highest between series association. As a standard statistical practice we carried out the diagnostic check of the residuals to determine the suitability of the fitted model. To this end, we start with obtaining the Q-Q plot given in Figure 5 for the residuals of each series to verify the normality assumption. The Q-Q plot in Figure 5 indicates the residuals violating normality assumption as they do not align with the straight line. Further, we moved on with studying the non-linearity of the residuals and presence of heteroscedasticity each at univariate (individual residuals) and multivariate (all residuals together) framework. The results in Table 5 point that residuals series are non-linear in

Table 2: Correlation matrix of the stationary series

	Moong	Masur	Urad	Arhar
Moong	1	0.364	0.519	0.457
Masur	0.364	1	0.412	0.550
Urad	0.519	0.412	1	0.593
Arhar	0.457	0.550	0.593	1

Table 3: Descriptive statistics of the stationary series

Statistics	Moong	Masur	Urad	Arhar
Mean	0.403	0.429	0.476	0.371
Standard Deviation	2.942	3.066	4.601	4.388
Kurtosis	5.337	3.597	7.579	9.003
Skewness	0.119	0.251	1.098	-0.393

nature as the null hypothesis of linearity is not accepted for BDS test. Then, the presence of heteroscedasticity is confirmed for both univariate and multivariate framework by the ARCH- Lagrange multiplier (LM) and Multivariate ARCH-LM tests respectively. For both the tests, null hypothesis of homocedasticity is not accepted as evident from Table 5. Finally, we conclude the residuals to be non-normal, non-linear and heteroscedastic (volatility clustering) in nature. These characteristics of the residuals demands for further modelling using MGARCH model in general and DCC-MGARCH model in particular as we aim at understanding the dynamic co-movement in the volatility structure.

Table 4: Estimates of VAR(1) model

Series	Moong	Masur	Urad	Arhar
Moong (-1)	0.347 (0.064)	0.003 (0.066)	-0.022 (0.044)	0.091 (0.049)
Masur (-1)	0.153 (0.065)	0.316 (0.067)	0.012 (0.045)	0.058 (0.049)
Urad (-1)	0.202 (0.088)	-0.130 (0.091)	0.428 (0.061)	0.190 (0.067)
Arhar (-1)	0.293 (0.095)	0.178 (0.097)	-0.027 (0.065)	0.219 (0.072)
C	0.231 (0.1565)	0.217 (0.158)	0.156 (0.215)	0.112 (0.231)
AIC	8.598			

Values in the parenthesis are Standard Errors.

3.3. Fitting DCC-MGARCH models

The stationary series were tested for the presence of dynamic conditional correlation using Engle and Sheppard Test of Dynamic Correlation (Engle and Sheppard, 2001). The test statistic value (9.257) was rejected at 5% level of significance, confirming the presence of dynamic properties of the series. Thus we decided for the application of DCC model. To begin the implementation of the DCC model we have to first identify the volatility model,

in this study we have used DCC-MGARCH model in 5 with an assumption of the errors to be multivariate normal and the multivariate t distribution. The parameters were estimated both by the MLE and Bayesian framework.

The results of the DCC model estimated using MLE with multivariate Normal and multivariate t errors are reported in Table 6. It is clear from the Table 6 that ARCH (α) parameters for Masur, Urad and Arhar along with the DCC_α are not significant at 5% when estimated using MLE for both the error distributions. However, the lower value of BIC for DCC model with multivariate t error suggests its superiority over the other model for modelling the pulses series. This result cast a doubt in the appropriateness of the model fitted.

Then, we turned our attention into the Bayesian estimation procedure. Essentially, Bayesian estimation starts with defining prior distribution for all the parameters of the model. In this study we have defined the priors based on results obtained by Fioruci *et al.* (2014) and Shiferaw (2019). We use truncated Normal priors on the GARCH parameters α and β as follows:

$$p(\alpha) \sim \phi N_2(\alpha|\mu, \Sigma) 1\{\alpha \in R_+^2\}$$

$$p(\beta) \sim \phi N_1(\beta|\mu, \Sigma) 1\{\beta \in R_+\}$$

where μ and Σ are the hyperparameters, $1\{\cdot\}$ is the indicator function and ϕN_d is the d -dimensional Normal density. The values of the hyperparameters are kept fixed. The prior distributions are normal owing to its conjugate nature. The ω , DCC_α and DCC_β are assigned $N \sim (0, 10)$ priors, whereas the priors for α and β are varied with a total of 5 different combinations. The prior combinations are as follows: The prior combinations are as follows:

Prior 1 : $\alpha \sim N(0.1, 0.01), \beta \sim N(0.4, 0.01)$

Prior 2 : $\alpha \sim N(0.2, 0.03), \beta \sim N(0.5, 0.03)$

Prior 3 : $\alpha \sim N(0.3, 0.05), \beta \sim N(0.6, 0.05)$

Prior 4 : $\alpha \sim N(0.4, 0.07), \beta \sim N(0.7, 0.07)$

Prior 5 : $\alpha \sim N(0.5, 0.09), \beta \sim N(0.8, 0.09)$.

