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Almost Unbiased Dual Exponential Type Estimators of Population Mean Using Auxiliary Information

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Abstract

In this paper dual exponential type estimators of population mean have been proposed. The large sample properties of the proposed estimators have been studied by obtaining the bias and mean square error (MSE) expressions. The proposed estimators under optimum conditions were found to be unbiased and more efficient than sample mean, ratio estimator of Cochran (1940), product estimator of Robson (1957), exponential ratio and product estimators of Bahl and Tuteja (1991), exponential ratio estimators of Singh *et al.* (2009) and the exponential product type estimators of Onyeka (2013). A numerical study has also been carried out to support the theoretical findings of the paper.

Key words: Dual estimator; Exponential estimator; Auxiliary variable; Unbiased estimator; Mean square error.

AMS Subject Classifications: 62K05, 05B05

1. Introduction

Whenever a researcher intends to get the precise estimates, the best choice is to make the wise use of auxiliary information. The auxiliary information can be used either at the design stage or at the estimation stage or at both stages. The ratio, product, difference and regression estimators are defined by using the available auxiliary information at the estimation stage. At this stage the auxiliary information may be available in the form of correlation coefficient, mean, median, coefficient of variation, skewness, kurtosis etc. The pioneer work for estimation of population mean using auxiliary information was done by Cochran (1940) while proposing classical ratio estimator, used when there is a high positive correlation between study variable (Y) and auxiliary variable (X) with the regression line passing through origin. If the correlation between Y and X is negative high, product method of estimation proposed by Robson (1957) can be used. While as the linear regression estimator is preferred when there is a very high (positive or negative) correlation between X and Y and the regression line of Y on X has intercept on y-axis. Many researchers such as

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Sisodia and Dwivedi (1981), Singh and Tailor (2003), Khoshnevisan *et al.* (2007), Sharma and Bhatnagar (2008), Sharma *et al.* (2010), Yadav and Kadilar (2013), Kumar *et al.* (2018) and others proposed modified ratio or product type estimators by utilizing different known values of the parameters of auxiliary variable and these estimators have gained relevance in estimation theory because of their improved precision than conventional ratio and product estimators. The modified ratio and product estimators can work well only when correlation between Y and X is high, therefore Bahl and Tuteja (1991) proposed exponential ratio and product type estimators and these estimators can be employed even when there is not a high degree of correlation between X and Y. Later Singh *et al.* (2007, 2009), Onyeka (2013), Yasmeen *et al.* (2016), Panigrahi and Mishra (2017) and Hussain *et al.* (2021) proposed some improved versions of the exponential ratio and product type estimators.

On taking a note of the above discussion, it was observed that most of the estimators available in the literature are biased. They lack the very first property of a good estimator which may lead to over or under estimation of the population mean. Therefore, the authors Singh and Singh (1993), Yadav *et al.* (2012), Singh *et al.* (2016) and others worked in this direction and proposed almost unbiased estimators of population mean. Further, it is also observed that for positively correlated variables ratio estimators are used and for negatively correlated variables product estimators are used. So the authors Singh *et al.* (2009*a*), Tailor and Sharma (2009), Sharma and Tailor (2010), Tailor *et al.* (2012) and others proposed ratio cum product estimators which can be employed for both positively and negatively correlated variables. It is also observed that the exponential estimators can also be employed for low degree of correlation. By keeping the stated points in view, two almost unbiased dual exponential type estimators of population mean have been proposed in the paper.

Consider a finite population containing N number of units in total and draw a random sample of size n $(n < N)$ by simple random sampling without replacement (SRSWOR) sampling scheme. Associated with every unit, there are two variables Y and X, the population mean of X is assumed to be known. The sample mean of Y and X i.e $\bar{y} = \frac{1}{n}\sum_i^n y_i$ and $\bar{x} = \frac{1}{n}\sum_i^n x_i$ are the unbiased estimates of $\bar{X} = \frac{1}{N}\sum_i^N X_i$ and $\bar{Y} = \frac{1}{N}\sum_i^N Y_i$ respectively. Other formula and notations that are used in the paper (John and Inyang (2015)) are as

Study Variable	**Auxiliary Variable**
$C_y = \frac{S_y}{\bar{Y}}$ is the coefficient of variation .	: $C_x = \frac{S_x}{\bar{X}}$ is the coefficient of variation.
$S_y^2 = \frac{1}{N-1}\sum_{i=1}^N (Y_i - \bar{Y})^2$ is the population mean square.	: $S_x^2 = \frac{1}{N-1}\sum_{i=1}^N (X_i - \bar{X})^2$ is the population mean square.
$s_y^2 = \frac{1}{n-1}\sum_{i=1}^n (y_i - \bar{y})^2$ is the sample mean square.	: $s_x^2 = \frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{x})^2$ is the sample mean square.

