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A Hybrid ARIMA-GARCH Type Copula Approach for Agricultural Price Forecasting

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Abstract

Agricultural commodity prices frequently exhibit inherent noise and volatility, attributable to market dynamics. This paper undertakes a comprehensive analysis of price volatility concerning key oil seed crops (Safflower, Mustard, Groundnut) and pulses (Lentil, Chickpea, Green gram) across two markets for each commodity in the Indian agricultural sector. The present study aims to improve the accuracy of price forecasting by utilizing the Bivariate Auto Regressive Integrated Moving Average (ARIMA)-Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) type-Copula model. Monthly agricultural commodity price datasets for key oil seed crops and pulse crops spanning January 2010 to December 2022 have been used to evaluate the predictive performance of this model. Comparative evaluations are carried out against conventional time series models, namely Multivariate GARCH (MGARCH)-Dynamic Conditional Correlation (DCC) model and the Univariate ARIMA-GARCH model. Empirical findings demonstrate that the Bivariate ARIMA-GARCH type-Copula model outperformed the conventional time series models considered in forecasting performance. This superiority is evidenced by evaluation metrics, including Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). Moreover, this study utilized the Diebold–Mariano test to highlight the predictive accuracy of the Bivariate ARIMA-GARCH type-Copula model for the dataset under consideration, surpassing conventional time series models. The integration of Copulas with the ARIMA-GARCH type model shows promise for enhancing price forecasting accuracy, offering valuable insights for researchers and policymakers navigating the dynamic agricultural market landscape in India.

Key words: Gaussian Copula; Simulation; Student t-Copula; Time Series; Volatility.

AMS Subject Classifications: 62K05, 05B05

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1. Introduction

Agriculture is a crucial component of the Indian economy, supporting over 47% of the population's livelihood. As stated in the 2022-23 Indian Economic Survey, the agricultural sector has demonstrated consistent growth, averaging an annual rate of 4.6% over the past six years. Agricultural commodities price data are often characterized by inherent noise and volatility due to the nature of the market. This is largely due to the rapid response of these prices to changes in supply and demand conditions, as well as the impact of weather-related factors on farm production. Moreover, asymmetric phenomena can also arise in price series, where prices tend to behave differently during economic downturns as opposed to periods of growth. It is common for agricultural price series to exhibit periods of stability, followed by periods of high volatility. These fluctuations are a common feature of the agricultural commodity market. Monitoring volatility in agricultural commodity prices can have a significant impact on a nation's overall economic performance. As such, agricultural commodity price forecasts are essential in enabling decision-makers to formulate economic policies and strategies that are in line with anticipated changes (Bhardwaj *et al.* (2014)).

One of the predominant statistical methodologies employed in forecasting price series is the Auto Regressive Integrated Moving Average (ARIMA) model as established by Box and Jenkins (1970). Nevertheless, the inherent assumptions of linearity and homoscedastic error variance within the ARIMA framework might not adequately accommodate the forecasting challenges posed by volatile agricultural commodity prices. In recognition of this limitation, Engle (1982) introduced the Auto Regressive Conditional Heteroscedastic (ARCH) model, subsequently refined by Bollerslev (1986) into the Generalized ARCH (GARCH) model. Volatility within agricultural commodity price series often exhibit both symmetric and asymmetric patterns. Although the GARCH model effectively captures the magnitude of shocks, it may not fully capture the directional characteristics of these shocks. Consequently, alternative asymmetric GARCH-type models have been devised, such as the Exponential GARCH (EGARCH) model proposed by Nelson (1991), the GJR-GARCH model introduced by Glosten *et al.* (1993), and the Asymmetric Power ARCH (APARCH) model presented by Ding *et al.* (1993). Various studies have endeavored to apply both ARIMA and GARCH models in forecasting agricultural commodity prices. Examples of such investigations include those conducted by Paul *et al.* (2009), Bhardwaj *et al.* (2014) and Dinku (2021). Moreover, the integration of ARIMA and GARCH methodologies, known as ARIMA-GARCH models, has emerged as a viable approach for forecasting agricultural commodity prices. This fusion has been demonstrated in research conducted by (Mitra and Paul (2017) and Merabet *et al.* (2021)).

The dynamics of agricultural price volatilities exhibit interdependency across commodities and markets, prompting an increased scholarly emphasis on quantifying the interdependence within agricultural price series data. However, conventional Time Series (TS) models, such as ARIMA and GARCH models, often neglect the pivotal aspect of interdependency among different series. To address this deficiency, the Vector Auto Regressive (VAR) model was introduced, enabling the exploration of linear interrelationships among multiple TS. VAR model's efficacy in capturing the volatile nature of TS data is limited. In response, the Multivariate GARCH (MGARCH) model emerged as a potential solution to this challenge.

A variety of MGARCH models have been developed over time. Engle and Kroner (1995) introduced the BEKK (Baba, Engle, Kraft, and Kroner) model, which represents a multivariate extension of the GARCH model and offers substantial flexibility in modeling. Bollerslev (1990) proposed the Constant Conditional Correlation (CCC) model, providing a relatively flexible approach that combines univariate GARCH models while assuming constant correlation among series over time. Additionally, Engle (2002) introduced the Dynamic Conditional Correlation (DCC) model, a novel class of Multivariate GARCH (MGARCH) model that combines the flexibility of univariate GARCH models with a parsimonious parametric framework for modeling correlations. Several studies have demonstrated the superiority of MGARCH models compared to univariate GARCH models in forecasting agricultural commodity prices (Wang and Wu (2012); Aziz and Iqbal (2016)). The application of MGARCH models for modeling the degree of interactions among various volatile agricultural commodities and markets is widely documented in the literature (Musunuru (2014); Sanjuán-López and Dawson (2017)).

MGARCH models often rely on assumptions of Multivariate Normal (MVN) distribution or Multivariate t (MV-t) distributions for the innovations. MVN distributions assume that each variable follows a univariate normal distribution, which may not hold true in many real world situations where variables exhibit non-normal distributions or complex relationships. Additionally, the Pearson correlation coefficient used in MVN distribution assumes linearity in the relationships between variables, limiting its ability to capture non-linear relationships that are often present. This limitation extends to MV-t distributions as well. To address these shortcomings, Copula-GARCH models have been introduced, where GARCH model combined with Copula model. The Copula is employed to capture dependency between related TS by focusing on their joint distribution and offering flexibility in modeling complex nonlinear dependencies. Sklar (1959)'s theorem is central to the theory of Copulas which states that "any multivariate distribution function can be represented as a composition of its univariate marginal distributions and a Copula", where the Copula captures all the dependencies in the joint distribution. In other words, Copula-based modelling provides the capacity to isolate the dependence structure from marginal distributions of the related TS. Various applications of Copula-GARCH model for portfolio risk estimation on financial TS data can be found in Weiß (2013); Lu *et al.* (2014); Karmakar (2017).