These combinations allow us to test the sensitivity of parameter estimates of the model with regards to change in distribution or the parameters of the distribution. As, in our investigation we have used normal priors, hence we judge the sensitivity by varying mean and variance within a specified range. The prior settings used for simulation study can be broadly classified into two groups. In the first group (Prior 1, Prior 2 and Prior 3) where the sum of α, β are < 1 and the second group (Prior 4 and Prior 5) where the sum of α, β are > 1 . These groups are formed intentionally to study the impact of α, β on the overall parameter estimation of the model. Along with the sensitivity of the priors we have also evaluated the effect of various sample sizes of MCMC draws on parameter estimates. The results reported in Table 6 are of prior 2 specifications with 10,000 MCMC draws. Results are encouraging as parameters estimate along with its standard error clearly indicates its statistical significance. Moreover, the DCC model with multivariate t errors has lower DIC values as compared with the multivariate Normal errors. This indicates the superiority of the model over the other one. The superiority of DCC-MARCH model with multivariate t

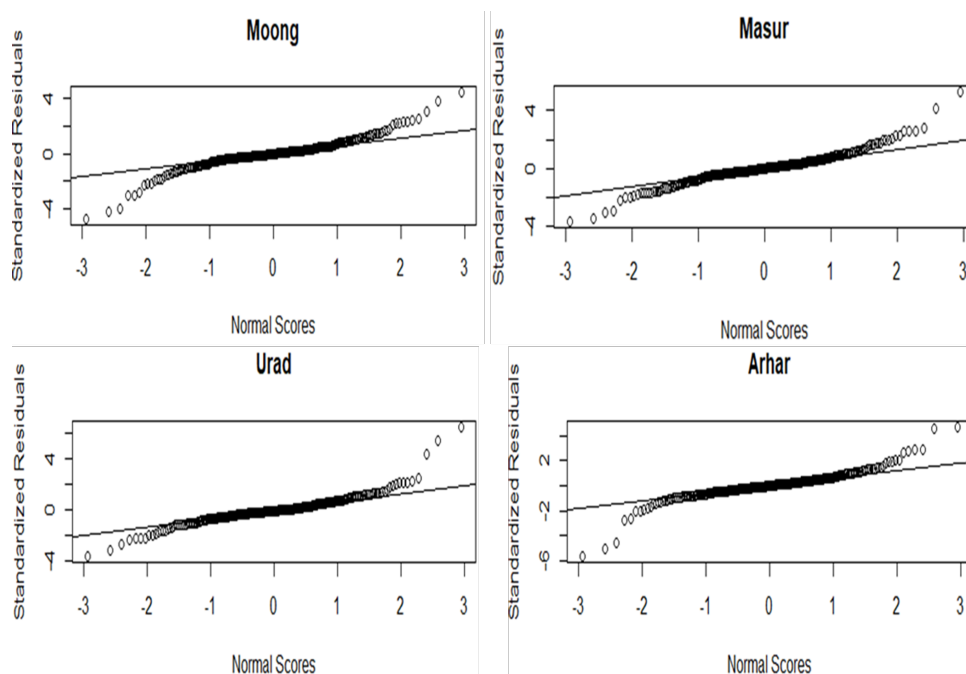


Figure 5: Q-Q plots of the residuals after fitting VAR(1) model

distribution to model leptokurtic pulses price series goes with the findings in literature (Ku, 2008).

Table 5: Residual diagnostics of the VAR (1) model

Test	Moong	Masur	Urad	Arhar
ARCH-LM	90.942*	13.482*	23.776*	23.776*
BDS	6.290*	3.726*	5.441*	4.161*
MARCH		241.467*		

*Significant at 5% level of significance

4. Discussions

The series under consideration were modelled following standard statistical procedures with VAR-DCC-MARCH models. VAR model was first applied for capturing the mean dependency of the multivariate framework. Residuals obtained from VAR model was used as an input for the DCC-MGARCH model for further analysis. In this study we have used the variant of DCC model after obtaining a positive result of DCC test. The DCC model parameters were estimated using the classical MLE method as well as the Bayesian method. The results from MLE estimates were not encouraging as all the parameters were not statistically significant. Thus result raises a very vital question upon the reliability of the model to explain the dynamic nature of correlation present between the series. Thus, to have further insight we decided to estimate the parameters using the Bayesian method. The results obtained through the Bayesian framework were very positive. As, looking into the parameters value along with its standard errors we can validate its statistical significance. The standard errors of each parameter under Bayesian framework are smaller as compared