Further,
$S_{yx} = \frac{1}{N-1}\sum_{i=1}^N (Y_i - \bar{Y})(X_i - \bar{X})$ is the population covariance between Y and X.
$s_{yx} = \frac{1}{n-1}\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})$ is the sample covariance between y and x.
$\rho = \frac{S_{xy}}{\sqrt{S_x^2 S_y^2}}$ is the population correlation coefficient between X and Y.
$\theta = \frac{a\bar{X}}{2(a\bar{X}+b)}$ and $\gamma = \frac{1-f}{n}$, where the sampling fraction $f = \frac{n}{N}$.

The Percent relative efficiency (PRE) of the estimators is obtained using the formula as PRE $= \frac{MSE\ of\ existing\ estimator}{MSE\ of\ proposed\ estimator} \times 100$

2. Review of Some Existing Ratio and Product Type Estimators

The sample mean estimator is

$$t_1 = \frac{1}{n}\sum_{i=1}^{n} y_i.$$

With the Bias and MSE are as

$$Bias(t_1) = 0. \tag{1}$$

$$MSE(t_1) = \gamma \bar{Y}^2 C_y^2. \tag{2}$$

The ratio estimator proposed by Cochran (1940) which is more efficient than the estimator t_1, if $\frac{C_x}{2C_y} < \rho \leq +1$ is

$$t_2 = \bar{y}\frac{\bar{X}}{\bar{x}}.$$

The expressions of Bias and MSE for the estimator t_2 are as

$$Bias(t_2) = \gamma \bar{Y}(C_x^2 - C_{yx}). \tag{3}$$

$$MSE(t_2) = \gamma \bar{Y}^2(C_y^2 + C_x^2 - 2C_{yx}). \tag{4}$$

When the variables X and Y are negatively correlated and $-1 \leq \rho < -\frac{C_x}{2C_y}$, Robson (1957) proposed product estimator of population. The main advantage of this estimator is that the exact expressions of Bias and mean squared error were obtained. The proposed estimator is given as

$$t_3 = \bar{y}\frac{\bar{x}}{\bar{X}}.$$

The Bias and MSE expressions are as

$$Bias(t_3) = \gamma \bar{Y} C_{yx}. \tag{5}$$

$$MSE(t_3) = \gamma \bar{Y}^2(C_y^2 + C_x^2 + 2C_{yx}). \tag{6}$$

The exponential ratio (t_4) and product (t_5) type estimators proposed by Bahl and Tuteja (1991) which are efficient even when there is a low degree of correlation between X and Y are as

$$t_4 = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right),$$

with the Bias and MSE as

$$Bias(t_4) = \gamma \bar{Y}\left(\frac{3}{8}C_x^2 - \frac{1}{2}C_{yx}\right), \tag{7}$$

$$MSE(t_4) = \gamma \bar{Y}^2\left(C_y^2 + \frac{C_x^2}{4} - C_{yx}\right), \tag{8}$$

and

$$t_5 = \bar{y} \exp\left(\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}}\right),$$

with the Bias and MSE as

$$Bias(t_5) = \gamma \bar{Y}\left(\frac{1}{2}C_{yx} - \frac{1}{8}C_x^2\right), \tag{9}$$

$$MSE(t_5) = \gamma \bar{Y}^2\left(C_y^2 + \frac{C_x^2}{4} + C_{yx}\right). \tag{10}$$

A class of modified exponential ratio estimators proposed by Singh *et al.* (2009) using the constants a($\neq 0$) and b, where a and b are either the real number or functions of some known parameters of auxiliary variable such as coefficient of variation, skewness, correlation etc. The proposed estimators are as

$$t_6 = \bar{y} \exp\left[\frac{(a\bar{X} + b) - (a\bar{x} - b)}{(a\bar{X} + b) + (a\bar{x} - b)}\right].$$