Previous studies utilized the ARIMA-GARCH copula model, initially fitting individual TS data and then employing residuals to model Copulas for joint distributions. These copula models used to analyze correlations among different TS, exploring various statistical measures such as skewness, kurtosis, and fat-tails (Li *et al.* (2020); Shahriari *et al.* (2023)). However, an evident research gap exists as copula models have not been utilized for forecasting future data points. Understanding the price dynamics is crucial, especially in agriculture. The present study pioneers using the Bivariate Copula-GARCH type model for forecasting agricultural prices. After fitting individual TS data to the ARIMA-GARCH type model, residuals are used to fit copula models for joint distributions. Future data points are forecasted by simulating observations from the estimated bivariate distribution function. This advanced modeling technique enhances forecasting accuracy for the data under consideration.

The rest of the manuscript is organized as follows: Section 2 presents a description of the models utilized, Section 3 discusses empirical findings, and Section 4 offers concluding remarks.

2. Material and methods

2.1. ARIMA model

The Box Jenkins ARIMA model, represented in Eq. (1), stands as the predominant technique for forecasting TS data:

$$\phi_p(B)(1-B)^d y_t = c + \theta_q(B)\varepsilon_t \tag{1}$$

where,

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$$

$$\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$$

Here, y_t represents the value of current time; c is the constant term; B denotes backward shift operator; ε_t represents the error term; $\phi_1, \phi_2, \ldots, \phi_p$ denote Auto-Regressive (AR) coefficients of order p; $\theta_1, \theta_2, \ldots, \theta_q$ represent Moving Average (MA) coefficients of order q; d is the order of differencing.

2.2. ARCH and GARCH models

ARIMA models are limited in their ability to capture the volatility inherent in TS data and cannot adequately describe changes in conditional variances observed in real-world datasets. To address the inadequacies of ARIMA model, Engle (1982) proposed Auto Regressive Conditional Heteroscedastic (ARCH) model represented in Eq. (2). The ARCH model for the series $\{\varepsilon_t\}$ is characterized by defining the conditional distribution of ε_t given the information available up to time $t-1$, denoted as Ψ_{t-1}. The ARCH model for the series ε_t can be expressed as:

$$\varepsilon_t | \Psi_{t-1} \sim N(0, h_t) \quad \text{and} \quad \varepsilon_t = \sqrt{h_t}\nu_t$$

where h_t is conditional variance, ν_t is identically and independently distributed (iid) innovations with zero mean and unit variance. The conditional variance h_t is defined as

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \tag{2}$$

The conditions of $\alpha_0 > 0, \alpha_i \geq 0 \forall i$ and $\sum_{i=1}^{q} \alpha_i < 1$ are necessary and sufficient to guarantee non-negativity and a finite conditional variance for h_t. Here, α_i denotes the coefficients indicating the impact of past shocks on the current volatility.

In response to certain shortcomings of the ARCH model, such as the rapid decay of the unconditional autocorrelation function of squared residuals, non-parsimony *etc.*, Bollerslev (1986) introduced the Generalized ARCH (GARCH) model. The variance equation of GARCH model is represented in Eq. 3 as:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{3}$$

where $\alpha_0 > 0, \alpha_i \geq 0 \forall i, \beta_j \geq 0 \forall j$. Here, β_j denotes the coefficients indicating the impact of past volatilities on the current volatility. While the GARCH model excels at capturing overall volatility in TS, it falls short when it comes to asymmetric impacts of positive and negative events. To address this limitation, various asymmetric GARCH-type of models have evolved namely EGARCH, GJR-GARCH and APARCH model stated subsequently.

2.3. Asymmetric GARCH-type models

2.3.1. EGARCH model

EGARCH model addresses asymmetric volatility without parameter constraints. It models the conditional variance, h_t, as an asymmetric function of lagged disturbances, defined by Eq. (4).

$$\ln(h_t) = \alpha_0 + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) + \sum_{i=1}^{q} \left(\alpha_i \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \lambda_i \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right) \tag{4}$$

where λ_i represents the asymmetric parameter, capturing asymmetric effects due to external shocks.

2.3.2. GJR-GARCH model

GJR-GARCH model considers the impact of ε_{t-1}^2 on the conditional variance, depending on the sign of ε_{t-1}. They introduced an indicator variable to capture this sign dependence. The GJR-GARCH model is represented in Eq. (5).

$$h_t = \alpha_0 + \sum_{j=1}^{p} \beta_j h_{t-j} + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \tag{5}$$

where γ $(-1 < \gamma < 1)$ denote the asymmetric parameter, and I_{t-1} is the indicator variable, such that

$$I_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0 \\ 0 & \text{if } \varepsilon_{t-1} \geq 0 \end{cases}$$

2.3.3. APARCH model

The APARCH model incorporates asymmetric power into the conditional variance, specified as represented in Eq. (6).

$$h_t^{\delta/2} = \alpha_0 + \sum_{j=1}^{p} \beta_j h_{t-j}^{\delta/2} + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma\varepsilon_{t-i}\right)^{\delta} \tag{6}$$

where γ $(-1 < \gamma < 1)$ denotes the asymmetric parameter, and $\delta(> 0)$ denotes the power term parameter. An application of different asymmetric GARCH type models can be found in Rakshit *et al.* (2021).

2.4. ARIMA-GARCH type models

ARIMA-GARCH type models integrate ARIMA for capturing linear dynamics and various GARCH models (e.g., GARCH, EGARCH, GJR-GARCH, and APARCH) to address volatility clustering. The ARIMA component accounts for linear behaviour in the first stage, thereby leaving nonlinear components in residuals. Paul *et al.* (2014) developed formulae for out-of-sample forecast using ARIMA-GARCH model. Paul (2015) applied ARIMA-GARCH model for forecasting volatility in agricultural crop yield.

