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# Bayesian Estimation for the Class of Life-Time Distributions Under Different Loss Functions

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**Abstract**

In this paper, the class of life-time distributions is considered for Bayesian analysis. The expressions for Bayes estimators of the parameter have been derived under four different prior distributions assuming four different loss functions and the comparison between estimators is made by using the mean square error through generated different sample sizes by using simulation technique.

*Keywords***:** Bayesian Estimation, Informative prior's, Non- informative prior's, Squared Error loss function, Al-Bayatti's loss function, Weighted loss function, LINEX loss function

## 1. Introduction

In Bayesian inference prior information about the parameter of a model is represented by probability function. So in case of assortment of prior information we must be cautious. In more general sense, prior information is a way to recapitulate the available information. There is no exclusive way for the assortment of a prior distribution so the consequence may be negligible and there is always a possibility of obtaining the final answer with the help of distorted prior information. In case of very little explanatory information about the unknown parameter we use non-informative prior. However, if one has sufficient information about the parameter(s), it is better to choose informative prior. In order to handle such situation Laplace, Jeffreys, Lindley etc provides different approaches. In the present study we consider two non-informative (Jeffery's and Quasi) and two informative (Inverse exponential and Pareto Type II) priors.

The class of life-time distributions is very imperative concept when we study the reliability of the system. Schnabel (1991) commenced the Bayesian ideas life testing and reliability analysis under symmetric loss function, Pander and Rai (1992), Dey et al. (2010) explains the Bayesian estimation under different loss factions, Siu and Kelly (1998), Nigm et al. (2003), Murthy et al. (2004) explained different cases of generalize Weibull distribution, Gupta

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& Kundu (2003) noted that the two-parameter EE distribution provides a better fit than the two-parameter Weibull distribution for some specific data. Ahmad et al. (2014) studied the Bayesian analysis of exponentiated inverted Weibull distribution under asymmetric loss functions. Kazimi et al. (2012) explains the preference of prior of class of life time distributions using Jeffery's, Gamma and Gumbell type-II priors.

In this paper, we compare the Bayesian estimators of the parameter of the class of Life time distribution using four different prior distributions (Jeffery, Quasi, inverse Exponential and Pareto 1) distributions under four different loss functions (Squared error, Al-Bayatti's, LINEX and Weighted), the performance of the obtained estimators are compared by using the mean square error, through generated many sample sizes by using simulation technique.

Let us consider a random sample $\underline{x} = (x_1, x_2, ..., x_n)$ of size n taken from the class of life time distributions (suggested by Prakash and Singh (2010)) with unknown parameter $\theta$, then the probability density function is given as

$$f(x) = \frac{\alpha}{\Gamma\beta} \frac{1}{\theta^{\beta}} x^{\alpha\beta-1} e^{-\frac{x^{\alpha}}{\theta}} , \qquad \theta > 0,\ 0 \le x < \propto \tag{1.1}$$

For different values of constants $\alpha$ and $\beta$ the different distribution is

| $\alpha$ | $\beta$ | Distribution |
|---|---|---|
| 1 | | Gamma distribution |
| 1 | 1 | Negative exponential distribution |
| 1 | Positive integer | Erlang distribution |
| | 1 | Weibull distribution |
| 2 | 1 | Rayleigh distribution |
| 2 | 3/2 | Maxwell distribution |

The likelihood function of (1.1) is given as

$$L = \left(\frac{\alpha}{\Gamma\beta}\right)^n \frac{1}{\theta^{\beta n}} \prod_{i=1}^{n} x_i^{\alpha\beta-1} \exp\left( - \frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta} \right) \tag{1.2}$$

and the log likelihood function is given as

$$\log L = n \log\alpha - n\log\Gamma\beta - n\beta\log\theta + (\alpha\beta - 1)\sum_{i=1}^{n}\log x_i - \frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta} \tag{1.3}$$

## 2. Aim of the Bayesian Estimation for the Class of Life time Distributions

The aim of this study is to show which prior (informative or non-informative) is more preferable for our considered class of lifetime model under different loss functions. In this section we studied Bayes estimators under four different loss functions. One is symmetric (squared error) loss function and the others are asymmetric (LINEX, Al-Bayatti's and Weighted) loss functions. Posterior distribution is obtained when prior information is combined with the likelihood. Therefore prior information is necessary for Bayesian approach. The prior information is a purely subjective assessment of an expert before any data has been observed. So here we employ two non- informative (the Jeffrey's and the Quasi) priors along with two informative (the inverse exponential and the Pareto type I) priors for the class of life time distributions.

## 3. Bayesian Analysis using Jeffery's Prior

The Jeffreys' prior proposed by Al-Kutubi (2005) is given as

$$g_1(\theta) \propto \sqrt{I(\theta)}$$

Where $[I(\theta)] = -nE\left[\frac{\partial^2 \log f(x;\theta,\alpha)}{\partial\theta^2}\right]$ is the Fisher's information matrix. For the model (1.1),

$$g_1(\theta) = \frac{1}{\theta} \tag{3.1}$$

Combining the prior distribution in (2.1) and the likelihood function, the posterior density of $\theta$ is derived as follows:

$$\pi_1(\theta \mid x) \propto \left(\frac{\alpha}{\Gamma\beta}\right)^n \frac{1}{\theta^{\beta n}} \prod_{i=1}^{n} x_i^{\alpha\beta-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta}\right) \frac{1}{\theta} \tag{3.2}$$

$$\pi_1(\theta \mid x) = K \frac{1}{\theta^{\beta n+1}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta}\right)$$

where $K^{-1} = \frac{\Gamma(\beta n)}{\left(\sum_{i=1}^{n} x_i^{\alpha}\right)^{\beta n}}$

Hence the posterior distribution using Jeffery's prior is given by

$$\pi_1(\theta \mid x) = \frac{\left(\sum_{i=1}^{n} x_i^{\alpha}\right)^{\beta n}}{\Gamma(\beta n)} \frac{1}{\theta^{\beta n+1}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta}\right) \tag{3.3}$$

which is the probability density function inverse gamma distribution with parameters $\beta n$ and $T = \left(\sum_{i=1}^{n} x_i^{\alpha}\right)$

