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Stochastic Modelling and Analysis of a Parallel Mixer-Crane System of Steel Melting Shop of an Integrated Steel Plant

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Abstract

This paper deals with the stochastic modelling and analysis of a parallel mixer-crane system, which plays an important role in the steel melting shop area which is comprises of two parallel mixer-crane system. This mixer- crane system is helpful in the process of blooms which are the raw materials in the manufacturing of rails and structural. Two mixers are installed in this area that work parallel and helps to maintain the temperature and composition of pig iron which is obtained from the blast furnace area in liquid form. One crane is attached to each mixer which helps in pouring the liquid pig iron into the mixer. Once the composition of the hot metals is changed inside the mixer, it is sent for further purification. Besides simple repair, cold repair, capital repair and shut down repairs are also performed. Failure time distribution are taken to be negative exponential whereas repair time distribution are taken to be arbitrary. Several system characteristics which are useful for the system managers and designers are evaluated. Finally, some graphs are also plotted in order to highlight the important results.

Keywords: Mean time to system failure, Availability, Busy period of repairman, Preparation time for repair.

1 Introduction

Reliability is both desirable as well as a necessary factor in the present-day technology for achieving healthy economic progress of a nation. Now-a-days, most of the old industries are under modernization in order to face the competition of the international market. In each industry, main aim is to maximize the profit and reduce the maintenance cost. System engineers and managers are interested in the study of those models that really exist in the industries. During the last three decades, reliability technology has been developed in order to use it in technological fields. It is most frequently used in the development of electrical and electronic equipment's. Further, now a days this technology is very useful in the study of most of the machine problems of most of the industries. Most of the researchers of field of reliability theory have studied only theoretical models such as parallel, series, complex, K out of n:F, k out of n:G,

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etc. But in order to make the production up-to-date practical models have to be taken into consideration.

A lot of work has been done in the field of reliability analyzing models based on producing different kind of products. Kumar et al.(1992) obtained the availability of crystallization system in sugar industry under common cause failure. Later on, Kaushik and Singh (1994) performed the reliability analysis of the naphtha fuel oil and water system under priority repair used in thermal power plant. Singh et al.(1995) have studied a stone crushing system having one apron feeder, one grizzly, one primary Gyratory crusher. This group of equipments is used to get iron ores from stones in mining crushing plants. Kumar et al. (1997) have analysed ash handling system in a thermal power plant. Singh et al.(1999) have studied configurational modelling and analysis of a coke pushing system of coke oven area of an integrated steel plant. They have obtained various parameters of the systems which are useful to the system managers and engineers. Gupta and Shivakar (2003) performed the stochastic analysis of a cloth weaving system model. Gupta et al. (2005) discussed the reliability and availability analysis of serial processes of butter oil plant and behavior analysis of the cement industry. Gupta and Kumar (2007) carried out the cost-benefit analysis of a distillery plant. Kumar and Tiwari (2008) presented the analysis for evaluating the performance measures for co-sift conversion system model in a fertilizer plant. Khanduja et al. (2012) analyzed a bleaching system model of a paper plant regarding the steady-state behavior and maintenance planning. More recently, some of the other industrial system models producing different products have been already analyzed by Damghani et al. (2013), Kumar, et al. (2013), Devet al. (2013) and Vaynas and Peng (2014).

Keeping in view the importance of analyzing a real existing industrial system model, this paper deals with the Hot metal crane system of steel melting shop area of integrated steel Plant of Bhilai, India. Bhilai Steel Plant being one of the leading steel plants of India is having the following major units:

1. Coke-ovens 2. Sintering plant No. 1 3. Sintering plant No.
4. Blast furnaces 5. Steel melting shop 6. Oxygen Blow Converter
7. Continuous Casting Plant 8. Blooming Mill 9. Billet Mill
10. Rail and Structural Mill 11. Merchant Mill 12. Wire rod mill
13. Plate Mill

2. Model

This model deals with the stochastic modeling and analysis of a **Mixer-Crane system** which plays an important role in the steel melting shop area which is comprised of mixer, furnace, stripper yard and mould yards. This mixer-crane system is helpful in the process of making blooms which is the raw material in the manufacturing of rails and structural. Two mixers are installed in this area that work parallel and help to maintain the temperature and composition of pig iron which is obtained from the blast furnace area in liquid form. One crane is attached to each mixer which helps in pouring the liquid pig-iron into the mixer. Once the composition of the hot metal is changed inside the mixer, it is sent for further purification. Besides regular repair; cold repair, capital repair and shut down repairs are also performed.