The presence of serial autocorrelation in residuals from the ARIMA model is typically assessed using the Ljung-Box test, a statistical test proposed by Ljung and Box (1978). Meanwhile, the existence of heteroscedasticity in these residuals is evaluated through the ARCH Lagrange Multiplier (LM) test, introduced by Engle (1982). If serial correlation and heteroscedasticity are detected in the residuals based on the results of the Ljung-Box test and ARCH-LM tests, respectively, the residuals are then subjected to a GARCH model. GARCH is employed to model these residual patterns comprehensively, thereby capturing both mean and volatility dynamics effectively. This approach ensures a thorough analysis of both linear and nonlinear components in the data, enhancing the overall modelling accuracy and robustness. The schematic representation of ARIMA-GARCH type model is illustrated in Figure 1.

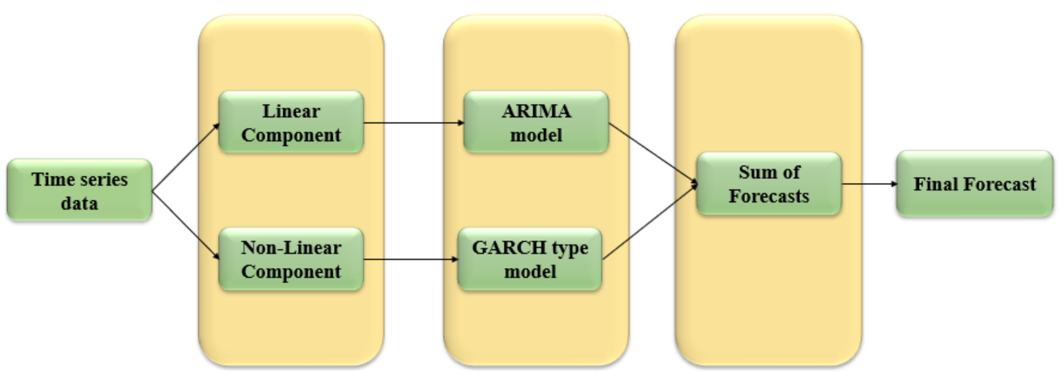

Figure 1: ARIMA-GARCH type model

2.5. Copula

Copulas have been introduced by applied mathematician, Sklar (1959). Copula comes from the Latin word "copulature" which means "to join together". Copulas are handled by utilizing Probability Integral Transformation (PIT) and Inverse Probability Integral Transformation (Inverse PIT) which are described subsequently.

PIT: Suppose that a random variable X has a continuous distribution for which the Cumulative Distribution Function (CDF) is F_X. Then the random variable U defined by PIT as $U = F_X(X)$ has a standard uniform distribution.

Inverse PIT: Given a continuous standard uniform variable U and an invertible CDF G_X^{-1}, the random variable X defined by Inverse PIT $X = G_X^{-1}(U)$ has distribution function G_X.

Accordingly, the formal definition of Copula is as follows: Let $X = (X_1, X_2, \ldots, X_d)$ be a vector of random variables with their marginal CDFs $F_1, F_2, \ldots, F_d$ as continuous functions. By applying the PIT to each component, obtain the U vector containing $U_1, U_2, \ldots, U_d$ random variables; here each variable will follow standard uniform distribution as

$$U = (U_1, U_2, \ldots, U_d) = [F_1(X_1), F_2(X_2), \ldots, F_d(X_d)]$$

Then, Copula C is a joint cumulative distribution function of d random variables given by

$$C(U_1, U_2, \ldots, U_d) = H[G_1^{-1}(U_1), G_2^{-1}(U_2), \ldots, G_d^{-1}(U_d)]$$

To overcome the limitation of Pearson correlation coefficient, Copula modeling utilizes Spearman's rank correlation coefficient, a nonparametric measure of correlation. It avoids distributional assumptions and linear relationships. Nonparametric correlation measures allow flexible analysis, accommodating non-linear patterns and non-normal data.

2.5.1. Bivariate gaussian copula

Let Φ_{xy} be the distribution function of a standardised bivariate normal CDF and Φ^{-1} be the inverse of standard normal CDF, and ρ is the Spearman rank correlation coefficient (i.e. dependence parameter) between the components. Then the bivariate Gaussian Copula CDF is expressed as shown in Eq. (7).

$$C_\rho(u_1, u_2) = \Phi_{xy}[\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho] \tag{7}$$

Let $s = \Phi^{-1}(u_1)$ and $t = \Phi^{-1}(u_2)$, then the Gaussian Copula density is given by Eq. (8).

$$c_\rho(u_1, u_2) = \frac{1}{\sqrt{1-\rho^2}} \exp\left\{\frac{-(\rho^2 s^2 + \rho^2 t^2 - 2\rho s t)}{2(1-\rho^2)}\right\} \tag{8}$$

2.5.2. Bivariate Student-t copula

When the interest focuses on modelling data which exhibits heavy-tailed behaviour, the Student-t Copula may be used instead of the Gaussian Copula.

Let t_{xy} be the distribution function of a standardised bivariate Student-t CDF and t^{-1} be the inverse of standard Student's t CDF with η degrees of freedom and ρ dependence parameter is the Spearman rank correlation coefficient between the components, then the bivariate Student-t Copula CDF is expressed as shown in Eq. (9).

$$C_{\rho\eta}(u_1, u_2) = t_{xy}[t_\eta^{-1}(u_1), t_\eta^{-1}(u_2); \rho] \tag{9}$$

Let $s = t_\eta^{-1}(u_1)$ and $r = t_\eta^{-1}(u_2)$, then the Student-t Copula density is given by Eq. (10) as follows:

$$c_{\eta\rho}(u_1, u_2) = \frac{\Gamma\left(\frac{\eta+2}{2}\right)\Gamma\left(\frac{\eta}{2}\right)}{\sqrt{1-\rho^2}\Gamma^2\left(\frac{\eta+1}{2}\right)} \left\{\left(1+\frac{s^2}{\eta}\right)\left(1+\frac{r^2}{\eta}\right)\right\}^{(\eta+1)/2} \left(1+\frac{s^2+r^2-2\rho sr}{\eta(1-\rho^2)}\right)^{-(\eta+2)/2} \tag{10}$$

2.6. Bivariate ARIMA-GARCH type-Copula model

2.6.1. ARIMA-GARCH type model selection

ARIMA-GARCH type models, *viz.*, ARIMA-GARCH, ARIMA-EGARCH, ARIMA-GJR-GARCH and ARIMA-APARCH, are fitted to the two TS data independently. The optimal ARIMA-GARCH type model is selected based on minimum value of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for the two TS. From the optimal ARIMA-GARCH type model, mean and volatility forecasts are obtained for both TS. While these models capture temporal dependency within the individual TS, a Copula model is employed to capture dependency between two TS.