**3.1. Bayes estimator using Jeffrey's prior under SELF**

The squared error loss function (SELF) was proposed by Legendre (1805). By using squared error loss function $l(\hat{\theta},\theta)=c(\hat{\theta}-\theta)^2$ for some constant c the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^{\infty} c(\hat{\theta}-\theta)^2 \frac{T^{\beta n}}{\Gamma(\beta n)} \frac{1}{\theta^{\beta n+1}} \exp\left(-\frac{T}{\theta}\right) d\theta$$

$$= \frac{cT^{\beta n}}{\Gamma(\beta n)}\left[\hat{\theta}^2 \frac{\Gamma(\beta n)}{T^{\beta n}} - 2\hat{\theta}\frac{\Gamma(\beta n-1)}{T^{\beta n-1}} + \frac{\Gamma(\beta n-2)}{T^{\beta n-2}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T}{(\beta n-1)} \tag{3.4}$$

**3.2. Estimator using Jeffrey's prior under Al-Bayyati's loss function**

Al-Bayyati, (2002) introduced a new loss function using Weibull distribution. By using Al-Bayyati's loss function $l(\theta,\hat{\theta}) = \theta^{c_2}(\hat{\theta}-\theta)^2$, $c_2 \in R$ the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^{\infty} \theta^{c_2}(\hat{\theta}-\theta)^2 \frac{T^{\beta n}}{\Gamma(\beta n)} \frac{1}{\theta^{\beta n+1}} \exp\left(-\frac{T}{\theta}\right) d\theta$$

$$= \frac{T^{c_2}}{\Gamma(\beta n)}\left[\hat{\theta}^2\,\Gamma(\beta n-c_2) - 2\hat{\theta}T\,\Gamma(\beta n-c_2-1) + T^2\,\Gamma(\beta n-c_2-2)\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T}{(\beta n-c_2-1)} \tag{3.5}$$

### 3.3. Estimator using Jeffrey's prior under Weighted loss function

By using weighted loss function $l(\hat{\theta},\theta)=\frac{(\theta-\hat{\theta})^2}{\theta}$ , the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty \frac{(\theta-\hat{\theta})^2}{\theta}\frac{T^{\beta n}}{\Gamma(\beta n)}\frac{1}{\theta^{\beta n+1}}\exp\left(-\frac{T}{\theta}\right)d\theta$$

$$=\frac{T^{\beta n}}{\Gamma(\beta n)}\left[\frac{\Gamma(\beta n-1)}{T^{\beta n-1}}-2\hat{\theta}\frac{\Gamma(\beta n)}{T^{\beta n}}+\hat{\theta}^2\frac{\Gamma(\beta n+1)}{T^{\beta n+1}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}=\frac{T}{\beta n} \tag{3.6}$$

### 3.4. Estimator using Jeffrey's prior under LINEX loss function

The LINEX loss function was introduced by Klebanov (1972) and used by Varian (1979) in the context of real life assessment. By using LINEX loss function $l(\hat{\theta},\theta)=\exp a\left(\frac{\hat{\theta}}{\theta}-1\right)-a\left(\frac{\hat{\theta}}{\theta}-1\right)-1$ , the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty\left(\exp a\left(\frac{\hat{\theta}}{\theta}-1\right)-a\left(\frac{\hat{\theta}}{\theta}-1\right)-1\right)\frac{T^{\beta n}}{\Gamma(\beta n)}\frac{1}{\theta^{\beta n+1}}\exp\left(-\frac{T}{\theta}\right)d\theta$$

$$=\frac{T^{\beta n}}{\Gamma(\beta n)}\left[\frac{e^{-a}\,\Gamma(\beta n)}{(T-a\hat{\theta})^{\beta n}}-a\hat{\theta}\frac{\Gamma(\beta n+1)}{T^{\beta n+1}}+a\frac{\Gamma(\beta n)}{T^{\beta n}}-\frac{\Gamma(\beta n)}{T^{\beta n}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}=\frac{T}{a}\left\{1-\exp\left(-\frac{a}{\beta n+1}\right)\right\} \tag{3.7}$$

## 4. Bayesian Analysis using Quasi Prior

When there is no information about the parameter $\theta$, one may use the quasi density as given by:

$$g_2(\theta)=\frac{1}{\theta^d}\quad,\theta>0,d>0 \tag{4.1}$$

The quasi prior leads to diffuse prior when $d$=0 and to a non informative prior for a case when $d$=1.

Combining the prior distribution in (4.1) and the likelihood function, the posterior density of $\theta$ is derived as follows:

$$\pi_2(\theta \mid x) \propto \left(\frac{\alpha}{\Gamma\beta}\right)^n \frac{1}{\theta^{\beta n}} \prod_{i=1}^{n} x_i^{\alpha\beta-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta}\right) \frac{1}{\theta^d} \tag{4.2}$$

$$\pi_2(\theta \mid x) = K \frac{1}{\theta^{\beta n+d}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta}\right)$$

where $K^{-1} = \dfrac{\Gamma(\beta n + d - 1)}{\left(\sum_{i=1}^{n} x_i^{\alpha}\right)^{\beta n+d-1}}$

Hence the posterior distribution using Quasi prior is given by

$$\pi_2(\theta \mid x) = \frac{\left(\sum_{i=1}^{n} x_i^{\alpha}\right)^{\beta n+d-1}}{\Gamma(\beta n + d - 1)} \frac{1}{\theta^{\beta n+d}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta}\right) \tag{4.3}$$

which is the probability density function inverse gamma distribution with parameters $\beta n + d - 1$ and $T = \left(\sum_{i=1}^{n} x_i^{\alpha}\right)$