Table 6: Parameter estimates of the DCC-MGARCH (1, 1) model

Parameters	Multivariate Normal error distribution		Multivariate t error distribution	
	MLE	Bayesian	MLE	Bayesian
ω_{Moong}	0.020 (0.024)	0.169 (0.007)	0.020 (0.024)	0.213 (0.014)
α_{Moong}	0.150 (0.037)	0.193 (0.005)	0.150 (0.037)	0.153 (0.011)
β_{Moong}	0.846* (0.046)	0.785 (0.005)	0.846* (0.046)	0.821 (0.010)
ω_{Masur}	0.059 (0.141)	0.573 (0.042)	0.059 (0.141)	0.511 (0.036)
α_{Masur}	0.06 (0.048)	0.229 (0.017)	0.06 (0.048)	0.163 (0.012)
β_{Masur}	0.937* (0.068)	0.729 (0.017)	0.937* (0.068)	0.774 (0.012)
ω_{Urad}	0.109 (0.506)	0.723 (0.029)	0.109 (0.505)	0.839 (0.072)
α_{Urad}	0.067 (0.099)	0.151 (0.006)	0.067 (0.099)	0.177 (0.009)
β_{Urad}	0.930* (0.152)	0.813 (0.007)	0.931* (0.151)	0.75 (0.013)
ω_{Arhar}	0.051 (0.078)	0.753 (0.037)	0.051 (0.078)	1.504 (0.089)
α_{Arhar}	0.189 (0.044)	0.285 (0.015)	0.189 (0.044)	0.247 (0.020)
β_{Arhar}	0.805* (0.043)	0.668 (0.014)	0.805* (0.043)	0.654 (0.011)
DCC_{α}	0.003 (0.007)	0.029 (0.002)	0.003 (0.008)	0.02 (0.002)
DCC_{β}	0.914* (0.024)	0.282 (0.011)	0.923* (0.043)	0.31 (0.037)

*Significant at 5% significance level. Values in the parenthesis are standard errors.

to that of MLE estimates as reported in Table 6.

Though we have modelled four series together, we can isolate the bivariate combinations for better interpretation of the DCC among them. A total of 6 such combinations were formed between them, namely Moong-Masur, Moong-Urad, Moong-Arhar, Masur-Urad, Masur-Arhar and Urad-Arhar. The dynamic correlation between each of the six combinations was studied under MLE and Bayesian framework. The mean, minimum and maximum values for each dynamic correlation are reported in Table 7 obtained from DCC-MGARCH model with multivariate t error distribution. The mean values for each combination are similar to 1 decimal place for MLE and Bayesian estimates but the maximum and minimum values differ noticeable for the two estimation procedure. The wider ranges of the dynamic correlation estimated under Bayesian DCC model indicates the efficiency of the model in capturing the dynamic co-movement among the series. Figure 6 depicts the DCC plots for all 6 combination of the series estimated under both techniques using multivariate t error. The figure highlights the inability of MLE estimates in capturing the co-movement efficiently over time between all the series as the correlation estimated do not fluctuate much over

time, unlike the fluctuations as noticed in the original series. This is mainly due to model parameters not being statistically significant and most importantly the DCC_{α} . Whereas the dynamic correlation exhibited by the Bayesian model seems to have explained the inherent fluctuations present in the original series. For instance, the Moong-Masur prices have fluctuated considerably between the period April, 2010 and March, 2016 similarly if we look into the DCC plot in Figure 6 a similar pattern of highly variable correlation is found between the series for the period. In Table 2 the dynamic correlation was found to be highest between Moong-Urad series (0.600) followed by Urad-Arhar (0.529), Masur-Arhar (0.483) and lowest among Moong-Masur (0.288). These dynamic correlation patterns fit with the linear correlation obtained between the stationary series.