2.6.2. Copula modeling

The residuals of the fitted ARIMA-GARCH type models for the two TS are employed for Copula modeling. The Spearman rank correlation coefficient ρ is utilized to assess the relationship between the residuals of the two TS. If the residuals of the two fitted TS are not significantly correlated, then the forecasts from the optimal ARIMA-GARCH type models for each TS are considered the final predictions. However, if the residuals are significantly correlated, Copula modeling is employed. In Copula modeling, the residuals of both TS are transformed using the PIT. The transformed values and estimated dependence parameter ρ of copula are then utilized to fit both Gaussian Copula and Student-t Copula functions. The optimal Copula function is selected based on the AIC and BIC criteria. The schematic representation of the methodology of Bivariate ARIMA-GARCH type-Copula model is illustrated in Figure 2.

2.6.3. One day ahead forecast through simulation

The optimal Copula function used to obtain the bivariate distribution (joint distribution) of residuals from an ARIMA-GARCH-type model is applied to two TS. By simulating a large number of observations from the estimated bivariate distribution function through random sampling, multiple potential future scenarios are generated. These scenarios incorporate uncertainty, variability, and the complex relationships between the residuals of the two TS, helping capture the range of possible future outcomes more comprehensively. The step

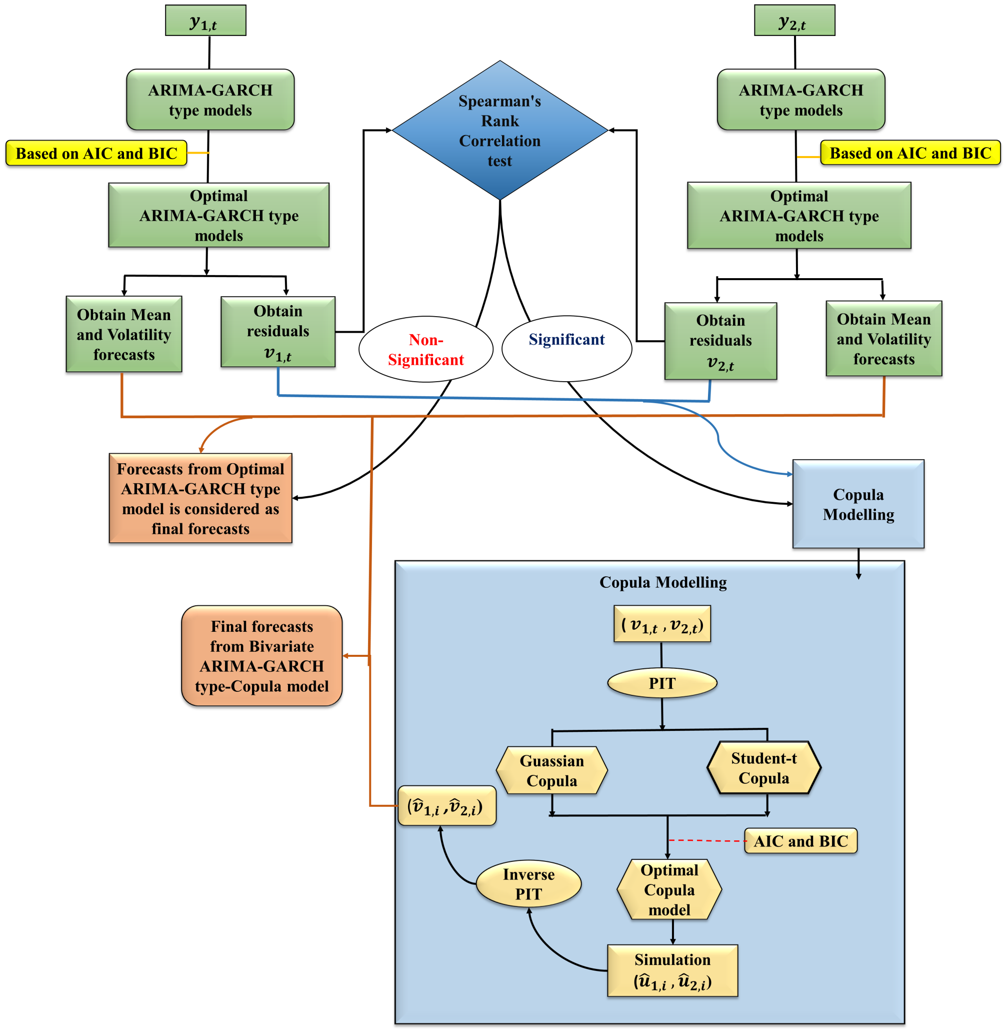

Figure 2: Flow chart of Bivariate ARIMA-GARCH type-Copula

by step algorithm to obtain one day ahead forecast through simulation can be summarized as follows:

1. Simulate n pairs of random samples $(\hat{u}_{1,i}, \hat{u}_{2,i})$ from the estimated Optimal Copula function. Here $\hat{u}_{1,i}$ and $\hat{u}_{2,i}$ denote the simulated values for the residuals of the optimal ARIMA-GARCH type models of the first and second TS, respectively, where $1 \leq i \leq n$.

2. To ensure that the simulated values of residuals are in their respective original scales, inverse PIT is applied to obtain transformed values $(\hat{v}_{1,i}, \hat{v}_{2,i})$.

3. Multiply $\hat{v}_{1,i}$ by the respective predicted one-day ahead volatility $\sqrt{h_{1,t}}$ from the optimal GARCH type model for the first TS, and multiply $\hat{v}_{2,i}$ by the respective predicted one-day ahead volatility $\sqrt{h_{2,t}}$ from the optimal GARCH type model for the second TS.

$$\hat{\varepsilon}_{1,i} = \hat{v}_{1,i}\sqrt{h_{1,t}} \quad \text{and} \quad \hat{\varepsilon}_{2,i} = \hat{v}_{2,i}\sqrt{h_{2,t}}$$

4. Obtain $(\hat{\mu}_{1,i}, \hat{\mu}_{2,i})$ by adding the mean forecast from the ARIMA model for the first and second TS to the $\hat{\varepsilon}_{1,i}$ and $\hat{\varepsilon}_{2,i}$ respectively.

$$\hat{\mu}_{1,i} = \hat{\mu}_{1,t} + \hat{\varepsilon}_{1,i} \quad \text{and} \quad \hat{\mu}_{2,i} = \hat{\mu}_{2,t} + \hat{\varepsilon}_{2,i}$$

5. Take average to obtain one day ahead forecasts $(\hat{k}_{1,t}, \hat{k}_{2,t})$ of both TS

$$\hat{k}_{1,t} = \frac{1}{n}\sum_{i=1}^{n}\hat{\mu}_{1,i} \quad \text{and} \quad \hat{k}_{2,t} = \frac{1}{n}\sum_{i=1}^{n}\hat{\mu}_{2,i}$$

$\hat{k}_{1,t}$ and $\hat{k}_{2,t}$ are considered as one day ahead forecasts from Bivariate ARIMA-GARCH type-Copula model for first and second TS, respectively.