### 4.1. Bayes estimator using Quasi prior under SELF

By using squared error loss function $l(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2$ for some constant c the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^{\infty} c(\hat{\theta}-\theta)^2 \frac{T^{\beta n+d-1}}{\Gamma(\beta n + d - 1)} \frac{1}{\theta^{\beta n+d}} \exp\left(-\frac{T}{\theta}\right) d\theta$$

$$= \frac{cT^{\beta n+d-1}}{\Gamma(\beta n + d - 1)} \left[\hat{\theta}^2 \frac{\Gamma(\beta n + d - 1)}{T^{\beta n+d-1}} - 2\hat{\theta} \frac{\Gamma(\beta n + d - 2)}{T^{\beta n+d-2}} + \frac{\Gamma(\beta n + d - 3)}{T^{\beta n+d-3}}\right]$$

Now solving $\dfrac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T}{(\beta n + d - 2)} \tag{4.4}$$

### 4.2. Estimator using Quasi prior under Al-Bayyati's loss function

By using Al-Bayyati's loss function $l(\theta,\hat{\theta}) = \theta^{c_2}(\hat{\theta}-\theta)^2$, $c_2 \in R$ the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty \theta^{c_2}(\hat{\theta}-\theta)^2 \frac{T^{\beta n+d-1}}{\Gamma(\beta n+d-1)} \frac{1}{\theta^{\beta n+d}} \exp\left(-\frac{T}{\theta}\right) d\theta$$

$$= \frac{T^{\beta n+d-1}}{\Gamma(\beta n+d-1)}\left[\hat{\theta}^2 \frac{\Gamma(\beta n+d-c_2-1)}{T^{\beta n+d-c_2-1}} - 2\hat{\theta}\frac{\Gamma(\beta n+d-c_2-2)}{T^{\beta n+d-c_2-2}} + \frac{\Gamma(\beta n+d-c_2-3)}{T^{\beta n+d-c_2-3}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T}{(\beta n+d-c_2-2)} \tag{4.5}$$

### 4.3. Bayes estimator using Quasi prior under Weighted loss function

By using weighted loss function $l(\hat{\theta},\theta) = \frac{(\theta-\hat{\theta})^2}{\theta}$, the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty \frac{(\theta-\hat{\theta})^2}{\theta} \frac{T^{\beta n+d-1}}{\Gamma(\beta n+d-1)} \frac{1}{\theta^{\beta n+d}} \exp\left(-\frac{T}{\theta}\right) d\theta$$

$$= \frac{T^{\beta n+d-1}}{\Gamma(\beta n+d-1)}\left[\frac{\Gamma(\beta n+d-2)}{T^{\beta n+d-2}} - 2\hat{\theta}\frac{\Gamma(\beta n+d-1)}{T^{\beta n+d-1}} + \hat{\theta}^2\frac{\Gamma(\beta n+d)}{T^{\beta n+d}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T}{(\beta n+d-1)} \tag{4.6}$$

### 4.4. Estimator using Quasi prior under LINEX loss function

By using LINEX loss function $l(\hat{\theta},\theta) = \exp a\left(\frac{\hat{\theta}}{\theta}-1\right) - a\left(\frac{\hat{\theta}}{\theta}-1\right) - 1$, the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty \left(\exp a\left(\frac{\hat{\theta}}{\theta}-1\right) - a\left(\frac{\hat{\theta}}{\theta}-1\right) - 1\right) \frac{T^{\beta n+d-1}}{\Gamma(\beta n+d-1)} \frac{1}{\theta^{\beta n+d}} \exp\left(-\frac{T}{\theta}\right) d\theta$$

$$= \frac{T^{\beta n+d-1}}{\Gamma(\beta n+d-1)}\left[\frac{e^{-a}\,\Gamma(\beta n+d-1)}{(T-a\hat{\theta})^{\beta n+d-1}} - a\hat{\theta}\frac{\Gamma(\beta n+d)}{T^{\beta n+d}} + a\frac{\Gamma(\beta n+d-1)}{T^{\beta n+d-1}} - \frac{\Gamma(\beta n+d-1)}{T^{\beta n+d-1}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T}{a}\left\{1 - \exp\left(-\frac{a}{\beta n + d}\right)\right\} \tag{4.7}$$

## 5. Bayesian Analysis using Inverse exponential Prior

It is assumed that the prior distribution of $\theta$ is the Inverse exponential distribution with hyper parameter *a* is given as:

$$g_3(\theta) = \frac{a_2}{\theta^2}\exp\left(-\frac{a_2}{\theta}\right); \quad \theta > 0, a_2 > 0 \tag{5.1}$$

Combining the prior distribution in (5.1) and the likelihood function, the posterior density of $\theta$ is derived as follows:

$$\pi_3(\theta \mid x) \propto \left(\frac{\alpha}{\Gamma\beta}\right)^n \frac{1}{\theta^{\beta n}} \prod_{i=1}^{n} x_i^{\alpha\beta-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta}\right) \frac{a_2}{\theta^2}\exp\left(-\frac{a_2}{\theta}\right) \tag{5.2}$$

$$\pi_3(\theta \mid x) = K\frac{1}{\theta^{\beta n+2}} \exp\left(-\frac{(a_2 + \sum_{i=1}^{n} x_i^{\alpha})}{\theta}\right)$$

Where $K^{-1} = \frac{\Gamma(\beta n + 1)}{\left(a_2 + \sum_{i=1}^{n} x_i^{\alpha}\right)^{\beta n+1}}$

Hence the posterior distribution using Inverse Exponential prior is given by

$$\pi_3(\theta \mid x) = \frac{(a_2 + \sum_{i=1}^{n} x_i^{\alpha})^{\beta n+1}}{\Gamma(\beta n + 1)} \frac{1}{\theta^{\beta n+2}} \exp\left(-\frac{(a + \sum_{i=1}^{n} x_i^{\alpha})}{\theta}\right) \tag{5.3}$$

which is the probability density function inverse gamma distribution with parameters $\beta n + 1$ and $T_1 = \left(a_2 + \sum_{i=1}^{n} x_i^{\alpha}\right)$ .