Bayesian estimation comes with obvious advantages over the classical approach and has been put forward by our empirical illustration. As the Bayesian estimates are dependent upon the prior distribution and MCMC algorithm for generating posterior estimates, it is important to assess the sensitivity of the estimates with respect to the priors used and the MCMC samples drawn. In our implementation we have considered both these aspects. A total of 5 different prior settings under normal distribution has been tried with an increment of 0.1 in mean and 0.02 in the variance of the normal distribution. This has yielded us two fold results, first the sensitivity has been tested and second we could identify the optimal prior setting for our implementation. We document that the model estimates are quite robust with respect to the priors as we could only obtain slight changes in the estimates for prior 1 and prior 2 and rest of the prior settings (prior 3, prior 4 and prior 5) provided similar results as that of prior 2. Thus we have reported in Table 8 the estimates obtained from prior 1 and prior 2 settings only. After obtaining the optimal prior settings (prior 2) based on lower DIC value, we concentrate our efforts towards the sample size of MCMC draws. Two different sample sizes were tried 10,000 and 15,000 with 20% burn-in samples *i.e.*, 2000 and 3000 respectively. In the similar lines we could not find any significant difference among the estimates obtained. Hence, all the results presented for Bayesian estimates are obtained from 10,000 MCMC draws with 2,000 samples as burn-in.

As forecasting is essential for time series problems, however assessing forecasting performance in volatility problems is difficult as we are forecasting conditional covariance or correlation (Livingston and Nur, 2023). Hence we can compare the forecasted values of conditional variances with the true values and compute correlation among them as a measure of association among them (Table 9). The correlation between the original series and the forecasted conditional variances were found to be 0.67, -0.98, -0.67 and 0.89 respectively for Moong, Masur, Urad and Arhar.

5. Conclusions

In this study we have employed the VAR-DCC-MARCH model to volatile pulses price. In the first step the multivariate series is modelled using VAR model to capture the linear dependency and DCC-MGARCH model for capturing the dynamic co-movement among them. We document the superiority of the multivariate t innovations for modelling the leptokurtic behaviour of the series over the multivariate normal assumption of the errors. Our comparative analysis between the MLE and Bayesian estimation approach for DCC-MGARCH model provided comprehensive results highlighting the appropriateness of the Bayesian DCC-MGARCH model in terms of lower DIC values and standard errors of the