The Bias and MSE expressions of t_6 are as

$$Bias(t_6) = \gamma \bar{Y}(\theta^2 C_x^2 - \theta C_{yx}). \tag{11}$$

$$MSE(t_6) = \gamma \bar{Y}^2(C_y^2 + \theta^2 C_x^2 - 2\theta C_{yx}). \tag{12}$$

Onyeka (2013) proposed a class of product type estimators as

$$t_7 = \bar{y} \exp\left[\frac{(a\bar{X} + b) - (a\bar{x} + b)}{(a\bar{X} + b) + (a\bar{x} + b)}\right].$$

The expressions of Bias and MSE for the estimator t_7 are as

$$Bias(t_7) = \gamma \bar{Y}\left(\frac{1}{2}\theta C_{yx} - \frac{1}{8}\theta^2 C_x^2\right). \tag{13}$$

$$MSE(t_7) = \gamma \bar{Y}^2\left(C_y^2 + \frac{1}{4}\theta^2 C_x^2 + \theta C_{yx}\right). \tag{14}$$

3. Proposed Estimators

The proposed dual type exponential estimators are as

$$t_{de1} = \bar{y}\left[\alpha \exp\left(\frac{\bar{X} - \bar{x}}{p\bar{X}}\right) + (1-\alpha)\exp\left(\frac{\bar{x} - \bar{X}}{p\bar{X}}\right)\right] \quad \text{and}$$

$$t_{de2} = \bar{y}\left[\beta \exp\left(\frac{\bar{X} - \bar{x}}{q\bar{x}}\right) + (1-\beta)\exp\left(\frac{\bar{x} - \bar{X}}{q\bar{x}}\right)\right].$$

Where p and q are non zero constants whose value is chosen such that the estimators t_{de1} and t_{de2} should be unbiased. The value of constants α and β are chosen such that the MSE of t_{de1} and t_{de2} is minimum. The expressions of Bias and MSE are obtained through the

following methodology.
Let

$$e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \qquad ; \qquad e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}.$$

Therefore, the following expected values are obtained

$$E(e_0) = E(e_1) = 0.$$
$$E(e_0^2) = \gamma C_y^2; \qquad E(e_1^2) = \gamma C_x^2; \qquad E(e_0 e_1) = \gamma \rho C_y C_x.$$

On writing the estimator t_{de1} and t_{de2} in terms of $e_i (i = 0, 1)$, the following equations are obtained as

$$t_{de1} = \bar{Y}(1 + e_0) \left[\alpha \exp\left(\frac{-e_1}{p} \right) + (1 - \alpha) \exp\left(\frac{e_1}{p} \right) \right]. \tag{15}$$

$$t_{de2} = \bar{Y}(1 + e_0) \left[\beta \exp\left(\frac{-e_1}{q}(1 + e_1)^{-1} \right) + (1 - \beta) \exp\left(\frac{e_1}{q}(1 + e_1)^{-1} \right) \right]. \tag{16}$$

After solving equation (15) & (16) and keeping the terms up 2^{nd} degree, the equations reduce to

$$t_{de1} = \bar{Y} \left[1 + e_0 + (1 - 2\alpha)\frac{e_1}{p} + \frac{e_1^2}{2p^2} + (1 - 2\alpha)\frac{e_0 e_1}{p} \right]$$

$$\Rightarrow t_{de1} - \bar{Y} = \left[e_0 + (1 - 2\alpha)\frac{e_1}{p} + \frac{e_1^2}{2p^2} + (1 - 2\alpha)\frac{e_0 e_1}{p} \right]. \tag{17}$$

$$t_{de2} = \bar{Y} \left[1 + e_0 + (1 - 2\beta)\frac{e_1}{q} + \left(\frac{1}{2q} + 2\beta - 1 \right) \frac{e_1^2}{q} + (1 - 2\beta)\frac{e_0 e_1}{q} \right]$$

$$\Rightarrow t_{de2} - \bar{Y} = \left[e_0 + (1 - 2\beta)\frac{e_1}{q} + \left(\frac{1}{2q} + 2\beta - 1 \right) \frac{e_1^2}{q} + (1 - 2\beta)\frac{e_0 e_1}{q} \right]. \tag{18}$$