### 5.1. Bayes estimator using Inverse exponential prior under SELF

By using squared error loss function $l(\hat{\theta},\theta)=c(\hat{\theta}-\theta)^2$ for some constant c the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty c(\hat{\theta}-\theta)^2\,\frac{T_1^{\beta n+1}}{\Gamma(\beta n+1)}\frac{1}{\theta^{\beta n+2}}\,\exp\left(-\frac{T_1}{\theta}\right)\,d\theta$$

$$=\frac{cT_1^{\beta n+1}}{\Gamma(\beta n+1)}\left[\hat{\theta}^2\,\frac{\Gamma(\beta n+1)}{T_1^{\beta n+1}}-2\hat{\theta}\frac{\Gamma(\beta n)}{T_1^{\beta n}}+\frac{\Gamma(\beta n-1)}{T_1^{\beta n-1}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}=\frac{T_1}{\beta n} \tag{5.4}$$

### 5.2. Estimator using Inverse exponential prior under Al-Bayyati's loss function

By using Al-Bayyati's loss function $l(\theta,\hat{\theta})=\theta^{c_2}(\hat{\theta}-\theta)^2$, $c_2\in R$ the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty \theta^{c_2}(\hat{\theta}-\theta)^2\,\frac{T_1^{\beta n+1}}{\Gamma(\beta n+1)}\frac{1}{\theta^{\beta n+2}}\,\exp\left(-\frac{T_1}{\theta}\right)\,d\theta$$

$$=\frac{T_1^{c_2}}{\Gamma(\beta n+1)}\left[\hat{\theta}^2\,\Gamma(\beta n-c_2+1)-2\hat{\theta}T_1\,\Gamma(\beta n-c_2)+T_1^{\,2}\,\Gamma(\beta n-c_2-1)\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}=\frac{T_1}{\beta n-c_2} \tag{5.5}$$

### 5.3. Bayes estimator using Inverse exponential prior under Weighed loss function

By using weighted loss function $l(\hat{\theta},\theta)=\frac{(\theta-\hat{\theta})^2}{\theta}$ , the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty \frac{(\theta-\hat{\theta})^2}{\theta}\frac{T_1^{\beta n+1}}{\Gamma(\beta n+1)}\frac{1}{\theta^{\beta n+2}}\,\exp\left(-\frac{T_1}{\theta}\right)\,d\theta$$

$$=\frac{T_1^{\beta n+1}}{\Gamma(\beta n+1)}\left[\hat{\theta}^2\,\frac{\Gamma(\beta n)}{T_1^{\beta n}}-2\hat{\theta}\frac{\Gamma(\beta n+1)}{T_1^{\beta n+1}}+\hat{\theta}^2\,\frac{\Gamma(\beta n+2)}{T_1^{\beta n+2}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T_1}{\beta n + 1} \tag{5.6}$$

**5.4. Estimator using Inverse exponential prior under LINEX loss function**

By using LINEX loss function $l(\hat{\theta},\theta) = \exp a\left(\frac{\hat{\theta}}{\theta} - 1\right) - a\left(\frac{\hat{\theta}}{\theta} - 1\right) - 1$, the risk function is given by

$$R(\hat{\theta},\theta) = \int_0^\infty \left\{ \exp a\left(\frac{\hat{\theta}}{\theta} - 1\right) - a\left(\frac{\hat{\theta}}{\theta} - 1\right) - 1 \right\} \frac{T_1^{\beta n+1}}{\Gamma(\beta n+1)} \frac{1}{\theta^{\beta n+2}} \exp\left(-\frac{T_1}{\theta}\right) d\theta$$

$$= \frac{T_1^{\beta n+1}}{\Gamma(\beta n+1)} \left[ \frac{e^{-a}\,\Gamma(\beta n+1)}{(T_1 - a\hat{\theta})^{\beta n+1}} - a\hat{\theta}\frac{\Gamma(\beta n+2)}{T_1^{\beta n+2}} + a\frac{\Gamma(\beta n+1)}{T_1^{\beta n+1}} - \frac{\Gamma(\beta n+1)}{T_1^{\beta n+1}} \right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T_1}{a}\left\{1 - \exp\left(-\frac{a}{\beta n + 2}\right)\right\} \tag{5.7}$$

## 6. Bayesian Analysis using Pareto I Prior

It is assumed that the prior distribution of $\theta$ is the Inverse exponential distribution with hyper parameter $a$ is given as:

$$g_4(\theta) = b\, a_1^{\,b}\, \theta^{-(b+1)} \;; \quad \theta > b, \quad a_1, b > 0 \tag{6.1}$$

Combining the prior distribution in (6.1) and the likelihood function, the posterior density of $\theta$ is derived as follows:

$$\pi_4(\theta \mid x) \propto \left(\frac{\alpha}{\Gamma\beta}\right)^n \frac{1}{\theta^{\beta n}} \prod_{i=1}^{n} x_i^{\alpha\beta-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta}\right) b\, a_1^{\,b}\, \theta^{-(b+1)} \tag{6.2}$$

$$\pi_4(\theta \mid x) = K \frac{1}{\theta^{\beta n+b+1}} \exp\left( -\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta} \right)$$

where $K^{-1} = \dfrac{\Gamma(\beta n + b)}{\left(\sum_{i=1}^{n} x_i^{\alpha}\right)^{\beta n+b}}$

Hence the posterior distribution using Pareto type I prior is given by

$$\pi_4(\theta \mid x) = \frac{(\sum_{i=1}^{n} x_i^{\alpha})^{\beta n+b}}{\Gamma(\beta n+b)} \frac{1}{\theta^{\beta n+b+1}} \exp\left( -\frac{\sum_{i=1}^{n} x_i^{\alpha}}{\theta} \right) \tag{6.3}$$

which is the probability density function inverse gamma distribution with parameters $\beta n + 1$ and $T = \left( \sum_{i=1}^{n} x_i^{\alpha} \right)$