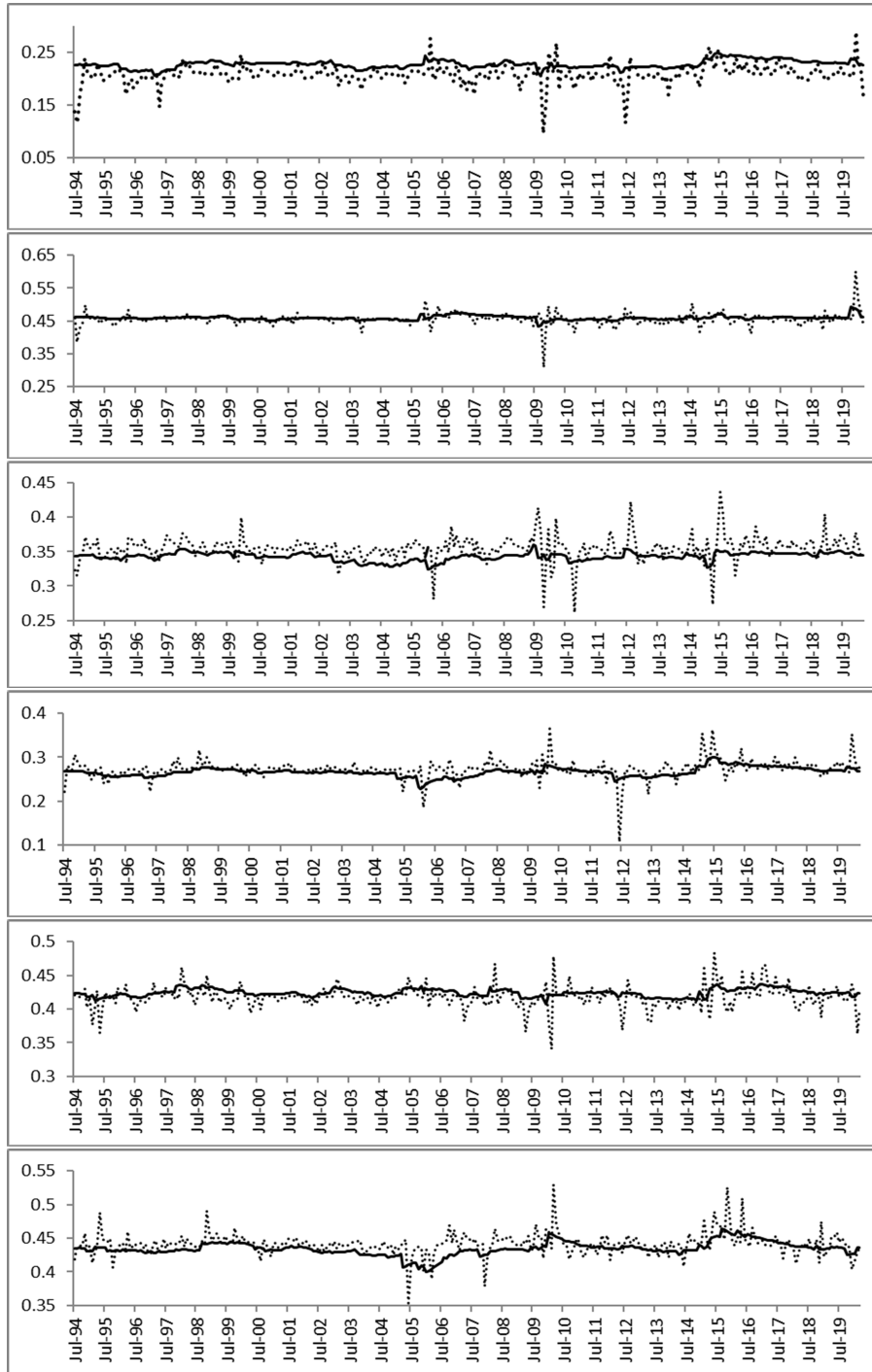


Figure 6: Dynamic conditional correlation plots obtained from MLE (in bold) and Bayesian (in dotted) using multivariate t error distribution of Moong-Masur, Moong-Urad, Moong-Arhar, Masur-Urad, Masur-Arhar, Urad and Arhar (top to bottom)

Table 7: Descriptive statistics of the dynamic conditional correlation

Series	Descriptive Statistics					
	Mean		Minimum		Maximum	
	MLE	Bayesian	MLE	Bayesian	MLE	Bayesian
Moong-Masur	0.227	0.208	0.205	0.095	0.249	0.288
Moong-Urad	0.458	0.455	0.433	0.307	0.492	0.600
Moong-Arhar	0.342	0.354	0.324	0.261	0.359	0.436
Masur- Urad	0.266	0.273	0.229	0.107	0.301	0.365
Masur- Arhar	0.424	0.417	0.407	0.340	0.437	0.483
Urad-Arhar	0.434	0.441	0.399	0.353	0.463	0.529

Table 8: Individual GARCH (1,1) estimates under different prior settings for Bayesian DCC-MGARCH model

Prior	Distribution	α_{Moong}	β_{Moong}	α_{Masur}	β_{Masur}	α_{Urad}	β_{Urad}	α_{Arhar}	β_{Arhar}	DIC
Prior 1	MVN	0.208	0.771	0.248	0.717	0.146	0.818	0.279	0.660	5708.382
Prior 2		0.193	0.785	0.229	0.729	0.151	0.813	0.285	0.668	5705.396
Prior 1	MVT	0.153	0.821	0.163	0.774	0.177	0.750	0.247	0.654	5553.357
Prior 2		0.153	0.821	0.163	0.774	0.177	0.750	0.247	0.654	5553.357

Table 9: Values of the original and forecasted conditional variance

Months	Original Values				Forecasted Conditional variance			
	Moong	Masur	Urad	Arhar	Moong	Masur	Urad	Arhar
Apr-20	161.9	166.6	180.4	155.4	20.1	43.8	42.6	12.7
May-20	167.1	167.8	187.5	156.7	19.5	41.6	40.6	13.6
Jun-20	163.7	172.6	183.8	158.3	19.0	39.7	38.9	14.3
Jul-20	161.4	175.3	181.5	159.8	18.5	37.8	37.3	14.9
Aug-20	158.4	175.9	180.7	160.6	18.1	36.2	35.9	15.4
Sep-20	152.7	177.0	177.4	165.5	17.7	34.7	34.7	15.8
Oct-20	151.2	181.2	181.3	173.3	17.3	33.3	33.6	16.1
Nov-20	153.5	185.1	187.9	174.2	16.9	32.0	32.6	16.5
Dec-20	156.6	182.3	190.8	168.8	16.6	30.8	31.7	16.8
Jan-21	155.0	184.1	193.8	166.5	16.3	29.7	30.9	17.0
Feb-21	157.8	185.1	196.4	172.9	16.0	28.7	30.1	17.3
Mar-21	157.3	187.4	196.5	174.7	15.7	27.7	29.4	17.5

model parameters. To add into the results, we also obtained the optimal prior setting for the dataset which is robust to the change in the distributional parameters and the sample size of the MCMC algorithm. The pulses series were found to have first order regressive relation among themselves. DCC-MGARCH models provided exciting insights into the dynamicity of the pulses. Results of this investigation have revealed the existence of dynamic co-movement among them with highest between Moong-Urad and lowest in Moong-Masur combinations. The findings of this study can better assist the stakeholders in formulating policy decisions focusing on enhancing income of the pulse cultivators of India. This study can be extended using different advanced MCMC algorithms and other multivariate volatility models.