Failure time distributions are taken to be negative exponential, whereas repair time distributions are taken to be arbitrary. Using regenerative point technique, several system characteristics, which are useful to the system managers and designers are evaluated. At last some graphs are plotted in order to highlight the important results. Using regenerative point technique, few operating characteristics of system model are evaluated to carry out the profit analysis.

Using regenerative point technique, following measures of system effectiveness are obtained to carry out the profit analysis:

(i) Steady state transition probabilities and Mean sojourn times in different states;
(ii) Mean time to system failure;
(iii) Availability of the system in (0,t] and in steady state;
(iv) Expected busy period of the repairman in repair in (0,t] and in steady state;
(v) Expected busy period of the repairman in Capital repair in (0,t] and in steady state;
(vi) Expected busy period of the repairman in Cold repair in (0,t] and in steady state;
(vii) Expected busy period of the repairman Preparation in (0,t] and in steady state;
(viii) Expected busy period of the repairman in Shut down in (0,t] in steady state;
(ix) Expected profit earned by the system in (0,t] and in steady state.

At last some particular cases are also discussed and graphs are also plotted to highlight the important results.

(a) Description of the System

1. There are two subsystems working in parallel. Each subsystem consists of one mixer and crane.
2. After a random time, system may leave for cold repair / capital repair provided all the units in the system are in good and working condition.
3. Any of the subsystem undergoes repair if any of the mixer fails provided crane of the same subsystem is already under repair.
4. Shut down occurs in the following cases:
   (i) mixer or crane of any subsystem fails provided one mixer is already under repair.
   (ii) mixer / crane of any subsystem fails provided one subsystem is under repair.
   (iii) if a subsystem is under preparation and in the second one, mixer fails when the crane is already under repair.
   (iv) if whole system is working with a single mixer and failure of that mixer leads to shut-down.
   (v) any of the mixer or crane fails provided a subsystem is kept either in capital repair or cold repair.
   (vi) both the cranes fail.
5. After shut-down / capital repair / cold repair / simple repair, subsystems undergo for fresh preparation.
6. Failure time distribution of each unit is taken as exponentially distributed, whereas all the repair time distributions are taken as arbitrarily distributed.
7. After repair units work as good as new.

State Transition Diagram and Graphs are shown in Figures1.1,1.2,1.3 and 1.4, respectively.

(b) Notations

E_1 : Set of regenerative states $\{S_0 - S_9\}$

E_2 : Set of non-regenerative states $\{S_{10}, S_{11}\}$

$\boxed{c}$: Laplace Convolution

$\boxed{s}$: Laplace Stieltjes Convolution

a(t),A(t) : pdf and cdf of cold repair time.

b(t),B(t) : pdf and cdf of capital repair time.

λ : Constant failure rate of Mixer.

β : Constant failure rate of Crane.

$g_i(t), G_i(t)$, $i = 1,5$: pdf and cdf of repair time of mixer/crane.

$g_i(t), G_i(t)$, $i = 2,3,4,6$: pdf and cdf of cold / shut-down / capital / sub-system under repair time.

$B_i^j(t)$:Probability that failed unit is under simple/cold/shut down repair for j = 2,3,4,5 or subsystem under preparation for j=6 respectively starting from regenerative state i.