**6.1. Bayes estimator using Pareto I prior under SELF**

By using squared error loss function $l(\hat{\theta}, \theta) = c(\hat{\theta} - \theta)^2$ for some constant c the risk function is given by

$$R(\hat{\theta}, \theta) = \int_0^{\infty} c(\hat{\theta} - \theta)^2 \frac{T^{\beta n+b}}{\Gamma(\beta n+b)} \frac{1}{\theta^{\beta n+b+1}} \exp\left( -\frac{T}{\theta} \right) d\theta$$

$$= \frac{cT^{\beta n+b}}{\Gamma(\beta n+b)} \left[ \hat{\theta}^2 \frac{\Gamma(\beta n+b)}{T^{\beta n+b}} - 2\hat{\theta} \frac{\Gamma(\beta n+b-1)}{T^{\beta n+b-1}} + \frac{\Gamma(\beta n+b-2)}{T^{\beta n+b-2}} \right]$$

Now solving $\dfrac{\partial R(\hat{\theta}, \theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T}{(\beta n + b - 1)} \tag{6.4}$$

**6.2. Bayes estimator using Pareto I prior under Al-Bayyati's loss function**

By using Al-Bayyati's loss function $l(\theta,\hat{\theta})=\theta^{c_2}(\hat{\theta}-\theta)^2$, $c_2\in R$ the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty \theta^{c_2}(\hat{\theta}-\theta)^2\frac{T^{\beta n+b}}{\Gamma(\beta n+b)}\frac{1}{\theta^{\beta n+b+1}}\exp\left(-\frac{T}{\theta}\right)d\theta$$

$$=\frac{T^{\beta n+b}}{\Gamma(\beta n+b)}\left[\hat{\theta}^2\int_0^\infty\frac{1}{\theta^{\beta n+b-c_2+1}}\exp\left(-\frac{T}{\theta}\right)d\theta-2\hat{\theta}\int_0^\infty\frac{1}{\theta^{\beta n+b-c_2}}\exp\left(-\frac{T}{\theta}\right)d\theta+\int_0^\infty\frac{1}{\theta^{\beta n+b-c_2-1}}\exp\left(-\frac{T}{\theta}\right)d\theta\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}=\frac{T}{\beta n+b-c_2-1} \tag{6.5}$$

**6.3. Bayes estimator using Pareto I prior under Weighted loss function**

By using weighted loss function $l(\hat{\theta},\theta)=\frac{(\theta-\hat{\theta})^2}{\theta}$, the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty\frac{(\theta-\hat{\theta})^2}{\theta}\frac{T^{\beta n+b}}{\Gamma(\beta n+b)}\frac{1}{\theta^{\beta n+b+1}}\exp\left(-\frac{T}{\theta}\right)d\theta$$

$$=\frac{T^{\beta n+b}}{\Gamma(\beta n+b)}\left[\frac{\Gamma(\beta n+b-1)}{T^{\beta n+b-1}}-2\hat{\theta}\frac{\Gamma(\beta n+b)}{T^{\beta n+b}}+\hat{\theta}^2\frac{\Gamma(\beta n+b+1)}{T^{\beta n+b+1}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, we obtain the Bayes estimator as

$$\hat{\theta}=\frac{T}{(\beta n+b)} \tag{6.6}$$

**6.4. Estimator using Pareto I prior under LINEX loss function**

By using LINEX loss function $l(\hat{\theta},\theta)=\exp a\left(\frac{\hat{\theta}}{\theta}-1\right)-a\left(\frac{\hat{\theta}}{\theta}-1\right)-1$, the risk function is given by

$$R(\hat{\theta},\theta)=\int_0^\infty\left\{\exp a\left(\frac{\hat{\theta}}{\theta}-1\right)-a\left(\frac{\hat{\theta}}{\theta}-1\right)-1\right\}\frac{T^{\beta n+b}}{\Gamma(\beta n+b)}\frac{1}{\theta^{\beta n+b+1}}\exp\left(-\frac{T}{\theta}\right)d\theta$$

$$=\frac{T^{\beta n+b}}{\Gamma(\beta n+b)}\left[\frac{e^{-a}\,\Gamma(\beta n+b)}{(T-a\hat{\theta})^{\beta n+b}}-a\hat{\theta}\frac{\Gamma(\beta n+b+1)}{T^{\beta n+b+1}}+a\frac{\Gamma(\beta n+b)}{T^{\beta n+b}}-\frac{\Gamma(\beta n+b)}{T^{\beta n+b}}\right]$$

Now solving $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, we obtain the Bayes estimator as

$$\hat{\theta} = \frac{T}{a}\left\{1 - \exp\left(-\frac{a}{\beta n + b + 1}\right)\right\} \tag{6.7}$$

## 7. Real data analysis

This section presents numerical example for the proposed estimates based on a real data set. For illustration of our proposed estimates, we consider 55 observations of burning velocity (cm/sec) of different chemical materials with $\alpha$ =1.0 and $\beta$ = 1.0, the data is 68, 61, 64, 55, 51, 68, 44, 82, 60, 89, 61, 54, 166, 66, 50, 87, 48, 42, 58, 46, 67, 46, 46, 44, 48, 56, 47, 54, 47, 80, 38, 108, 46, 40, 44, 312, 41, 31, 40, 41, 40, 56, 45, 43, 46, 46, 46, 46, 52, 58, 82, 71, 48, 39, 41. The source of the above explained data related to the burning velocity of different chemical materials for the year 2005 is available on the website (http://www.cheresources.com/mists.pdf). By using different non-informative priors i.e.Jeffreys and Quasi priors and non informative i.e invese Exponential and Pareto 1 prior under different Loss functions i.e. Square Error loss function, Albayyati loss function, Weighted loss function and LINEX loss function, the Bayes estimates and Posterior variance of the posterior distribution are as follows where posterior variances are in parentheses.