Acknowledgements

The authors are grateful to the editor and anonymous reviewers for their valuable comments and suggestions.

Conflict of interest

The authors do not have any financial or non-financial conflict of interest to declare for the research work included in this article.

References

- Bauwens, L., Laurent, S., and Rombouts, J. V. K. (2006). Multivariate GARCH models: a survey. *Journal of Applied Econometrics*, **21**, 79–109.
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, **31**, 307–327.
- Bollerslev, T. (1990). Modeling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economic and Statistics*, **72**, 498–505.
- Chen, Y., Zheng, B., and Qu, F. (2020). Modeling the nexus of crude oil, new energy and rare earth in China: an asymmetric VAR-BEKK (DCC)-GARCH approach. *Resources Policy*, **65**, 101545.
- Chevallier, J. (2012). Time-varying correlations in oil, gas and CO2 prices: An application using BEKK, CCC and DCC-MAGARCH models. *Applied Economics*, **44**, 4257–4274.
- Engle, R. (2002). Dynamic conditional correlation. *Journal of Business & Economic Statistics*, **20**, 339–350.
- Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of U.K. inflation. *Econometrica*, **50**, 987–1008.
- Engle, R. F. and Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. *Econometric Theory*, **11**, 122–150.
- Engle, R. F. and Sheppard, K. (2001). Theoretical and empirical properties of dynamic conditional correlation multivariate garch. Working paper, National Bureau of Economic Research.
- Fama, E. F. (1965). The behavior of stock-market prices. *The Journal of Business*, **38**, 34.
- Fioruci, J. A., Ehlers, R. S., and Filho, M. G. A. (2014). Bayesian multivariate GARCH models with dynamic correlations and asymmetric error distributions. *Journal of Applied Statistics*, **41**, 320–331.
- Gupta, R., Kanda, P. T., Tiwari, A. K., and Wohar, M. E. (2019). Time-varying predictability of oil market movements over a century of data: The role of US financial stress. *The North American Journal of Economics and Finance*, **50**, 100994.
- Hall, P. and Yao, Q. (2003). Inference in ARCH and GARCH models with heavy-tailed errors. *Econometrica*, **71**, 285–317.
- Hull, J. and White, A. G. (1998). Incorporating volatility updating into the historical simulation method for value at risk. *Journal of Risk*, **1**, 5–19.

- Ku, Y.-H. H. (2008). Student-t distribution based VAR-MGARCH: an application of the DCC model on international portfolio risk management. *Applied Economics*, **40**, 1685–1697.
- Lama, A., Jha, G. K., Gurung, B., Paul, R. K., and Sinha, K. (2016). VAR-MGARCH models for volatility modelling pulses prices. *Journal of the Indian Society of Agricultural Statistics*, **70**, 145–151.
- Lama, A., Jha, G. K., Paul, R. K., and Gurung, B. (2015). Modelling and forecasting of price volatility: An application of GARCH and EGARCH models. *Agricultural Economics Research Review*, **28**, 73–82.
- Lean, H. H. and Teng, K. T. (2013). Integration of world leaders and emerging powers into the Malaysian stock market: A DCC-MGARCH approach. *Economic Modelling*, **32**, 333–342.
- Lin, B. and Li, J. (2015). The spillover effects across natural gas and oil markets: Based on the VEC-MGARCH framework. *Applied Energy*, **155**, 229–241.
- Livingston, G. C. and Nur, D. (2023). Bayesian inference of multivariate-GARCH-BEKK models. *Statistical Papers*, **64**, 1749–1774.
- Miazhynskaia, T. and Dorffner, G. (2006). A comparison of Bayesian model selection based on MCMC with an application to GARCH-type models. *Statistical Papers*, **47**, 525–549.
- Mohammadi, H. and Tan, Y. X. (2015). Return and volatility spillovers across equity markets in mainland China, Hong Kong and the United States. *Econometrics*, **3**, 215–232.
- Shiferaw, Y. A. (2019). Time-varying correlation between agricultural commodity and energy price dynamics with Bayesian multivariate DCC-GARCH models. *Physica A: Statistical Mechanics and its Applications*, **526**, 120807.
- Sims, C. A. (1977). Macroeconomics and reality. *Econometrica*, **48**, 1–48.