3. Data and empirical findings

3.1. Data description

In this study, we collected monthly agricultural commodity price data for three oilseed crops and three pulse crops from two primary markets for each commodity. The data was obtained from the AGMARKNET portal of the Ministry of Agriculture and Farmers Welfare, Government of India (`https://agmarknet.gov.in/`), covering the period from January 2010 to December 2022. The selection of major markets was based on their significant arrival quantities. The chosen markets are detailed below:

Oilseeds:

- Safflower: Latur (Maharashtra) and Kalaburagi (Karnataka)
- Mustard: Sri Ganganagar (Rajasthan) and Satna (Madhya Pradesh)
- Groundnut: Gondal (Gujarat) and Bikaner (Rajasthan)

Pulses:

- Lentil: Banda (Uttar Pradesh) and Narsinghpur (Madhya Pradesh)
- Chickpea: Hinganghat (Maharashtra) and Dewas (Madhya Pradesh)
- Green gram: Bhagat Ki Kothi (Rajasthan) and Kalaburagi (Karnataka)

Each agricultural commodity price dataset contained 156 observations, the series was divided into training and testing sets. The training set consisted of 144 months of observations, which were used for model building. The last 12 months of observations were

Table 1: Descriptive statistics of monthly agricultural commodity price data

Commodity	Markets	Mean	S.D.	C.V (%)	Skew	Kurt	Minimum	Maximum
Safflower	Latur	3301.18	844.03	25.57	0.81	0.08	1985.71	5755.00
	Kalaburagi	3236.95	940.20	29.05	0.71	-0.12	1795.95	5944.72
Mustard	Sri Ganganagar	3922.62	1268.29	32.33	1.16	0.72	2153.15	7679.16
	Satna	3690.72	1246.44	33.77	1.18	0.72	1900.00	7397.23
Groundnut	Gondal	4518.99	1065.89	23.59	0.28	-0.90	2796.08	6933.18
	Bikaner	4055.43	951.53	23.46	0.49	-0.41	2456.59	6540.67
Lentil	Banda	4328.20	1220.39	28.20	0.45	-0.76	2279.00	7277.69
	Narsinghpur	4255.75	1183.62	27.81	0.45	-0.93	2466.35	7022.16
Chickpea	Hinganghat	3627.42	1090.69	30.07	0.89	1.81	1835.11	7629.75
	Dewas	3859.89	1260.67	32.66	1.13	2.84	1835.09	8871.43
Green gram	Bhagat Ki Kothi	5413.40	1267.25	23.41	-0.18	-0.75	2025.00	8270.67
	Kalaburagi	5254.95	1125.54	21.42	-0.21	-0.68	2612.50	8132.74

S.D.: Standard Deviation, C.V.: Coefficient of Variation, Skew: Skewness, Kurt: Kurtosis

kept for validating the model. The Table 1 presents key statistics for various commodities across different markets.

Green gram in Bhagat Ki Kothi market stands out with the highest mean price, while safflower in Kalaburagi market records the lowest. Mustard in Satna market exhibits the highest coefficient of variation (C.V.), indicating considerable price variability, while green gram in Kalaburagi shows the lowest. Dewas for Chickpea reports the highest maximum price, and safflower in Kalaburagi reflects the lowest minimum. Positively skewed distributions are observed in most of the agricultural commodity markets except the green gram market in Bhagat Ki Kothi and Kalaburagi, which display negative skewness. Leptokurtic distributions are evident in Mustard in Sri Ganganagar, Satna, Chickpea in Hinganghat and Dewas, while Safflower in Latur exhibit approximately mesokurtic distributions. The remaining commodities markets demonstrate platykurtic distributions.

3.2. Test for normality

To evaluate normality of agricultural commodity price data, most widely used statistical tests, *viz.*, Jarque-Bera test (Jarque and Bera (1987)) and Shapiro-Wilk test (Shapiro and Wilk (1965)) were employed. The results of these normality tests are presented in Table 2, indicating that the majority of agricultural commodity markets show significant deviations from normality at 1% level as evidenced by low p-values (<0.01), the Green gram prices in the Bhagat Ki Kothi and Kalaburagi markets are significant at the 5% level with p-value below 0.05 from the Jarque-Bera test and Shapiro-Wilk tests. Hence all the agricultural commodity markets price data considered were non-normal.

3.3. Test for stationarity

The stationarity of data is a crucial property of TS analysis. A series is considered stationary if it maintains a constant mean and variance over time. To assess stationarity, several statistical tests namely the Augmented Dickey-Fuller (ADF) test (Dickey and Fuller (1979)), the Phillips-Perron (PP) test (Phillips and Perron (1988)) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski *et al.* (1992)) were employed. The null hypothesis for the ADF and PP tests states that the series is non-stationary, while for the KPSS test, it suggests that the series is stationary. Table 3 presents stationarity test results

Table 2: Normality test results of agricultural commodity price data of different markets

Commodity	Markets	Jarque-Bera test		Shapiro-Wilk test	
		Statistic	p-value	Statistic	p-value
Safflower	Latur	17.322	0.0002	0.936	< 0.0001
	Kalaburagi	13.489	0.0012	0.944	< 0.0001
Mustard	Sri Ganganagar	39.849	< 0.0001	0.871	< 0.0001
	Satna	40.732	< 0.0001	0.861	< 0.0001
Groundnut	Gondal	17.056	0.0029	0.966	0.0008
	Bikaner	17.397	0.0025	0.967	0.0009
Lentil	Banda	8.978	0.0113	0.957	0.0002
	Narsinghpur	10.663	0.0048	0.944	< 0.0001
Chickpea	Hinganghat	44.062	< 0.0001	0.932	< 0.0001
	Dewas	89.206	< 0.0001	0.915	< 0.0001
Green gram	Bhagat Ki Kothi	8.185	0.0124	0.978	0.0154
	Kalaburagi	8.948	0.0138	0.978	0.0168

for agricultural commodity price data across various markets. All agricultural commodity markets price series are deemed non-stationary, as indicated by the *p*-values.

