

## Two-Sided Neighbours for Block Designs Through Projective Geometry

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### Abstract

This paper is concerned with the construction of neighbour designs for OS2 series with parameters  $v = s^2 + s + 1 = b$ ,  $r = s + 1 = k$  and  $\lambda = 1$ , through projective geometry, for different values of  $s$ , whether  $s$  is a prime number or a prime power. For these constructed designs, a method has been developed to find out both-sided neighbours of a treatment. These both-sided neighbours have some common neighbours forming a series, and have property of circularity also.

*Keywords:* Projective Geometry; OS2 Series; Border Plots; Neighbour Design; Two-Sided Neighbours

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### 1 Introduction

Designs over finite fields were introduced by Thomas in 1987. He constructed the first nontrivial 2-designs, over a finite field which were the designs with parameters  $2-(v, 3, 7; 2)$  for  $v \geq 7$  &  $v \equiv \pm 1 \pmod{6}$  and had used a geometric construction in a projective plane. In 1990 Suzuki extended Thomas family of 2-designs to a family of designs with parameters  $2-(v, 3, s^2 + s + 1; s)$  admitting a Singer cycle. In 1995 Miyakawa, Munemasa and Yoshiara gave a classification of  $2-(7, 3, \lambda, s)$  designs for  $s = 2, 3$  with small  $\lambda$ . In 2005 Kerber, Laue and Braun published the first 3-design over a finite field, a  $3-(8, 4, 11; 2)$  design admitting the normalize of a Singer cycle, as well as the smallest 2-design known as design with parameters  $2-(6, 3, 3; 2)$ .

A *balanced incomplete block design (BIBD)* is an ordinary  $2-(v, k, \lambda)$  design i.e. a set of  $k$ -subsets of an  $v$ -set such that each 2-subset is contained in exactly  $\lambda$  blocks. Then *balanced incomplete block design (BIBD)*, *symmetric balanced incomplete block design (SBIBD)*, *finite projective and affine planes* were defined by Dembowski (1968) and Hall (1967). The concept of neighbour designs was introduced by Rees (1967) in serology and defined it as a collection of circular blocks in which any two distinct treatments appear as neighbours equally often. Das & Saha (1976) provided some methods of construction of such designs in which every pair of treatments (viruses) occurs as neighbours equally ensuring a balanced situation. Tomar et al.(2005) had obtained a series of totally balanced block designs for competition effects. Jaggi, Gupta & Ashraf(2006) suggested general method of construction of complete block designs partially balanced for neighbouring competition effects and had suggested a series of incomplete block designs partially balanced for neighbour effects. Pateria, Jaggi, and Varghese (2007) considered a series of block designs by putting  $N-1$  MOLS with  $N$  treatments one below another and had obtained designs for  $N=5$  &  $N=7$  with necessary analytical methods. Arpan Bhowmik et al.(2012(a)) considered block model with one-sided interference effect arising from the immediate left neighbouring unit incorporating linear trend component, for estimating direct as well as interference effect. A construction of trend free incomplete block

design balanced for interference effect has been discussed and its characterization properties have been studied. In 2012(b), they have also derived information matrices for some classes of balanced and strongly balanced block designs with second order interference effect with reference to both complete and incomplete blocks. Their characterization properties have also been studied.

Though several methods of constructions of neighbour designs are discussed by several researchers, here we consider the construction of neighbour designs using projective geometry. In the following sections, construction of BIBD for OS2 series with parameters  $v = s^2 + s + 1 = b$ ,  $r = s + 1 = k$  and  $\lambda = 1$  using finite projective geometry has been considered.