Now taking expectation on both sides of (17) and (18), the bias of the estimators t_{de1} and t_{de2} is obtained as

$$Bias(t_{de1}) = \gamma \bar{Y} \left[\frac{1}{2p^2} C_x^2 + \frac{1}{p}(1 - 2\alpha)\rho C_x C_y \right] \text{ and} \tag{19}$$

$$Bias(t_{de2}) = \gamma \bar{Y} \frac{1}{q} \left[\left(\frac{1}{2q} + 2\beta - 1 \right) C_x^2 + (1 - 2\beta)\rho C_x C_y \right] \text{ respectively.} \tag{20}$$

On squaring the equations (17) & (18) and taking expectation on both sides. After solving and retaining the terms up to 2^{nd} degree only, the mean square error of t_{de1} and t_{de2} is obtained as

$$MSE(t_{de1}) = \gamma \bar{Y}^2 \left[C_y^2 + (1 - 2\alpha)^2 \frac{C_x^2}{p^2} + \frac{2}{p}(1 - 2\alpha)\rho C_x C_y \right] \text{ and} \tag{21}$$

$$MSE(t_{de2}) = \gamma \bar{Y}^2 \left[C_y^2 + (1 - 2\beta)^2 \frac{C_x^2}{q^2} + \frac{2}{q}(1 - 2\beta)\rho C_x C_y \right] \text{ respectively.} \tag{22}$$

The estimator t_{de1} is unbiased, if

$$p = \frac{C_x}{2(2\alpha - 1)\rho C_y}, \tag{23}$$

and the estimator t_{de2} is unbiased, if

$$q = \frac{C_x}{2(2\beta - 1)(\rho C_y - C_x)}. \tag{24}$$

Substituting the values of (23) and (24) in equations (21) and (22) respectively, the following equations are obtained as

$$MSE(t_{de1}) = \gamma \bar{Y}^2 \left[C_y^2 + 4(1-2\alpha)^4 \rho^2 C_y^2 - 4(1-2\alpha)^2 \rho^2 C_y^2\right]. \tag{25}$$

$$MSE(t_{de2}) = \gamma \bar{Y}^2 \left[C_y^2 + 4(1-2\beta)^4 (\rho C_y - C_x)^2 - 4(1-2\beta)^2 (\rho C_y - C_x)\rho C_y\right]. \tag{26}$$

For obtaining optimum value of α and β differentiate equations (25) and (26) with respect to α and β respectively and equating to zero.

$$\text{Therefore, } \alpha = 0.146, 0.854 \qquad \text{and} \qquad \beta = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{\rho C_y}{2(\rho C_y - C_x)}}.$$

The value of unknown quantity (C_y) used to find the values of p, q and β can be obtained quite accurately from some previous survey or from the experience of the researcher (See Reddy (1974), Singh and Vishwakarma (2008), Singh and Kapre (2010)). Now by using the values of α (0.146 or 0.854) and β $\left(\frac{1}{2} + \frac{1}{2}\sqrt{\frac{\rho C_y}{2(\rho C_y - C_x)}} \text{ or } \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\rho C_y}{2(\rho C_y - C_x)}}\right)$ in equations (25) and (26) respectively, the minimum value of MSE of the estimators t_{de1} and t_{de2} is obtained as

$$MSE_{min}(t_{dei}) = \gamma \bar{Y}^2 C_y^2 (1 - \rho^2). \qquad i = 1, 2 \tag{27}$$

4. Efficiency Comparisons

From equations (2), (4), (6), (8), (10), (12), (14) and (27), the conditions under which the proposed estimators will be preferred for better precision are obtained as

$$MSE_{min}(t_{dei}) < V(t_1)$$

$$\Rightarrow \gamma \bar{Y}^2 C_y^2 (1 - \rho^2) < \gamma \bar{Y}^2 C_y^2, \ if \ \rho^2 \bar{Y}^2 > 0. \tag{28}$$

$$MSE_{min}(t_{dei}) < MSE(t_2)$$

$$\Rightarrow \gamma \bar{Y}^2 C_y^2 (1 - \rho^2) < \gamma \bar{Y}^2 (C_y^2 + C_x^2 - 2C_{yx}), \ if \ (\rho C_y - C_x)^2 > 0. \tag{29}$$