Table 3: Stationarity test results of agricultural commodity price data of different markets

Commodity	Markets	ADF test		PP test		KPSS test	
		Statistic	p-value	Statistic	p-value	Statistic	p-value
Safflower	Latur	-2.042	0.559	-4.202	0.874	1.788	< 0.01
	Kalaburagi	-2.012	0.572	-8.827	0.608	2.251	< 0.01
Mustard	Sri Ganganagar	-2.283	0.458	-8.068	0.652	2.179	< 0.01
	Satna	-2.357	0.427	-6.532	0.741	2.163	< 0.01
Groundnut	Gondal	-2.224	0.483	-17.24	0.125	1.566	< 0.01
	Bikaner	-2.232	0.479	-9.945	0.495	1.941	< 0.01
Lentil	Banda	-1.820	0.651	-9.350	0.578	1.586	< 0.01
	Narsinghpur	-1.789	0.663	-8.301	0.638	1.474	< 0.01
Chickpea	Hinganghat	-3.109	0.114	-16.948	0.142	1.201	< 0.01
	Dewas	-2.731	0.272	-14.232	0.298	1.111	< 0.01
Green gram	Bhagat Ki Kothi	-1.967	0.596	-16.988	0.195	0.985	< 0.01
	Kalaburagi	-2.918	0.193	-12.751	0.383	0.868	< 0.01

3.4. Residual analysis

Suitable ARIMA model is selected based on minimum AIC and BIC criteria and also observing the significance of autocorrelation and partial autocorrelation functions. Subsequently, the residuals from the ARIMA model undergo diagnostics measures.

3.4.1. Ljung-Box test for serial autocorrelation

The Ljung-Box test is utilized to assess the presence of serial autocorrelation in residuals from the ARIMA model. The null hypothesis suggests that the residuals exhibit no autocorrelation for a fixed number of lags. A rejection of this hypothesis indicates the presence of serial autocorrelation. Table 4 presents Ljung-Box test results, revealing significant autocorrelation in agricultural commodity price data from all specified markets.

3.4.2. ARCH lagrange multiplier (LM) test for heteroscedasticity

The ARCH LM test is employed to evaluate the existence of heteroscedasticity in residuals. It examines whether the variance of the residuals is constant over time or not. The null hypothesis states that the residuals are homoscedastic, while a rejection of null hypothesis suggests the presence of heteroscedasticity. In addition Table 4 presents ARCH-LM test results, indicating heteroscedasticity in agricultural commodity price data across all specified markets.

Table 4: Ljung-Box and ARCH-LM test statistic results of agricultural commodity price data of different markets

Commodity	Markets	Ljung-Box	ARCH-LM
Safflower	Latur	10.23	44.19
	Kalaburagi	14.51	37.82
Mustard	Sri Ganganagar	6.88	58.44
	Satna	6.95	78.67
Groundnut	Gondal	7.81	28.57
	Bikaner	12.91	41.36
Lentil	Banda	10.45	27.47
	Narsinghpur	10.94	46.33
Chickpea	Hinganghat	22.14	40.34
	Dewas	12.16	62.04
Green gram	Bhagat Ki Kothi	7.28	23.98
	Kalaburagi	14.76	20.36

Note: The test statistics provided in the table lead to p-values of less than 0.01 for all cases.

3.4.3. Broock-Dechert-Scheinkman (BDS) test for nonlinearity

The nonparametric Broock-Dechert-Scheinkman (BDS) test (Broock *et al.* (1996)) is utilized to test the nonlinearity of the residual series. This test assesses whether the residuals exhibit nonlinear dependence. The null hypothesis assumes that the residuals are independently and identically distributed (iid). A rejection of this hypothesis indicates nonlinearity in the residuals. The results of the BDS test, presented in Table 5, indicate the possible presence of nonlinear patterns in the residuals of the ARIMA model at 1% significance level in all the agricultural markets price series.

It is evident that autocorrelation, heteroscedasticity, and nonlinearity is detected in the residuals based on the results of the aforementioned tests, hence residuals are then subjected to a GARCH type models such as standard GARCH, EGARCH, GJR-GARCH, and APARCH. Through rigorous evaluation, the optimal ARIMA-GARCH model is selected based on criteria such as the AIC and the BIC, as outlined in Table 6. Subsequently, the estimated parameters of the best-fitted model are detailed in Table 7.

Table 5: BDS test results of agricultural commodity price data of different markets

Commodity	Markets	Dimension (m)			
		0.5σ	1.0σ	1.5σ	2.0σ
Safflower	Latur	10.73	8.40	7.50	5.48
	Kalaburagi	7.21	7.05	4.14	1.11
Safflower	Latur	16.45	10.01	8.34	6.58
	Kalaburagi	13.13	11.22	7.34	2.47
Mustard	Sri Ganganagar	5.18	7.14	7.46	5.18
	Satna	8.16	9.42	9.32	6.95
Mustard	Sri Ganganagar	9.25	10.86	10.57	7.89
	Satna	9.03	11.31	11.77	9.31
Groundnut	Gondal	10.56	5.81	3.95	2.54
	Bikaner	10.96	9.81	5.85	4.69
Groundnut	Gondal	21.56	11.31	8.45	6.83
	Bikaner	17.26	13.04	7.95	7.24
Lentil	Banda	9.94	7.43	5.70	3.59
	Narsinghpur	4.62	4.09	3.21	2.09
Lentil	Banda	13.45	9.84	7.60	5.14
	Narsinghpur	11.01	8.82	7.02	4.27
Chickpea	Hinganghat	20.41	10.74	7.21	6.58
	Dewas	14.29	9.36	7.43	6.70
Chickpea	Hinganghat	25.08	12.06	8.28	7.94
	Dewas	23.37	13.91	10.15	8.94
Green gram	Bhagat Ki Kothi	21.21	14.81	10.87	7.55
	Kalaburagi	5.71	5.38	5.21	5.64
Green gram	Bhagat Ki Kothi	28.27	18.66	12.74	8.44
	Kalaburagi	11.21	9.69	7.81	6.84

Note: The test statistics provided in the table lead to p-values less than 0.01 for all cases.

The correlation between the residuals of ARIMA-GARCH type models for two markets of the same agricultural commodity is examined through Spearman's rank correlation, and the results are shown in Table 8, indicating a significant correlation between the residuals of ARIMA-GARCH type models for two markets of all agricultural commodities at the one percent level. Subsequently, the residuals of the two markets were transformed via the PIT. The transformed values are then utilized to fit both Gaussian Copula and Student-t Copula models, and their AIC and BIC values are presented in Table 9. The results indicate that in all cases, the Student-t Copula model is the optimal Copula, with the lowest AIC and BIC values. This suggests that the Student-t Copula model provides a better goodness-of-fit compared to the Gaussian Copula.