## 2 Finite Projective Geometry

A finite projective geometry of  $n$  dimension  $PG(n, s)$  over  $GF(s)$ , Galois field of order  $s$ ,  $s$  is a prime power, consists of the ordered set  $(x_0, x_1, \dots, x_n)$  of points where  $x_i$  ( $i = 0, 1, \dots, n$ ) are elements of  $GF(s)$  and all of them are not simultaneously zero. For any  $\lambda \in GF(s)$  ( $\lambda \neq 0$ ), the points  $(\lambda_0, \lambda_1, \dots, \lambda_n)$  represents the same point as that of  $(x_0, x_1, \dots, x_n)$ . The total number of points lying on  $PG(n, s)$  is  $\frac{s^{n+1}-1}{s-1}$ . All those points which satisfy a set of  $(n-m)$  linearly independent homogeneous equations with coefficients from  $GF(s)$  (all of them are not simultaneously zero within the same equation) is said to represent a  $m$ -flat in  $PG(n, s)$ . In particular a 0-flat, a 1-flat, ..., and a  $(n-1)$ -flat respectively in  $PG(n, s)$  are known as a point, a line ... and a hyperplane of  $PG(n, s)$ . The number of points lying on a  $(m-1)$ -flat is  $\frac{s^m-1}{s-1}$  but the number of independent points lying on a  $(m-1)$ -flat is  $m$ . The number of  $(n-1)$ -flats within a  $PG(n, s)$  which contain a given  $(n-2)$ -flat is  $(s+1)$ .

## 3 Method of Construction of BIBD for OS2 Series

A BIBD of OS2 series is a finite projective plane of order  $s$  with parameters  $v = s^2 + s + 1 = b$ ,  $r = s + 1 = k$  and  $\lambda = 1$ , where  $s \geq 2$  is an integer, called the **order** of the projective plane. A projective plane of prime order  $s$  can be obtained using the following method of construction.

Create one point  $P$ .

Create  $s$  points, which are labelled as  $P(c) : c = 0, \dots, (s - 1)$ .

Create  $s^2$  points, which are labelled as  $P(r, c) : r, c = 0, \dots, (s - 1)$ .

On these points, construct the following lines:

One line  $L = \{ P, P(0), \dots, P(s - 1) \}$

$s$  lines  $L(c) = \{ P, P(0, c), \dots, P(s - 1, c) \}$ , where  $c = 0, \dots, (s - 1)$

$s^2$  lines  $L(r, c) = \{ P(c) \text{ and the points } P((r + ci) \bmod s, i) \}$ , where  $i, r, c = 0, 1, \dots, (s - 1)$ .

(It should be noted here that the expression " $(r + ci) \bmod s$ " will pass once through each value as  $i$  varies from 0 to  $s - 1$ , but only when  $s$  is prime.)

**i) when  $s=2$  i.e. a prime number**

Let us consider a case when  $s = 2$ , then parameters of OS2 series becomes  $v=b=7$ ,  $r=k=3$  and  $\lambda=1$ .

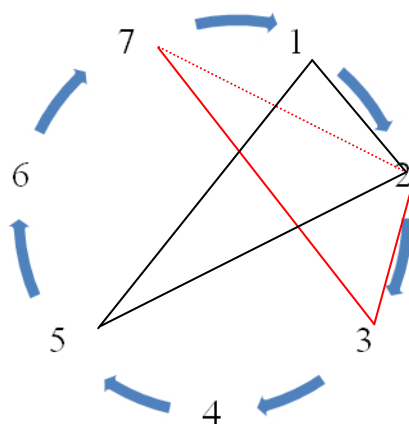


Fig-3.1 A cyclic way to represent a projective plane with seven points

For the construction of a finite projective plane of order 2, also known as Fano plane, create

- a) One point  $P$
- b)  $s = 2$  points, which we label as  $P(0), P(1)$
- c)  $s^2 = 4$  points, which we label as  $P(0,0), P(1,0), P(0,1), P(1,1)$

On these points, construct the following lines

- a) One line  $L = \{ P, P(0), P(1) \}$
- b)  $s = 2$  lines  $L(c) = \{ P, P(0,c), P(1,c) \} : c = 0, 1$  i.e.

when  $c=0$ :  $L(0) = \{ P, P(0,0), P(1,0) \}$  &

when  $c=1$ :  $L(1) = \{ P, P(0,1), P(1,1) \}$  .