$$MSE_{min}(t_{dei}) < MSE(t_3)$$

$$\Rightarrow \gamma \bar{Y}^2 C_y^2 (1 - \rho^2) < \gamma \bar{Y}^2 (C_y^2 + C_x^2 + 2C_{yx}), \ if \ (\rho C_y + C_x)^2 > 0. \tag{30}$$

$$MSE_{min}(t_{dei}) < MSE(t_4)$$

$$\Rightarrow \gamma \bar{Y}^2 C_y^2 (1 - \rho^2) < \gamma \bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} - C_{yx}\right), \ if \ (2\rho C_y - C_x)^2 > 0. \tag{31}$$

$$MSE_{min}(t_{dei}) < MSE(t_5)$$

$$\Rightarrow \gamma \bar{Y}^2 C_y^2(1-\rho^2) < \gamma \bar{Y}^2\left(C_y^2+\frac{C_x^2}{4}+C_{yx}\right),\ if\ (2\rho C_y+C_x)^2>0. \quad (32)$$

$$MSE_{min}(t_{dei}) < MSE(t_6)$$

$$\Rightarrow \gamma \bar{Y}^2 C_y^2(1-\rho^2) < \gamma \bar{Y}^2(C_y^2+\theta^2C_x^2-2\theta C_{yx}),\ if\ (\rho C_y-\theta C_x)^2>0. \quad (33)$$

$$MSE_{min}(t_{dei}) < MSE(t_7)$$

$$\Rightarrow \gamma \bar{Y}^2 C_y^2(1-\rho^2) < \gamma \bar{Y}^2\left(C_y^2+\frac{1}{4}\theta^2C_x^2+\theta C_{yx}\right),\ if\ (2\rho C_y-\theta C_x)^2>0. \quad (34)$$

The conditions (28) to (34) hold, therefore the estimators $t_{dei}(i=1,2)$ are more efficient than $t_1, t_2, t_3, t_4, t_5, t_6$ and t_7.

5. Numerical Illustration

The performance of the estimators proposed and considered for comparison in the paper have been evaluated by using the data of four populations P1, P2, P3 and P4 (See Table 1). In the populations P1 and P2, the variables X and Y are positively correlated while as for P3 and P4 are negatively correlated. The source of the population P1 is Sukhatme and Chand (1977) where the variable Y represents the apple trees of bearing age in 1964 and the variable X represents bushels harvested in 1964. The population P2 is from Murthy (1967), where the variable Y is fixed capital and the variable X is the output of 80 factories. The source of population P3 is Onyeka (2013) where the variable Y represents percentage of hives affected by disease and X the date of flowering of a particular summer species (no. of days from Jan. 1). The population P4 is from Gujarati (2004) where the variable Y is average miles per gallon and the variable X is top speed, miles per hour.

Table 1: Summary statistics of the population data sets.

Population	N	n	$\bar{Y}$	$\bar{X}$	C_y	C_x	ρ
P1	200	20	1031.82	2934.58	1.598	2.006	0.93
P2	80	20	11.264	51.826	0.750	0.354	0.94
P3	10	4	52	200	0.156	0.046	−0.94
P4	81	13	33.835	112.457	0.297	0.126	−0.69

Table 2: MSE, Bias and PRE of the estimators t_1, t_2, t_4, t_6 and t_{dei}.

Estimator	Population					
	P1			P2		
	MSE	$\mid Bias \mid$	PRE	MSE	$\mid Bias \mid$	PRE
t_1	122341.540	0.000	100.000	2.676	0.000	100.000
t_2	29476.060	48.421	415.054	0.898	0.052	297.995
t_4	27711.550	0.855	441.482	1.638	0.033	163.379
t_6	27745.680	22.518	440.939	1.644	0.039	162.773
t_{dei}	16528.340	0.000	740.192	0.311	0.000	860.450

It can be observed from Table-2 that the proposed estimators have minimum MSE amongst sample mean estimator (t_1) and the ratio estimators t_2, t_4, t_6 considered. The percent relative efficiency (PRE) of the proposed estimators is highest among all other estimators considered.