After fitting the Student-t Copula model to the residuals of the optimal ARIMA-GARCH models for the two considered markets, proceed to simulate $n = 1000$ pairs of random samples from the estimated Student-t Copula function. Next, obtain one-day-ahead forecasts from the Bivariate ARIMA-GARCH-Copula model using algorithm 2.6.3. Repeat this one-day-ahead forecast procedure for each day in the test dataset, employing algorithm 2.6.3.

Table 6: Optimal ARIMA-GARCH type model for different commodity markets

Commodity	Markets	Optimal ARIMA-GARCH type model	AIC	BIC
Safflower	Latur	ARIMA (2,1,1) - GARCH (1,1)	12.107	12.211
	Kalaburagi	ARIMA (2,1,0) - APARCH (1,0)	13.121	13.268
Mustard	Sri Ganganagar	ARIMA (1,1,0) - GARCH (1,1)	13.797	13.902
	Satna	ARIMA (2,1,1) - APARCH (1,0)	13.103	13.270
Groundnut	Gondal	ARIMA (2,1,0) - APARCH (1,0)	13.231	13.398
	Bikaner	ARIMA (1,1,0) - APARCH (1,0)	14.461	14.628
Lentil	Banda	ARIMA (2,1,1) - APARCH (1,1)	14.165	14.290
	Narsinghpur	ARIMA (2,1,0) - APARCH (1,1)	14.139	14.293
Chickpea	Hinganghat	ARIMA (2,1,0) - GARCH (1,1)	14.539	14.664
	Dewas	ARIMA (2,1,0) - GARCH (1,1)	14.838	14.922
Green gram	Bhagat Ki Kothi	ARIMA (2,1,1) - APARCH (1,1)	15.088	15.213
	Kalaburagi	ARIMA (2,1,1) - APARCH (1,1)	14.779	14.904

Table 7: Parameter estimates of ARIMA-GARCH type models

Commodity	Markets	ϕ_1	ϕ_2	θ_1	α_1	β_1	γ	δ
Safflower	Latur	0.364 (<0.001)	0.638 (<0.001)	0.747 (<0.001)	0.686 (<0.001)	0.206 (0.009)	- -	- -
	Kalaburagi	1.207 (<0.001)	-0.239 (0.003)	- -	0.418 (<0.001)	- -	0.178 (<0.001)	3.218 (<0.001)
Mustard	Sri Ganganagar	0.913 (<0.001)	- -	- -	0.362 (<0.001)	0.589 (<0.001)	- -	- -
	Satna	1.327 (<0.001)	-0.438 (<0.001)	0.104 (0.031)	0.633 (<0.001)	- -	0.057 (<0.001)	3.499 (<0.001)
Groundnut	Gondal	1.284 (<0.001)	-0.406 (<0.001)	- -	0.503 (<0.001)	- -	0.242 (<0.001)	3.072 (<0.001)
	Bikaner	0.887 (<0.001)	- -	- -	0.087 (<0.001)	- -	0.952 (<0.001)	3.500 (<0.001)
Lentil	Banda	0.885 (<0.001)	-0.148 (0.018)	0.510 (<0.001)	0.099 (<0.001)	0.567 (<0.001)	0.480 (<0.001)	3.127 (<0.001)
	Narsinghpur	0.978 (<0.001)	-0.189 (0.014)	- -	0.049 (<0.001)	0.493 (<0.001)	0.898 (<0.001)	3.358 (<0.001)
Chickpea	Hinganghat	1.198 (<0.001)	-0.294 (0.003)	- -	0.384 (<0.001)	0.581 (<0.001)	- -	- -
	Dewas	0.758 (<0.001)	0.249 (0.002)	0.911 (<0.001)	0.272 (<0.001)	0.702 (<0.001)	- -	- -
Green gram	Bhagat Ki Kothi	0.433 (<0.001)	0.570 (<0.001)	0.674 (<0.001)	0.340 (<0.001)	0.058 (<0.001)	0.254 (<0.001)	3.445 (<0.001)
	Kalaburagi	1.054 (<0.001)	-0.548 (<0.001)	-0.523 (0.046)	0.395 (<0.001)	0.108 (<0.001)	0.104 (<0.001)	3.268 (<0.001)

Note: The values in the parenthesis indicates the p-value

In evaluating the forecasting performance of the ARIMA-GARCH type-Copula, a comparative analysis is conducted against established traditional methodologies. Specifically, the efficacy of the ARIMA-GARCH-Copula model is compared with that of the Univariate ARIMA-GARCH type and MGARCH-DCC models. The assessment of accuracy utilizes key evaluation metrics, namely Root Mean Square Error (RMSE) (Eq.11), Mean Absolute Error (MAE) (Eq.12), and Mean Absolute Percentage Error (MAPE) (Eq.13), applied to the test dataset.

Table 8: Correlation analysis of ARIMA-GARCH type models

Commodity	Markets	Spearman's Rank Correlation Coefficient
Safflower	Latur and Kalaburagi	0.644
Mustard	Sri Ganganagar and Satna	0.554
Groundnut	Gondal and Bikaner	0.469
Lentil	Banda and Narsinghpur	0.674
Chickpea	Hinganghat and Dewas	0.577
Green gram	Bhagat Ki Kothi and Kalaburagi	0.527

Note: p-values of correlation coefficient are less than 0.01 for all cases.

Table 9: Comparison of Copula models

Commodity	Markets	Gaussian Copula		Student-t Copula	
		AIC	BIC	AIC	BIC
Safflower	Latur and Kalaburagi	-352.43	-346.33	**-354.36**	**-351.31**
Mustard	Sri Ganganagar and Satna	-476.47	-473.42	**-481.54**	**-475.44**
Groundnut	Gondal and Bikaner	-272.15	-269.10	**-278.62**	**-272.52**
Lentil	Banda and Narsinghpur	-444.05	-441.00	**-495.60**	**-489.50**
Chickpea	Hinganghat and Dewas	-351.78	-348.73	**-393.26**	**-387.16**
Green gram	Bhagat Ki Kothi and Kalaburagi	-224.19	-221.14	**-250.90**	**-244.80**

$$\text{RMSE} = \sqrt{\frac{1}{m}\sum_{t=1}^{m}(y_t - \hat{y}_t)^2} \tag{11}$$

$$\text{MAE} = \frac{1}{m}\sum_{t=1}^{m}|y_t - \hat{y}_t| \tag{12}$$

$$\text{MAPE} = \frac{1}{m}\sum_{t=1}^{m}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100 \tag{13}$$

where y_i and $\hat{y}_i$ represent the actual and predicted values, respectively, and m is the number of observations in test dataset.