ANNEXURE

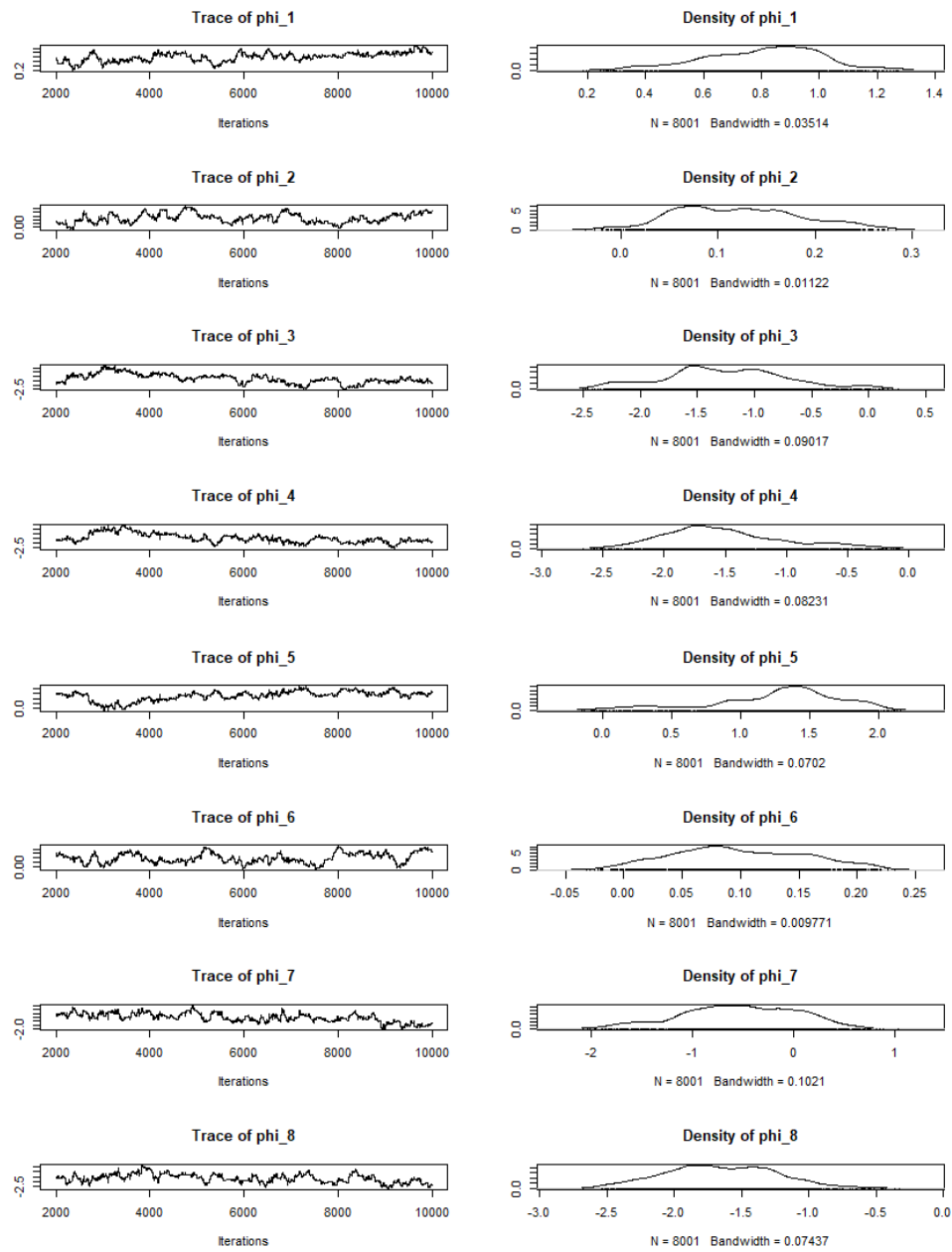


Figure 7: Trace plots

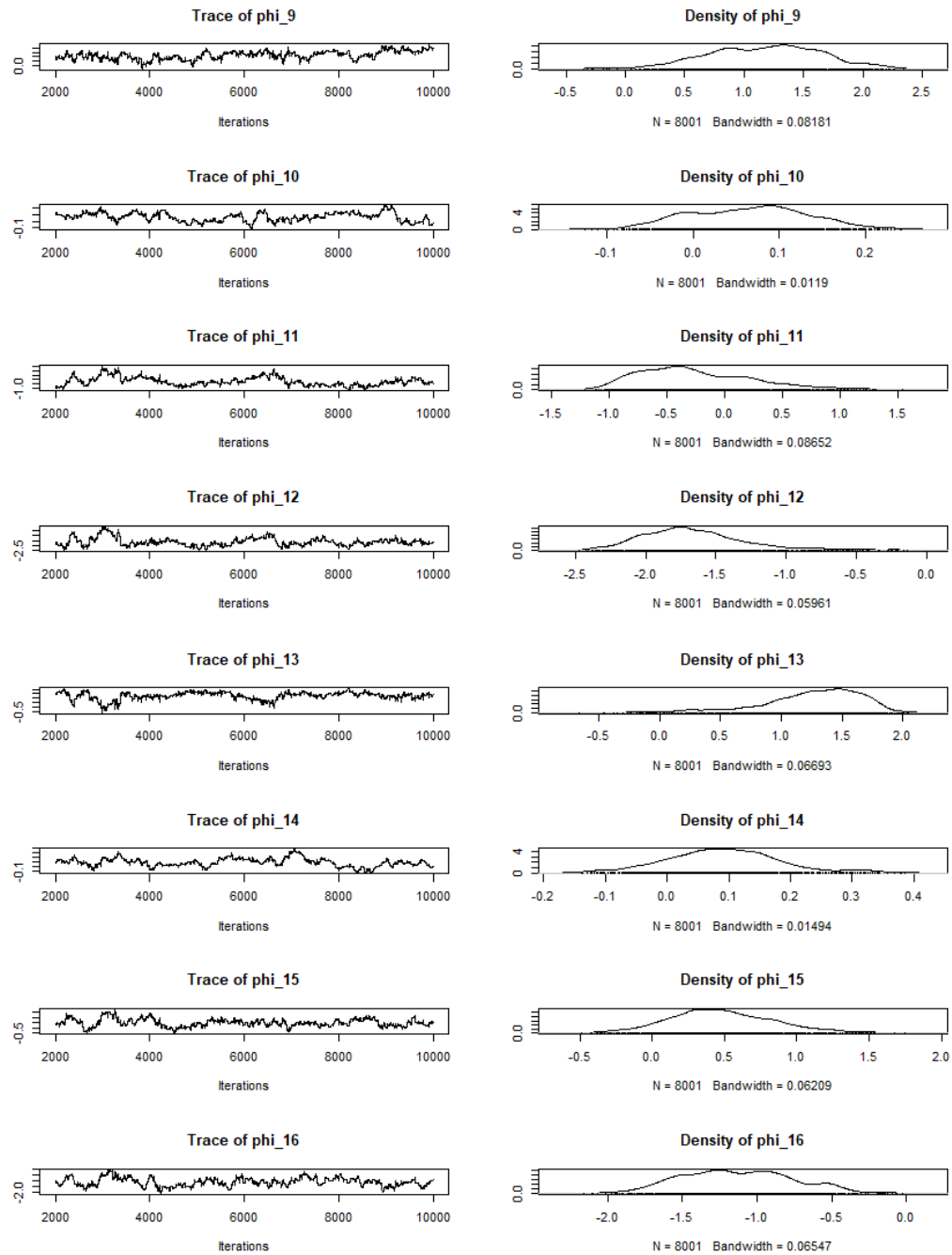