**Table 1.** Bayes estimates and Posterior variances

| Prior | SELF | ABL | | WL | LINEX | |
|---|---|---|---|---|---|---|
| | | $c_2$=2 | $c_2$= -2 | | a=0.5 | a= -0.5 |
| JP | 62.1296<br>(0.5630) | 64.5192<br>(0.6071) | 59.9107<br>(0.5235) | 61.00<br>(0.5427) | 59.6440<br>(0.5188) | 60.1789<br>(0.5282) |
| QP | 62.7102<br>(0.5735) | 65.1456<br>(0.6189) | 60.4504<br>(0.5329) | 61.5596<br>(0.5527) | 60.1789<br>(0.5282) | 60.7235<br>(0.5378) |
| IEP | 61.0090<br>(0.5427) | 63.3113<br>(0.5844) | 58.8684<br>(0.5053) | 59.9196<br>(0.5235) | 58.6109<br>(0.5008) | 59.1273<br>(0.5097) |
| PP | 61.5596<br>(0.5527) | 63.9047<br>(0.5956) | 59.3805<br>(0.5142) | 60.4504<br>(0.5329) | 59.1185<br>(0.5097) | 59.6440<br>(0.5188) |

JP=Jeffery's Prior, QP=Quasi prior, IEP=inverse Exponential prior, PP=Pareto prior, SELF= Squared error loss function, ABL= Al-Bayattis's loss function, WL=Weighted loss function, LINEX=linear exponential loss function.

On comparing the Bayes posterior variances of different loss functions, it is observed that the LINEX loss function has less Bayes posterior variance than other loss functions. According to the decision rule of less Bayes posterior variance we conclude that the LINEX loss function is more preferable loss function.

## 8. Simulation Study

This section shows how simulation can be helpful and illuminating way to approach problems in Bayesian analysis. In our simulation study, we chose a sample size of n=25, 50 and 100 to represent small, medium and large data set. The Bayes estimates are estimated for the class of Life time distributions using informative (Jeffrey's and Quasi) & non informative (inverse Exponential and Pareto 1) priors under different loss functions. In order to assess the statistical performances of these estimates, we conducted a simulation study. The mean square error using generated random samples of different sizes are computed for each estimator. The value for the loss parameters $c_2$ is ±2.0 and $a$ is ±0.5. The study has been carried out for different values of $\theta$ keeping $\alpha$ and $\beta$ fixed. This was iterated 5000 times. The results are presented in tables for different selections of the parameters.

**Table 2.** Mean square error using Jeffery's prior

| n | $\theta$ | $\alpha$ | $\beta$ | $\hat{\theta}_{SL}$ | $\hat{\theta}_{AL}$ | | $\hat{\theta}_{WL}$ | $\hat{\theta}_{LL}$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | $c_2$=2 | $c_2$=-2 | | a=0.5 | a=-0.5 |
| 25 | 0.5 | 0.5 | 0.5 | 0.2639 | 0.4928 | 0.1523 | 0.1986 | 0.1429 | 0.1624 |
| | 1.0 | | | 0.0835 | 0.1986 | 0.0697 | 0.0692 | 0.0615 | 0.0685 |
| | 0.5 | 1.0 | 1.0 | 0.0159 | 0.0097 | 0.0231 | 0.0194 | 0.0241 | 0.0222 |
| | 1.0 | | | 0.6540 | 0.7410 | 0.6024 | 0.6246 | 0.5978 | 0.6074 |
| | 0.5 | 2.0 | 1.0 | 0.0464 | 0.0380 | 0.0562 | 0.0514 | 0.0374 | 0.0550 |
| | 1.0 | | | 0.0715 | 0.1176 | 0.0500 | 0.0583 | 0.0486 | 0.0517 |
| 50 | 0.5 | 0.5 | 0.5 | 0.0219 | 0.0359 | 0.0147 | 0.0176 | 0.0142 | 0.0153 |
| | 1.0 | | | 0.0326 | 0.0404 | 0.0388 | 0.0343 | 0.0303 | 0.0375 |
| | 0.5 | 1.0 | 1.0 | 0.0046 | 0.0054 | 0.0047 | 0.0046 | 0.0046 | 0.0047 |
| | 1.0 | | | 0.0247 | 0.0327 | 0.0210 | 0.0224 | 0.0208 | 0.0213 |
| | 0.5 | 2.0 | 1.0 | 0.0836 | 0.0785 | 0.0885 | 0.0861 | 0.0811 | 0.0879 |
| | 1.0 | | | 0.1439 | 0.1248 | 0.1628 | 0.1534 | 0.1651 | 0.1605 |
| 100 | 0.5 | 0.5 | 0.5 | 0.0207 | 0.0276 | 0.0156 | 0.0179 | 0.0150 | 0.0161 |
| | 1.0 | | | 0.1605 | 0.0207 | 0.0193 | 0.0194 | 0.0192 | 0.0190 |
| | 0.5 | 1.0 | 1.0 | 0.0081 | 0.0100 | 0.0067 | 0.0074 | 0.0065 | 0.0068 |
| | 1.0 | | | 0.0099 | 0.0090 | 0.0114 | 0.0106 | 0.0117 | 0.0112 |
| | 0.5 | 2.0 | 1.0 | 0.0706 | 0.0681 | 0.0731 | 0.0718 | 0.0634 | 0.0728 |
| | 1.0 | | | 0.0255 | 0.0210 | 0.0304 | 0.0279 | 0.0210 | 0.0298 |