- c)  $s^2 = 4$  lines  $L(r, c): P(c)$  and the points  $P((r + ci) \bmod 2, i)$ , where  $i = 0, 1 : r, c = 0, 1$

when  $r,c=0,0$ :  $L(0,0) = \{ P(0), P((0+0.0) \bmod 2, 0), P((0+0.1) \bmod 2, 1) \}$

$= \{ P(0), P(0,0), P(0,1) \}$

when  $r,c=1,0$ :  $L(1,0) = \{ P(0), P((1+0.0) \bmod 2, 0), P((1+0.1) \bmod 2, 1) \}$

$= \{ P(0), P(1,0), P(1,1) \}$

when  $r,c=0,1$ :  $L(0,1) = \{ P(1), P((0+1.0) \bmod 2, 0), P((0+1.1) \bmod 2, 1) \}$

$= \{ P(1), P(0,0), P(1,1) \}$

when  $r,c=1,1$ :  $L(1,1) = \{ P(1), P((1+1.0) \bmod 2, 0), P((1+1.1) \bmod 2, 1) \}$

$= \{ P(1), P(1,0), P(0,1) \}$

Let us assign number to these points as follows:

Points	$P(0,0)$	$P(1,0)$	$P(0,1)$	$P(1,1)$	$P$	$P(0)$	$P(1)$
Point Number	1	2	3	4	5	6	7

after numbering and then arranging in ascending or descending order (here ascending) the resulting BIBD is:

**Table 3.1**

Blocks	Contents		
1	5	6	7
2	1	2	5
3	3	4	5
4	1	3	6
5	2	4	6
6	1	4	7
7	2	3	7

**ii) when  $s=4$  i.e. a prime power**

Let us now consider a case when  $s = 2^2 = 4$ , then parameters of OS2 series becomes  $v=b=21$ ,  $r=k=5$ ,  $\lambda=1$ . For the construction of a finite projective plane of order 4, create

- one point  $P$
- $s = 4$  points, which we label as  $P(0), P(1), P(\alpha), P(\alpha^2)$
- $s^2 = 16$  points, which we label as  $P(0,0), P(1,0), P(\alpha,0), \dots, P(\alpha, \alpha^2), P(\alpha^2, \alpha^2)$ .

On these points, construct the following lines

- One line  $L = \{P, P(0), P(1), P(\alpha), P(\alpha^2)\}$ ,
- $s = 4$  lines  $L(c) = \{P, P(0,c), P(1,c), P(\alpha,c), P(\alpha^2,c)\} : c = 0, 1, \alpha, \alpha^2$  i.e.

when  $c=0$ :  $L(0) = \{P, P(0,0), P(1,0), P(\alpha,0), P(\alpha^2,0)\}$ ,

when  $c=1$ :  $L(1) = \{P, P(0,1), P(1,1), P(\alpha,1), P(\alpha^2,1)\}$ ,

when  $c=\alpha$ :  $L(\alpha) = \{P, P(0,\alpha), P(1,\alpha), P(\alpha,\alpha), P(\alpha^2,\alpha)\}$  &

when  $c=\alpha^2$ :  $L(\alpha^2) = \{P, P(0,\alpha^2), P(1,\alpha^2), P(\alpha,\alpha^2), P(\alpha^2,\alpha^2)\}$

- $s^2 = 16$  lines  $L(r, c): P(c)$  and the points  $P((r + ci) \bmod 2, i)$ , where  $i = 0, 1, \alpha, \alpha^2 : r, c = 0, 1, \alpha, \alpha^2$  i.e.  $i, c$  are elements of  $G.F.(s)$

when  $r,c=0,0$ :  $L(0,0) = \{P(0), P(0,0), P(0,1), P(0,\alpha), P(0,\alpha^2)\}$

when  $r,c=1,0$ :  $L(1,0) = \{P(0), P(1,0), P(1,1), P(1,\alpha), P(1,\alpha^2)\}$

when  $r,c=\alpha,0$ :  $L(\alpha,0) = \{P(0), P(\alpha,0), P(\alpha,1), P(\alpha,\alpha), P(\alpha,\alpha^2)\}$

when  $r,c=\alpha^2,0$ :  $L(\alpha^2,0) = \{P(0), P(\alpha^2,0), P(\alpha^2,1), P(\alpha^2,\alpha), P(\alpha^2,\alpha^2)\}$