(c) Symbols Used for the States of the System

M_0 / M_r : Mixer in Operation / under repair

C_0 / C_r / C_{wr} / C_I / C_R : in operation/under repair /waiting for repair /in ideal state/repair in continuation from previous state

UP, UP_c, SD, SUR, UCR : Under preparation / Preparation continued from previous state/Shut down repair/Subsystem under repair /Subsystem under capital repair

3 Transition Probabilities and Sojourn Times

Simple probabilistic considerations yield the following expressions for non-zero transition probabilities (p_{ij})

$p_{01}(t) = \int_0^\infty 2\lambda \bar{A}(t)\bar{B}(t)e^{-2c(t)}\,dt$; $p_{02}(t) = \int_0^\infty 2\beta \bar{A}(t)\bar{B}(t)e^{-2c(t)}\,dt$; $p_{03} = \int_0^\infty a(t)\ \bar{B}(t)e^{-2c(t)}\,dt$

$p_{04} = \int_0^\infty b(t)\bar{A}(t)e^{-2c(t)}\,dt$; $p_{15}(t) = \int_0^\infty c\,\bar{G}_5(t)\bar{B}(t)e^{-c(t)}\,dt$; $p_{16}(t) = \int_0^\infty g_5(t)\ e^{-c(t)}\,dt$;

$$p_{20}(t) = \int_0^\infty g_1(t)\, e^{-\lambda t}\, dt$$

$$p_{26} = \int_0^\infty \beta \bar{G}_1(t) e^{-ct}\, dt\,;\; p_{27} = \int_0^\infty p_1\, \lambda\, \bar{G}_1(t) e^{-\lambda t}\, dt\,;\; p_{28} = \int_0^\infty p_2\, \lambda\, \bar{G}_1(t) e^{-\lambda t}\, dt$$

$$p_{35} = \int_0^\infty g_2(t)\; e^{-ct}\, dt\,;\;\; p_{36} = \int_0^\infty c\; \bar{G}_2(t)\; e^{-ct}\, dt\,;\;\; p_{45} = \int_0^\infty g_4(t)\; e^{-ct}\, dt$$

$$p_{46} = \int_0^\infty c\; \bar{G}_4(t)\; e^{-ct}\, dt\,;\;\; p_{50} = \int_0^\infty h(t)\; e^{-ct}\, dt\,;\; p_{56} = \int_0^\infty c\; \bar{H}(t)\; e^{-ct}\, dt$$

$$p_{01}(t) = 1;\;\; p_{75} = \int_0^\infty g_6(t)\; e^{-ct}\, dt\,;\; p_{76} = \int_0^\infty c\; \bar{G}_6(t)\; e^{-ct}\, dt\,;\; p_{86} = \int_0^\infty \lambda\; \bar{G}_5(t)\; e^{-\lambda t}\, dt$$

$$p_{89} = \int_0^\infty g_5(t)\; e^{-\lambda t}\, dt\,;\; p_{90v10} = \int_{u=0}^\infty h(u)\; e^{-\lambda u}\, du \int_{v=u}^\infty g_1(v) e^{-c(v-u)} dv$$

$$p_{96v10} = \int_{u=0}^\infty h(u)\; e^{-\lambda u}\, du \int_{v=u}^\infty \beta\, e^{-\beta v}\, \bar{G}_1(v) e^{-c(v-u)} dv\,;$$

$$p_{97v10} = \int_{u=0}^\infty h(u)\; e^{-\lambda u}\, du \int_{v=u}^\infty p_1 \lambda\; \bar{G}_1(v) e^{-c(v-u)} dv$$

$$p_{96v11} = \int_{u=0}^\infty g_1(u)\; e^{-\lambda u}\, du \int_{v=u}^\infty \frac{c}{(\lambda-\beta)} e^{-c(v-u)}\; \bar{H}(v)\, dv$$

where ; $\lambda + \beta = c$ (3.1-3.25)

Mean sojourn time μ_i is defined as the time that the system continues in state S_i before transiting to any other state.