**Table 3.** Mean square error using Quasi prior

| n | $\theta$ | $\alpha$ | $\beta$ | $\hat{\theta}_{SL}$ | $\hat{\theta}_{AL}$ | | $\hat{\theta}_{WL}$ | $\hat{\theta}_{LL}$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | $c_2$=2 | $c_2$=-2 | | a=0.5 | a=-0.5 |
| 25 | 0.5 | 0.5 | 0.5 | 0.1531 | 0.3139 | 0.0825 | 0.1107 | 0.0770 | 0.0884 |
| | 1.0 | | | 0.7909 | 1.5706 | 0.4356 | 0.5797 | 0.4071 | 0.4664 |
| | 0.5 | 1.0 | 1.0 | 0.0132 | 0.0215 | 0.0103 | 0.0112 | 0.0102 | 0.0104 |
| | 1.0 | | | 0.4681 | 0.6764 | 0.3275 | 0.3910 | 0.3135 | 0.3423 |
| | 0.5 | 2.0 | 1.0 | 0.0266 | 0.0179 | 0.0356 | 0.0311 | 0.0367 | 0.0345 |
| | 1.0 | | | 0.1810 | 0.2837 | 0.1187 | 0.1458 | 0.1130 | 0.1248 |
| 50 | 0.5 | 0.5 | 0.5 | 0.0963 | 0.1415 | 0.0664 | 0.0799 | 0.0635 | 0.0695 |
| | 1.0 | | | 0.1232 | 0.1994 | 0.0805 | 0.0986 | 0.0769 | 0.0845 |
| | 0.5 | 1.0 | 1.0 | 0.0050 | 0.0062 | 0.0048 | 0.0048 | 0.0048 | 0.0048 |
| | 1.0 | | | 0.0653 | 0.0884 | 0.0488 | 0.0563 | 0.0471 | 0.0505 |
| | 0.5 | 2.0 | 1.0 | 0.0734 | 0.0682 | 0.0784 | 0.0759 | 0.0790 | 0.0777 |
| | 1.0 | | | 0.0622 | 0.0912 | 0.0405 | 0.0505 | 0.0382 | 0.0428 |
| 100 | 0.5 | 0.5 | 0.5 | 0.0415 | 0.0531 | 0.0324 | 0.0367 | 0.0315 | 0.0335 |
| | 1.0 | | | 0.0295 | 0.0400 | 0.0237 | 0.0261 | 0.0232 | 0.0242 |
| | 0.5 | 1.0 | 1.0 | 0.0095 | 0.0116 | 0.0078 | 0.0086 | 0.0076 | 0.0080 |
| | 1.0 | | | 0.0099 | 0.0110 | 0.0098 | 0.0098 | 0.0098 | 0.0098 |
| | 0.5 | 2.0 | 1.0 | 0.0755 | 0.0730 | 0.0780 | 0.0768 | 0.0784 | 0.0777 |
| | 1.0 | | | 0.0201 | 0.0249 | 0.0165 | 0.0182 | 0.0162 | 0.0169 |

**Table 4.** Mean square error using inverse Exponential prior

| n | $\theta$ | $\alpha$ | $\beta$ | $\hat{\theta}_{SL}$ | $\hat{\theta}_{AL}$ | | $\hat{\theta}_{WL}$ | $\hat{\theta}_{LL}$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | $c_2$=2 | $c_2$=-2 | | a=0.5 | a=-0.5 |
| 25 | 0.5 | 0.5 | 0.5 | 0.1710 | 0.3182 | 0.0984 | 0.1285 | 0.0923 | 0.1049 |
| | 1.0 | | | 0.1749 | 0.3834 | 0.1004 | 0.1269 | 0.0962 | 0.1054 |
| | 0.5 | 1.0 | 1.0 | 0.0368 | 0.0574 | 0.0244 | 0.0298 | 0.0233 | 0.0256 |
| | 1.0 | | | 0.0546 | 0.0878 | 0.0413 | 0.0460 | 0.0407 | 0.0422 |
| | 0.5 | 2.0 | 1.0 | 0.0536 | 0.0433 | 0.0632 | 0.0584 | 0.0643 | 0.0620 |
| | 1.0 | | | 0.0794 | 0.1285 | 0.0544 | 0.0645 | 0.0525 | 0.0565 |
| 50 | 0.5 | 0.5 | 0.5 | 0.1146 | 0.1625 | 0.0815 | 0.0965 | 0.0782 | 0.0850 |
| | 1.0 | | | 0.0537 | 0.0862 | 0.0409 | 0.0454 | 0.0403 | 0.0417 |
| | 0.5 | 1.0 | 1.0 | 0.0111 | 0.0152 | 0.0084 | 0.0096 | 0.0081 | 0.0086 |
| | 1.0 | | | 0.0321 | 0.0437 | 0.0252 | 0.0281 | 0.0246 | 0.0258 |
| | 0.5 | 2.0 | 1.0 | 0.0558 | 0.0507 | 0.0607 | 0.0583 | 0.0613 | 0.0601 |
| | 1.0 | | | 0.0242 | 0.0320 | 0.0206 | 0.0220 | 0.0204 | 0.0209 |
| 100 | 0.5 | 0.5 | 0.5 | 0.0292 | 0.0379 | 0.0224 | 0.0256 | 0.0217 | 0.0232 |
| | 1.0 | | | 0.0399 | 0.0544 | 0.0303 | 0.0345 | 0.0295 | 0.0313 |
| | 0.5 | 1.0 | 1.0 | 0.0025 | 0.0029 | 0.0024 | 0.0024 | 0.0024 | 0.0024 |
| | 1.0 | | | 0.0109 | 0.0126 | 0.0101 | 0.0104 | 0.0100 | 0.0101 |
| | 0.5 | 2.0 | 1.0 | 0.0760 | 0.0735 | 0.0785 | 0.0772 | 0.0788 | 0.0781 |
| | 1.0 | | | 0.0114 | 0.0133 | 0.0103 | 0.0107 | 0.0102 | 0.0104 |