when  $r,c=0,1$ :  $L(0,1) = \{P(1), P(0,0), P(1,1), P(\alpha,\alpha), P(\alpha^2,\alpha^2)\}$

when  $r,c=1,1$ :  $L(1,1) = \{P(1), P(1,0), P(0,1), P(\alpha^2,\alpha), P(\alpha,\alpha^2)\}$

when  $r,c=\alpha,1$ :  $L(\alpha,1) = \{P(1), P(\alpha,0), P(\alpha^2,1), P(0,\alpha), P(1,\alpha^2)\}$

when  $r,c=\alpha^2,1$ :  $L(\alpha^2,1) = \{P(1), P(\alpha^2,0), P(\alpha,1), P(1,\alpha), P(0,\alpha^2)\}$

when  $r,c=0,\alpha$ :  $L(0,\alpha) = \{P(\alpha), P(0,0), P(\alpha,1), P(\alpha^2,\alpha), P(1,\alpha^2)\}$

when  $r,c=1,\alpha$ :  $L(1,\alpha) = \{P(\alpha), P(1,0), P(\alpha^2,1), P(\alpha,\alpha), P(0,\alpha^2)\}$

when  $r,c = \alpha, \alpha$ :  $L(\alpha, \alpha) = \{ P(\alpha), P(\alpha,0), P(0,1), P(1,\alpha), P(\alpha^2,\alpha^2) \}$

when  $r,c = \alpha^2, \alpha$ :  $L(\alpha^2, \alpha) = \{ P(\alpha), P(\alpha^2,0), P(1,1), P(0,\alpha), P(\alpha,\alpha^2) \}$

when  $r,c = 0, \alpha^2$ :  $L(0, \alpha^2) = \{ P(\alpha^2), P(0,0), P(\alpha^2,1), P(1,\alpha), P(\alpha,\alpha^2) \}$

when  $r,c = 1, \alpha^2$ :  $L(1, \alpha^2) = \{ P(\alpha^2), P(1,0), P(\alpha,1), P(0,\alpha), P(\alpha^2,\alpha^2) \}$

when  $r,c = \alpha, \alpha^2$ :  $L(\alpha, \alpha^2) = \{ P(\alpha^2), P(\alpha,0), P(1,1), P(\alpha^2,\alpha), P(0,\alpha^2) \}$

when  $r,c = \alpha^2, \alpha^2$ :  $L(\alpha^2, \alpha^2) = \{ P(\alpha^2), P(\alpha^2,0), P(0,1), P(\alpha,\alpha), P(1,\alpha^2) \}$

To construct BIBD give number to the points which are treatments of the designs:

<b>Points</b>	<b>P(0,0)</b>	<b>P(1,0)</b>	<b>P(<math>\alpha</math>,0)</b>	<b>P(<math>\alpha^2</math>,0)</b>	<b>P(0,1)</b>	<b>P(1,1)</b>	<b>P(<math>\alpha</math>,1)</b>
<b>Point Number</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>Points</b>	<b>P(<math>\alpha^2</math>,1)</b>	<b>P(0,<math>\alpha</math>)</b>	<b>P(1,<math>\alpha</math>)</b>	<b>P(<math>\alpha</math>,<math>\alpha</math>)</b>	<b>P(<math>\alpha^2</math>, <math>\alpha</math>)</b>	<b>P(0,<math>\alpha^2</math>)</b>	<b>P(1,<math>\alpha^2</math>)</b>
<b>Point Number</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>Points</b>	<b>P(<math>\alpha</math>,<math>\alpha^2</math>)</b>	<b>P(<math>\alpha^2</math>,<math>\alpha^2</math>)</b>	<b>P</b>	<b>P(0)</b>	<b>P(1)</b>	<b>P(<math>\alpha</math>)</b>	<b>P(<math>\alpha^2</math>)</b>
<b>Point Number</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>