Table 3: MSE, Bias and PRE of the estimators t_1, t_3, t_5, t_7 and t_{dei}.

Estimator	Population					
	P3			P4		
	MSE	$\mid Bias \mid$	PRE	MSE	$\mid Bias \mid$	PRE
t_1	9.871	0.000	100.000	6.521	0.000	100.000
t_3	5.257	0.053	187.769	3.877	0.056	168.197
t_5	7.349	0.028	134.318	4.906	0.033	132.919
t_7	8.556	0.014	115.369	5.639	0.015	115.641
t_{dei}	1.149	0.000	859.095	3.416	0.000	190.896

Table-3 depicts that the proposed estimators have minimum MSE amongst sample mean estimator (t_1) and product estimators t_3, t_5, t_7 considered and are also unbiased. The percent relative efficiency (PRE) of proposed estimators is highest among all other estimators considered.

Thus, it can be concluded from Table-2 and Table-3 that the proposed estimators are unbiased and work efficiently in estimating the population mean irrespective of negative or positive correlation between the study and auxiliary variable.

6. Discussion

The population mean can be estimated using the proposed dual exponential type estimators of population mean by plugging in the values of $p = \frac{C_x}{2(2\alpha-1)\rho C_y}$, $q = \frac{C_x}{2(2\beta-1)(\rho C_y - C_x)}$ and the optimum values of α (0.146 or 0.854) and β $\left(\frac{1}{2} + \frac{1}{2}\sqrt{\frac{\rho C_y}{2(\rho C_y - C_x)}} \text{ or } \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\rho C_y}{2(\rho C_y - C_x)}}\right)$ in the respective estimators t_{ue1} and t_{ue2}. The constants p, q, β are dynamic in nature and therefore depend upon the parameters of population data whereas the value of constant α is static and results an estimator as

$$t_{de1} = \bar{y}\left[0.854\exp\left(\frac{1.414\rho C_y(\bar{X}-\bar{x})}{C_x\bar{X}}\right) + 0.146\exp\left(\frac{1.414\rho C_y(\bar{x}-\bar{X})}{C_x\bar{X}}\right)\right].$$

7. Conclusion

- The proposed almost unbiased dual exponential type estimators of population mean are as

$$t_{de1} = \bar{y}\left[\alpha\exp\left(\frac{\bar{X}-\bar{x}}{p\bar{X}}\right) + (1-\alpha)\exp\left(\frac{\bar{x}-\bar{X}}{p\bar{X}}\right)\right].$$

$$t_{de2} = \bar{y}\left[\beta\exp\left(\frac{\bar{X}-\bar{x}}{q\bar{x}}\right) + (1-\beta)\exp\left(\frac{\bar{x}-\bar{X}}{q\bar{x}}\right)\right].$$

- The proposed estimators are always more efficient than sample mean, ratio estimator of Cochran (1970), product estimator of Robson (1957), exponential ratio and product estimators of Bahl and Tuteja (1991), exponential ratio estimators of Singh *et al.* (2009) and the exponential product type estimators of Onyeka (2013).

- The proposed estimators t_{de1} and t_{de2} are unbiased and can be used for both positively and negatively correlated variables equal efficiently.

References

Bahl, S. and Tuteja, R. K. (1991). Ratio and product type exponential estimators. *Information and Optimization Sciences*, **1**, 159-163.

Cochran, W. G. (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. *The Journal of Agricultural Science*, **30**, 262-275.

Gujarati, D. N. (2004). *Basic Econometrics.* McGraw-Hill Companies, New York.

Hussain, S., Sharma, M. and Bhat, M. I. J. (2021). Optimum exponential ratio type estimators for estimating the population mean. *Journal of Statistics Applications and Probability Letters*, **8**(**2**), 73-82.

John, E. E and Inyang, E. E. (2015). Efficient exponential ratio estimators for estimating the population mean in simple random sampling. *Hacettepe Journal of Mathematics and Statistics*, **44**(**3**), 689-705.

Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007). A general family of estimators for estimating population mean using known value of some population parameters. *Far East Journal of Theoretical Statistics*, **22**, 181–191.

Kumar, B., Kumar, M., Rizvi, S. E. H. and Bhat, M. I. J. (2018). Estimation of population mean through improved class of ratio type estimators using auxiliary information. *Journal of Relability and Statistical Studies*, **11**(**1**), 01-07.