**Table 5.** Mean square error using Pareto 1 prior

| n | $\theta$ | $\alpha$ | $\beta$ | $\hat{\theta}_{SL}$ | $\hat{\theta}_{AL}$ | | $\hat{\theta}_{WL}$ | $\hat{\theta}_{LL}$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | $c_2$=2 | $c_2$=-2 | | a=0.5 | a=-0.5 |
| 25 | 0.5 | 0.5 | 0.5 | 0.1391 | 0.2505 | 0.0897 | 0.1094 | 0.0859 | 0.0938 |
| | 1.0 | | | 0.1785 | 0.3232 | 0.1451 | 0.1533 | 0.1450 | 0.1459 |
| | 0.5 | 1.0 | 1.0 | 0.0151 | 0.0217 | 0.0133 | 0.0137 | 0.0133 | 0.0133 |
| | 1.0 | | | 0.0782 | 0.1170 | 0.0616 | 0.0677 | 0.0606 | 0.0628 |
| | 0.5 | 2.0 | 1.0 | 0.0421 | 0.0322 | 0.0516 | 0.0469 | 0.0528 | 0.0505 |
| | 1.0 | | | 0.2794 | 0.4061 | 0.1974 | 0.2339 | 0.1896 | 0.2057 |
| 50 | 0.5 | 0.5 | 0.5 | 0.0550 | 0.0812 | 0.0386 | 0.0458 | 0.0371 | 0.0402 |
| | 1.0 | | | 0.0682 | 0.1001 | 0.0566 | 0.0604 | 0.0561 | 0.0572 |
| | 0.5 | 1.0 | 1.0 | 0.0063 | 0.0078 | 0.0058 | 0.0059 | 0.0058 | 0.0058 |
| | 1.0 | | | 0.0420 | 0.0559 | 0.0331 | 0.0370 | 0.0323 | 0.0340 |
| | 0.5 | 2.0 | 1.0 | 0.0423 | 0.0373 | 0.0470 | 0.0447 | 0.0476 | 0.0465 |
| | 1.0 | | | 0.0669 | 0.0888 | 0.0513 | 0.0584 | 0.0497 | 0.0530 |
| 100 | 0.5 | 0.5 | 0.5 | 0.0176 | 0.0233 | 0.0135 | 0.0153 | 0.0130 | 0.0139 |
| | 1.0 | | | 0.0314 | 0.0410 | 0.0263 | 0.0284 | 0.0259 | 0.0267 |
| | 0.5 | 1.0 | 1.0 | 0.0028 | 0.0032 | 0.0027 | 0.0027 | 0.0027 | 0.0027 |
| | 1.0 | | | 0.0244 | 0.0299 | 0.0201 | 0.0221 | 0.0197 | 0.0206 |
| | 0.5 | 2.0 | 1.0 | 0.0561 | 0.0536 | 0.0586 | 0.0573 | 0.0589 | 0.0583 |
| | 1.0 | | | 0.0167 | 0.0203 | 0.0142 | 0.0153 | 0.0139 | 0.0144 |

In table 2-5, Bayes estimation with LINEX Loss function provides the smallest values in maximum cases especially when loss parameter a is 0.5. When the value of the parameters $\alpha = 2$ and $\beta = 1$ (i.e for Rayleigh distribution) Al-Bayatti's loss function provides least mean square. Similarly, the increased true parametric values impose a negative impact on the convergence of the estimates. Also among the priors the inverse exponential prior (informative) is compatible for the unknown parameter of the class of life-time distributions and preferable over all other competitive priors because of having less mean square error. Moreover, when the sample size increases from 25 to 100, the MSE decreases quite significantly.

## 9. Conclusion

We consider the Bayesian analysis of the class of life-time distributions using different informative and non-informative priors. After analysis we conclude that the inverse exponential prior (informative) is compatible for the unknown parameter of the class of life-time distributions and preferable over all other competitive priors because of having less posterior Variance and mean square error. As far as choice of loss function is concerned, one can easily observe based on evidence of different properties as discussed above that LINEX loss function has smaller mean square error. Further, as we increase sample size posterior variance and mean square error comes down.

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## References


Ahmad, A., Ahmad, S.P. and Ahmed, A. (2014). Bayesian estimation of exponentiated inverted Weibull distribution under asymmetric loss functions. *Journal of Statistics Applications and Probability*, **4(1)**, 183-192.

Al-Bayyati, (2002). *Comparing methods of estimating Weibull failure models using simulation*. Ph.D. Thesis, College of Administration and Economics, Baghdad University, Iraq.

Al-Kutubi, H.S. (2005). On comparison estimation procedures for parameter and survival function. *Iraqi Journal of Statistical Science*, **9**, 1-14.

Dey, S. and Sudhansu, S.M. (2010). Bayesian estimation of the parameter of Maxwell distribution under different loss functions. *J. Stat. Theory Pract*., **4**, 279-287.

Gupta, R.D. and Kundu, D. (2001). Exponentiated exponential family: An alternative to Gamma and Weibull distributions. *Biometrical journal*, **43**, 117-130.

Jeffery's, H. (1964). An invariant form for the prior probability in estimation problems. *Proceedings of the Royal Society of London*, Series. A., **186**, 453-461.

Kazmi, S.M., Aslam, M. and Ali, S. (2012). Preference of prior for the class of life-time distributions under different loss functions. *Pak. J. Statist*. **28(4)**, 467-484.

Klebanov, L.B. (1972).Universal loss function and unbiased estimation. Dokl. Akad. Nank SSSR Soviet-Maths. Dokl. T., **203**, 1249-1251.

Legendre, A. (1805). *New Methods for the determination of Orbits of Comets*. Courcier, Paris.

Murthy, D.N.P., Xie, M. and Jiang, R. (2004). *Weibull Models*. John Wiley and Sons, Inc., Hoboken, New Jersey.

Nigm, A.M., AL-Hussaini, E.K. and Jaheen, Z.F. (2003). Bayesian one sample prediction of future observations under Pareto distribution. *Statistics*, **37(6)**, 527-536.

Pandey, B.N. and Rai, O. (1992). Bayesian estimation of mean and square of mean of Normal distribution using LINEX loss function. *Commun. Statist, Theory and Meth.,* **21(2)**, 3369-3391.

Prakash, G. and Singh, D.C. (2010). Bayesian Shrinkage estimation in a class of life testing distribution. *Data Science Journal*, **8(30)**, 243-258.

Schabe, H. (1991). Bayes estimates under asymmetric loss. IEEE Trans. Reliab., **40(1)**, 63-67.

Siu, N.O. and Kelly, D.L. (1998). Bayesian parameter estimation in probabilistic risk assessment. Reliability Engineering and System Safety, **62**, 89-116.

Varian, H.R. (1979). A Bayesian approach to real state assessment, In: Studies in Bayesian Econometrics and Statistics in honour of L.J. Savage", Eds. Feinberg, S.E. and Zellner, A. (North Holland, Amsterdam), 195–208.