It should be noted that the natural numbers are assigned to the points in a sequence as  $P(0,0)$ ,  $P(1,0)$ , ...,  $P(r,0)$ ;  $P(0,1)$ ,  $P(1,1)$ , ...,  $P(r,1)$ ; ... ;  $P(0,c)$ ,  $P(1,c)$ , ...,  $P(r,c)$ ;  $P$ ;  $P(0)$ ,  $P(1)$ , ...,  $P(c)$ ; where  $r, c = 0, \dots, (s-1)$ ; from  $1, 2, \dots, s^2 + s + 1$  respectively. After numbering and then arranging in ascending or descending order (here ascending) the resulting BIBD is:

**Table – 3.2**

<b>Blocks</b>	<b>Contents</b>				
<b>1</b>	17	18	19	20	21
<b>2</b>	1	2	3	4	17
<b>3</b>	5	6	7	8	17
<b>4</b>	9	10	11	12	17
<b>5</b>	13	14	15	16	17
<b>6</b>	1	5	9	13	18
<b>7</b>	2	6	10	14	18
<b>8</b>	3	7	11	15	18
<b>9</b>	4	8	12	16	18
<b>10</b>	1	6	11	16	19
<b>11</b>	2	5	12	15	19
<b>12</b>	3	8	9	14	19
<b>13</b>	4	7	10	13	19
<b>14</b>	1	7	12	14	20
<b>15</b>	2	8	11	13	20
<b>16</b>	3	5	10	16	20
<b>17</b>	4	6	9	15	20
<b>18</b>	1	8	10	15	21
<b>19</b>	2	7	9	16	21
<b>20</b>	3	6	12	13	21
<b>21</b>	4	5	11	14	21

Using the above explained method, one can construct BIBD through Projective Geometry, of OS2 series for any value of  $s$  whether  $s$  is a prime number or a prime power.

#### 4 Construction of Neighbour Design

Neighbour design may be constructed by using the border plots for a BIBD i.e. one plot is added at each end of each block. Arrangements of treatments at border plots at either end of the block are same as the treatment on the interior plot at the other end of block and border plots are not used for measuring the response variable. Plots other than border plots are described as internal plots for neighbour designs. In this design, all the blocks shall be circular in the sense that the border treatments at either end of the block are the same as the treatment on the interior plot at the other end of block. For a design  $d$ ,  $d(i, j)$  denotes the treatment applied to plot  $j$  of block  $i$ , particularly,  $d(i, 0)$  and  $d(i, k+1)$  are the two treatments applied to the border plots of block  $i$  and the circularity condition implies that  $d(i, 0) = d(i, k)$  and  $d(i, k+1) = d(i, 1)$ ; where  $1 \leq i \leq b$  &  $1 \leq j \leq k$ . These extra parameters used for neighbour plots are needed for the effect of left and right neighbours. Now consider the construction of neighbour design for OS2 series for any value of  $s$  whether  $s$  is a prime number or a prime power.

i) **When  $s = 2$ :** BIBD which has parameters  $v=b=7$ ,  $r=k=3$  &  $\lambda=1$  has been constructed and given in Table 3.1. Neighbour design constructed from this BIBD, by using border plots shall be as follows:

**Table – 4.1**

7	5	6	7	5
5	1	2	5	1
5	3	4	5	3
6	1	3	6	1
6	2	4	6	2
7	1	4	7	1
7	2	3	7	2

In the design obtained here, no treatment is (i) immediate to itself and (ii) immediate to any other treatment more than once, which are the conditions for a neighbour design to be balanced either completely or partially.

ii) **When  $s = 4$ :** Now consider BIBD which has parameters  $v=b=21$ ,  $r=k=5$  &  $\lambda=1$  has been constructed and given in Table 3.2. Neighbour design constructed from this BIBD, by using border plots shall be as follows:

**Table – 4.2**

21	17	18	19	20	21	17
17	1	2	3	4	17	1
17	5	6	7	8	17	5
17	9	10	11	12	17	9
17	13	14	15	16	17	13
18	1	5	9	13	18	1
18	2	6	10	14	18	2
18	3	7	11	15	18	3
18	4	8	12	16	18	4
19	1	6	11	16	19	1
19	2	5	12	15	19	2
19	3	8	9	14	19	3
19	4	7	10	13	19	4
20	1	7	12	14	20	1
20	2	8	11	13	20	2
20	3	5	10	16	20	3
20	4	6	9	15	20	4
21	1	8	10	15	21	1
21	2	7	9	16	21	2
21	3	6	12	13	21	3
21	4	5	11	14	21	4

In the design obtained here, all the blocks are circular in the sense that the border treatments at either end of the blocks are same as the treatment on the interior plot at the other end of block. Similarly the neighbour designs for  $s = 5, 2^3=8, 3^2=9, 11$  and so on, whether  $s$  is a prime number or prime power, can be constructed easily.