Murthy, M. N. (1967). *Sampling Theory and Methods.* Calcutta India, Statistical Publishing Society.

Onyeka, A. C. (2013). A class of product-type exponential estimators of the population mean in simple random sampling scheme. *Statistics in Transition-New Series*, **14**(**2**), 189-200.

Panigrahi, A. and Mishra, G. (2017). Improved exponential product type estimators of finite population mean. *Open Acess International Journal of Science and Engineering*, **2**(**7**), 25-28.

Robson, D. S. (1957). Applications of multivariate polykays to the theory of unbiased ratio type estimation. *Journal of the American Statistical Association*, **52**, 511–522.

Reddy, V. N. (1974). On a transformed ratio method of estimation. *Sankhya*, **C36**, 59–70.

Sharma, B. and Tailor, R. (2010). A new ratio-cum-dual to ratio estimator of finite population mean in simple random sampling. *Global Journal of Science Frontier Research*, **10**(**1**), 27–31.

Sukhatme, B. and Chand, L. (1977), Multivariate ratio-type estimators in proceedings of the social statistics section. *American Statistical Association, Michigan*, 927–931.

Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007). Improvement in estimating the population mean using exponential estimator in simple random sampling. *Renaissance High Press Ann Arbor USA*, 33-40.

Sisodia, B. V. S. and Dwivedi, V. K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. *Journal of the Indian Society of Agricultural Statistics*, **33**(**2**), 13-18.

Singh, H. P. and Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population mean. *Statistics in Transition*, **6**(**4**), 555-560.

Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2009). Improvement in estimating the population mean using exponential estimators in simple random sampling. *Bulletin of Statistics and Economics*, **3**(**A09**), 13-18.

Singh, H. P., Upahayaya, L. N. and Tailor, R. (2009a). Ratio-cum-product type exponential estimator. *Statistica*, **69**(**4**), 299-310.

Singh, R., Gupta, S. B. and Malik, S. (2016). Almost unbiased estimator using known value of population parameter(s) in sample surveys. *Journal of Modern Applied Statistical Methods*, **15**(**1**), 600-615.

Singh, S. and Singh, R. (1993). A new method: Almost seperation of bias precipitates in sample survey. *Journal of the Indian Statistical Association*, **31**, 99-105.

Sharma, M. and Bhatnagar, S. (2008). A general class of estimators for estimating population mean through auxiliary information. *Journal of Indian Statistical Association*, **46**(**2**), 155-162.

Sharma, M., Bhatnagar, S. and Imran, K., (2010), A general class of improved ratio type estimators through auxiliary information. *International Journal of Agricultural and Statistical Sciences*, **6**(**1**), 115-118.

Singh, H. P. and Karpe, N. (2010). Estimation of mean, ratio and product using auxiliary information in the presence of measurement errors in sample surveys. *Journal of Statistical Theory and Practice*, **4**(**1**), 111–136.

Singh, H. P. and Vishwakarma, G. K. (2008). Some families of estimators of variance of stratified random sample mean using auxiliary information. *Journal of Statistical Theory and Practice*, **2(1)**, 21–43.

Tailor, R. and Sharma, B. K. (2009). A modified ratio-cum-product estimator of finite population mean using known coefficient of variation and coefficient of kurtosis. *Statistics in Transition-New Series*, **10**(**1**), 15–24.

Tailor, R., Tailor, R., Parmar, R. and Kumar, M. (2012). Dual to ratio-cum-product estimator using known parameters of auxiliary variables. *Journal of Reliability and Statistical Studies*, **5**(**1**), 65–71.

Yasmeen, U., Noor ul Amin, M. and Hanif, M. (2016). Exponential ratio and product type estimators of population mean. *Journal of Statistics and Management System*, **19**(**1**), 55-71.

Yadav, R., Upadhayaya, L. N., Singh, H. P. and Chatterjee, S. (2012). Almost unbiased ratio and product type exponential estimators. *Statistics in Transition-New Series*, **13**(**3**), 537-550.

Yadav, S. K. and Kadilar, C. (2013). Improved class of ratio and product estimators. *Applied Mathematics and Computation*, **219**(**22**), 10726-10731.