Table 10 provides a comparison of model forecasting performance considering RMSE, MAE and MAPE. The findings consistently reveal that the Bivariate ARIMA-GARCH type-Copula model outperforms both the MGARCH-DCC model and the Univariate ARIMA-GARCH type model across all agricultural commodity market price series. This superiority is underscored by the model's ability to achieve the lowest RMSE, MAE and MAPE values.

In addition to traditional accuracy metrics, the Diebold-Mariano (DM) test proposed by Diebold and Mariano (2002) is used to compare the forecasting performance of two competing models. The fundamental premise of the DM test lies in its null hypothesis, which posits that both forecasting models exhibit the same level of accuracy. By comparing the forecast errors of the Bivariate ARIMA-GARCH type–Copula model and benchmark models

Table 10: Comparison of forecasting performance of different models

Commodity	Markets	BAGC model			MGARCH-DCC model			UAGC model		
		RMSE	MAE	MAPE (%)	RMSE	MAE	MAPE (%)	RMSE	MAE	MAPE (%)
Safflower	Latur	147.43	102.34	1.96	208.72	159.94	3.07	415.67	354.97	6.78
	Kalaburagi	367.02	293.90	5.49	422.53	343.90	6.54	453.45	392.31	7.58
Mustard	Sri Ganganagar	279.00	212.94	3.36	474.58	418.43	7.43	1065.31	978.72	15.55
	Satna	257.06	195.56	3.19	355.44	310.18	4.98	447.56	358.06	6.01
Groundnut	Gondal	329.16	224.28	3.58	522.77	510.90	7.94	933.32	766.58	11.82
	Bikaner	462.97	368.28	6.72	642.34	479.93	8.93	1078.72	982.64	16.43
Lentil	Banda	179.07	147.05	2.29	331.77	281.12	4.46	538.12	431.15	6.53
	Narsinghpur	260.87	227.61	3.68	527.40	490.60	8.01	751.30	655.51	10.59
Chickpea	Hinganghat	221.58	167.26	3.95	300.89	263.71	6.29	608.56	463.61	10.94
	Dewas	206.01	167.42	3.76	422.29	345.91	8.01	589.72	475.63	10.26
Green gram	Bhagat Ki Kothi	283.54	239.46	3.86	388.31	352.34	5.51	761.36	632.49	9.89
	Kalaburagi	234.50	202.74	3.27	502.84	378.53	5.87	848.16	704.32	11.35

Note: BAGCM:Bivariate ARIMA-GARCH type -Copula model; MGARCH-DCC: Multivariate GARCH DCC model; UAGCM: Univariate ARIMA-GARCH type-Copula

Table 11: Diebold-Mariano test for pairwise comparison of Copula based model with benchmark models

Commodity	Markets	Benchmark Models	
		MGARCH-DCC model	Univariate ARIMA-GARCH type model
Safflower	Latur	-3.3075 (0.0052)	-8.5562 (<0.0001)
	Kalaburagi	-2.2874 (0.0385)	-7.2749 (<0.0001)
Mustard	Sri Ganganagar	-4.7645 (0.0003)	-6.5871 (<0.0001)
	Satna	-4.7203 (0.0004)	-8.467 (<0.0001)
Groundnut	Gondal	-4.862 (0.0003)	-10.504 (<0.0001)
	Bikaner	-5.5891 (<0.0001)	-11.713 (<0.0001)
Lentil	Banda	-3.1998 (0.0064)	-4.9542 (0.0002)
	Narsinghpur	-3.8288 (0.0018)	-6.0619 (<0.0001)
Chickpea	Hinganghat	-3.6141 (0.0028)	-8.5482 (<0.0001)
	Dewas	-2.1141 (0.0428)	-12.388 (<0.0001)
Green gram	Bhagat Ki Kothi	-3.6864 (0.0025)	-5.8317 (<0.0001)
	Kalaburagi	-2.1669 (0.0479)	-4.7079 (0.0004)

(MGARCH-DCC model and Univariate ARIMA-GARCH type model), the DM test evaluates whether there exists a statistically significant difference in their predictive capabilities. Table 11 presents the statistic values and their corresponding p-values (in parentheses) of the DM test, comparing the predictive accuracy of the Bivariate ARIMA-GARCH type–Copula model with benchmark models on the test datasets. The results suggest that the forecasting performance of the Bivariate ARIMA-GARCH type–Copula model significantly outperforms both the MGARCH-DCC model and the Univariate ARIMA-GARCH type model.

4. Conclusions

This study focused on analyzing the price volatility of oilseed crops *viz.*, safflower, mustard, and groundnut, as well as pulses *viz.*, lentil, chickpea, and green gram across two markets for each commodity. By employing the Bivariate ARIMA-GARCH type-Copula model, the accuracy of price forecasting in the agricultural sector was studied. This study highlights the importance of incorporating Copulas into advanced modeling techniques to capture the complex interdependencies and joint distributions of agricultural commodity prices. The research findings demonstrate that the Bivariate ARIMA-GARCH type-Copula model surpassed both the MGARCH-DCC model and the Univariate ARIMA-GARCH type model in terms of forecasting performance. The evaluation metrics *viz.*, RMSE, MAPE, and MAE, consistently indicated the superior predictive ability of the Bivariate ARIMA-GARCH type-Copula model across all agricultural commodity market price series. Furthermore, the Diebold-Mariano (DM) test results provided additional validation of the Bivariate ARIMA-GARCH type-Copula model's outperformance compared to the alternative models. This signifies the robustness and reliability of Bivariate ARIMA-GARCH type-Copula model in capturing the joint distribution of commodity prices and improving forecasting accuracy. In the dynamic realm of agriculture, understanding price dynamics and volatility drivers is paramount. Combining Copulas with the ARIMA-GARCH model holds promise for better price predictions. By using advanced modeling, researchers and policymakers can improve forecasting accuracy. This study underscores the importance of continuous monitoring and analysis of agricultural commodity prices to mitigate risks and optimize market strategies in the ever-evolving landscape of the Indian economy.

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Conflict of interest

The authors do not have any financial or non-financial conflict of interest to declare for the research work included in this article.

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