## **5 Two-Sided Neighbours of Treatment for Neighbour Designs of OS2 Series**

For the analysis of neighbour designs, left neighbours and right neighbours of a treatment must be known. Laxmi and Parmita (2010) have defined that the left neighbour of a treatment is immediately previous treatment, which may be written as  $i-1$  and the left common neighbours are defined as the series of 's' treatments which is immediately previous series of the series in which treatment number 'i' lies, assuming the treatment in circular way. Similarly, the right

neighbour of a treatment is immediately next treatment, which may be written as  $i+1$  and the right common neighbours are defined as the series of 's' treatments which is immediately next series of the series in which treatment number 'i' lies, assuming the treatment in circular way. For more detail, one may refer to Laxmi and Parmita (2011). Laxmi et al. (2013) has obtained two-sided neighbours of a treatment of a neighbour design of OS2 series when constructed by method of MOLS. In the similar fashion, two-sided neighbours of a treatment are observed for design given in Table 4.1 and these are summarized in the following table:

**Table-5.1**

Other Left Neighbour	Common Left Neighbour Series	Treatment Number (i)	Common Right Neighbour Series	Other Right Neighbour	Series In Which Treatment Number 'i' Lies
5 1	6, 7 6, 7	1 2	3, 4 3, 4	2 5	$1 \leq i \leq 2$
5 3	1, 2 1, 2	3 4	6, 7 6, 7	4 5	$3 \leq i \leq 4$
<b>4</b>	<b>2, 7</b>	<b>5</b>	<b>1, 3</b>	<b>6</b>	<b>i = 5</b>
5 6	3, 4 3, 4	6 7	1, 2 1, 2	7 5	$6 \leq i \leq 7$

From the above table we observe that there shall be  $s+1(=3)$  left neighbours and  $s+1(=3)$  right neighbours for each treatment of OS2 series and none of these two sided neighbours is common. Therefore, there must be  $2s+2(=6)$  neighbours in total when considering both sided neighbours simultaneously.

Now considering the Table 4.2 two-sided neighbours of a treatment of OS2 series for  $s=4$ , where  $s$  is a prime power can be obtained and easily be summarized in the following table:

**Table-5.2**

Other Left Neighbour	Common Left Neighbour Series	Treatment Number (i)	Common Right Neighbour Series	Other Right Neighbour	Series In Which Treatment Number 'i' Lies
17 1 2 3	18,19,20,21 18,19,20,21 18,19,20,21 18,19,20,21	1 2 3 4	5, 6, 7, 8 5, 6, 7, 8 5, 6, 7, 8 5, 6, 7, 8	2 3 4 17	$1 \leq i \leq 4$
17 5 6 7	1, 2, 3, 4 1, 2, 3, 4 1, 2, 3, 4 1, 2, 3, 4	5 6 7 8	9,10,11,12 9,10,11,12 9,10,11,12 9,10,11,12	6 7 8 17	$5 \leq i \leq 8$
17 9 10 11	5, 6, 7, 8 5, 6, 7, 8 5, 6, 7, 8 5, 6, 7, 8	9 10 11 12	13,14,15,16 13,14,15,16 13,14,15,16 13,14,15,16	10 11 12 17	$9 \leq i \leq 12$