Figure 8: Trace plots

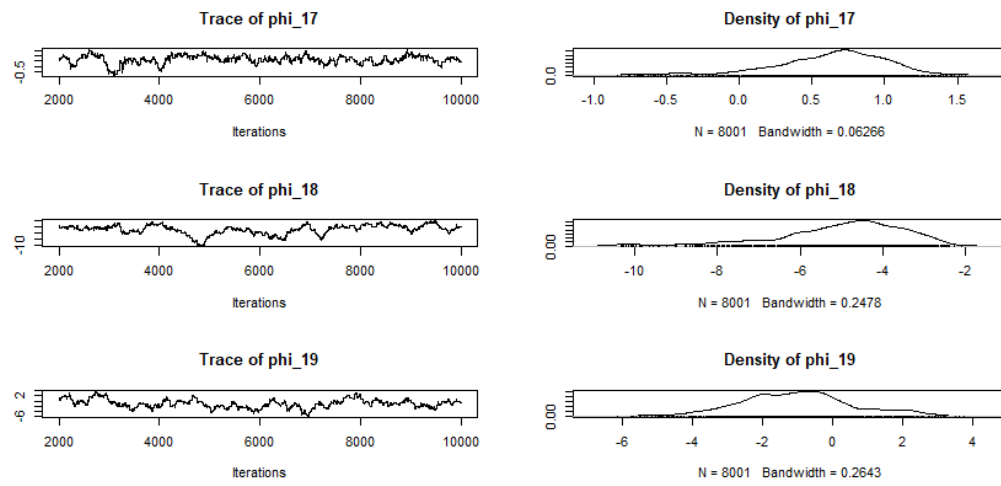


Figure 9: Trace plots