17	9,10,11,12	13	18,19,20,21	14	$13 \leq i \leq 16$
13	9,10,11,12	14	18,19,20,21	15	
14	9,10,11,12	15	18,19,20,21	16	
15	9,10,11,12	16	18,19,20,21	17	
<b>16</b>	<b>4, 8,12,21</b>	<b>17</b>	<b>1, 5, 9,13</b>	<b>18</b>	<b><math>i = 17</math></b>
17	13,14,15,16	18	1, 2, 3, 4	19	$18 \leq i \leq 21$
18	13,14,15,16	19	1, 2, 3, 4	20	
19	13,14,15,16	20	1, 2, 3, 4	21	
20	13,14,15,16	21	1, 2, 3, 4	17	

Neighbours for different values of  $s$ , can be obtained in the similar way and further be summarized easily in Tabular form. It is found that neighbours of a particular treatment are same whether a neighbour design for OS2 series is constructed either using method of MOLS or using method of Projective Geometry. The only difference is in their order of occurrence, which may be due to the shifting of the blocks in the neighbour designs for OS2 series when constructed using Projective Geometry. As discussed by Laxmi et al. (2013) both sided neighbours of a treatment can be obtained from the following table for any value of  $s$ , whether  $s$  is a prime number or prime power.

**Table-5.3**

Other Left Neighbour	Common Left Neighbour Series	Treatment Number (i)	Common Right Neighbour Series	Other Right Neighbour	Series In Which Treatment Number 'i' Lies
$s^2+1$	$s^2+2, \dots, s^2+s+1$	1	$s+1, \dots, 2s$	$i+1$	$1 \leq i \leq s$
$i-1$	$s^2+2, \dots, s^2+s+1$	2	$s+1, \dots, 2s$	$i+1$	
.	.	.	.	.	
.	.	.	.	.	
$i-1$	$s^2+2, \dots, s^2+s+1$	$s$	$s+1, \dots, 2s$	$s^2+1$	
$s^2+1$	$1, \dots, s$	$s+1$	$2s+1, \dots, 3s$	$i+1$	$s+1 \leq i \leq 2s$
$i-1$	$1, \dots, s$	$s+2$	$2s+1, \dots, 3s$	$i+1$	
.	.	.	.	.	
$i-1$	$1, \dots, s$	$2s$	$2s+1, \dots, 3s$	$s^2+1$	
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	

$s^2+1$	$(s-2)s+1, \dots, (s-2)s+s$	$(s-1)s+1$	$s^2+2, \dots, s^2+s+1$	$i+1$	$(s-1)s+1 \leq i \leq s^2$
$i-1$	$(s-2)s+1, \dots, (s-2)s+s$	$(s-1)s+2$	$s^2+2, \dots, s^2+s+1$	$i+1$	
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	
$i-1$	$(s-2)s+1, \dots, (s-2)s+s$	$(s-1)s+s = s^2$	$s^2+2, \dots, s^2+s+1$	$i+1 = s^2+1$	
<b><math>i-1 = s^2</math></b>	<b><math>s, 2s, \dots, (s-1)s \&amp; s^2+s+1</math></b>	<b><math>s^2+1</math></b>	<b><math>1, s+1, 2s+1, \dots, (s-1)s+1</math></b>	<b><math>i+1 = s^2+2</math></b>	<b><math>i = s^2+1</math></b>
$s^2+1$	$(s-1)s+1, \dots, s^2$	$s^2+2$	$1, \dots, s$	$i+1$	$s^2+2 \leq i \leq s^2+s+1$
$i-1$	$(s-1)s+1, \dots, s^2$	$s^2+3$	$1, \dots, s$	$i+1$	
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	
$i-1$	$(s-1)s+1, \dots, s^2$	$s^2+s+1$	$1, \dots, s$	$s^2+1$	

## 6 Steps To Find Two-Sided Neighbours For OS2 Series

One can directly obtain the two-sided neighbours of a treatment for the design constructed, using following steps:

a) *Observe the treatment number 'i', where  $i \neq s^2+1$ .*

b) *Then find the series in which the treatment number 'i' lies.*

The series is defined in such a way that the sequence of first 's' treatments of the design form the first series, the sequence of next 's' treatments i.e. 's+1' to '2s' form the second series and so on up to ' $s^2$ '. Thus have 's' series up to the treatment number ' $s^2$ '. The last series i.e. 's+1'-th series of 's' treatments always starts from treatment number ' $s^2+2$ ' instead of the treatment number ' $s^2+1$ ' and ended on treatment number ' $s^2+s+1$ ' for any 's' whether it is a prime number or prime power. Now the 's+2'-th series of next 's' treatments shall be ' $s^2+s+2$ ' to ' $s^2+2s+1$ ', which with mod(v) reduces to '1' to 's'. So the s+2-th series is again the first series of the design. This shows that the design is circular.

c) *Then find out the immediate common left neighbour series and immediate common right neighbour series for the treatment.*

d) *Other two neighbour treatments i.e. left neighbour and right neighbour can be find by taking left adjacent and right adjacent of the treatment respectively.*

e) *The last and first treatments of each series are respectively the left and right neighbours of the treatment number  $s^2+1$ .*

(This pattern of finding two-sided neighbours has been observed only for OS2 series of BIBD. It may work for other designs also).

In this study, a general method of construction of BIBD through projective geometry is considered and then neighbour design of this is obtained using border plots. A method has been developed to find out two sided neighbours of a treatment of OS2 series. It is a very shortcut and direct method for finding neighbours of a treatment. It is less time consuming as the experimenter can directly find out the neighbours of a particular treatment, using the given steps.

## References

- Arpan Bhowmik, Seema Jaggi, Cini Varghese and Eldho Varghese(2012a): Trend Free Block Design Balanced for Interference Effects. XI Biennial conference of the International Biometric Society (Indian Region) on Computational Statistics and Bio-Sciences, March 8-9, 2012
- Arpan Bhowmik, Seema Jaggi, Cini Varghese and Eldho Varghese(2012b): Block Designs Balanced for Second Order Interference Effects from Neighbouring Experimental Units. *Statistics and Applications*, **10**, Nos. 1&2, 2012 (New Series), pp. 1-12.
- Braun, M.; Kerber, A. and Laue, R. (2005): Systematic construction of q-analogs of  $t$ - $(v, k, \lambda)$  – designs. *Designs, Codes and Cryptography*, **34**(1), 55-70.
- Dembowski, P. (1968): *Finite geometries*. Springer-Verlag, New York. 105.
- Jaggi, Seema, Gupta, V.K., and Ashraf, Jawaid (2006): On block designs partially balanced for neighbouring competition effects. *Journal of the Indian Statistical Association*. **44**(1), 27-41.
- Laxmi, R.R. and Parmita(2010): Pattern of Left Neighbours for Neighbour Balanced Incomplete Block Designs. *Journal of Statistics Sciences*, **2**(2), 91-104.
- Laxmi, R.R. and Parmita(2011): Pattern of Right Neighbours for Neighbour Balanced Incomplete Block Designs. *Journal of Statistics Sciences*, **3**(1), 67-77.
- Laxmi, R.R.; Parmita and Rani, S.(2013): Two-Sided Neighbours for Block Designs with Neighbouring Effects. *IJEST*, **5**(3), 709-716.
- M. Hall, Jr. (1967): *Combinatory Theory*. Blaisdell Publishing Company, Waltham, Mass.
- Miyakawa, M., Munemasa, A. and Yoshiara, S.(1995):On a Class of Small 2-Designs over  $GF(q)$ . *Journal of Combinatorial Designs*, **3**, 61-77.
- Pateria, Dinesh Kumar, Jaggi, Seema, Varghese, Cini and Das, M.N. (2007): Incomplete non-circular block designs for competition effects. *Journal of Statistics & applications*, **5** Nos. 1&2, (New Series), 5-14.
- Rees, D. H. (1967): Some designs of use in serology. *Biometrics*, **23**, 779-791.
- Suzuki, H.,(1990): 2-Designs over  $GF(2^m)$ . *Graphs and Combinatorics*, **6**, 293-296.
- Thomas, S. (1987): Designs over Finite Fields. *Geometriae Dedicata*, **24**, 237-242.
- Tomar, J.S., Jaggi, Seema & Varghese, Cini (2005): On totally balanced block designs for competition effects. *Journal of Applied Statistics*, **32**(1), 87-97.

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