$$\mu_0 = \int_0^\infty e^{-ct}\bar{A}(t)\bar{B}(t)dt\,;\quad \mu_1 = \int_0^\infty \bar{G}_5(t)e^{-ct}dt\,;\quad \mu_2 = \int_0^\infty \bar{G}_1(t)e^{-ct}dt\,,\quad \mu_3 = \int_0^\infty \bar{G}_2(t)e^{-ct}dt\,;$$

$$\mu_4 = \int_0^\infty e^{-ct}G_4(t)dt\,;\quad \mu_5 = \int_0^\infty \bar{H}(t)e^{-ct}dt\,;\quad \mu_6 = \int_0^\infty \bar{G}_3(t)dt = 1\,;\quad \mu_7 = \int_0^\infty \bar{G}_6(t)e^{-ct}dt$$

$$\mu_8 = \int_0^\infty \bar{G}_5(t)e^{-\lambda t}dt\,;\quad \mu_9 = \int_0^\infty \bar{G}_1(t)\bar{H}e^{-\lambda t}dt$$

(3.26-3.35)

4 Mean Time to System Failure

Time to system failure can be regarded as first passage time to the failed state. To obtain it, we regard the down states as absorbing. Using arguments as for the regenerative process we obtain the following recursive relations for $\pi_j(t)$:

$$\pi_0(t) = \sum_{j=1,2,3,4} Q_{0j}(t) \boxed{s}\; \pi_j(t)\,;\;\; \pi_1(t) = Q_{15}(t) \boxed{s}\; \pi_{15}(t) + Q_{16}(t)\,;$$

$$\pi_2(t) = \sum_{j=0,7,8} Q_{2j}(t) \boxed{s}\; \pi_j(t) + Q_{26}\,;\quad \pi_3(t) = Q_{35}(t) \boxed{s}\, \pi_5(t) + Q_{36}(t)$$

$$\pi_4(t) = Q_{45}(t) \boxed{s}\, \pi_5(t) + Q_{46}(t)\,;\qquad \pi_5(t) = Q_{50}(t) \boxed{s}\, \pi_0(t) + Q_{56}(t)$$

$$\pi_7(t) = Q_{75}(t) \boxed{s} \pi_5(t) + Q_{76}(t); \qquad \pi_8(t) = Q_{89}(t) \boxed{s} \pi_9(t) + Q_{86}(t)$$

$$\pi_9(t) = Q_{90v10}(t) \boxed{s} \pi_0(t) + Q_{90v11}(t) \boxed{s} \pi_0(t) + Q_{97v10}(t) \boxed{s} \pi_7(t) + Q_{96}(t) + Q_{96v10}(t) + Q_{96v11}(t)$$

(4.1-4.9)

Taking Laplace-Stieltjes transforms of equations (4.1-4.9),the solution for $\tilde{\pi}_0(0)$, when the system starts from S_0, can be written in the following form by using the relation (3.1-3.25) & (3.26-3.35) of section 3:

$$E(T) = -\frac{d}{ds}\tilde{\pi}_0(s) \,|\, s = 0 = \frac{D'(0) - N'(0)}{D_1(0)} \tag{4.10}$$

$$E(T) = \frac{\mu_0 a_0 + \mu_1 a_1 + \mu_2 a_2 + \mu_3 a_3 + \mu_4 a_4 + \mu_5 a_5 + \mu_7 a_7 + \mu_8 a_8 + n_9 a_9}{1 - p_{02}p_{20} - p_{01}p_{15}p_{50} - p_{03}p_{35}p_{50} - p_{04}p_{45}p_{50} - p_{02}p_{50}B_{97}p_{89}p_{28}p_{75} - p_{50}p_{02}p_{75}p_{27} - B_{90}p_{89}p_{02}p_{28}}$$

[4.11]

$$a_0 = b_0 ; a_1 = p_{01}b_0 ; a_2 = p_{02}b_0 ; a_3 = p_{03}b_0 ; a_4 = p_{04}b_0$$
$$a_5 = -p_{01}p_{15} - p_{04}p_{45} + p_{27}p_{75} - p_{01}p_{16}p_{65} - p_{02}p_{26}p_{65} - p_{03}p_{36}p_{65} - p_{04}p_{46}p_{65}$$
$$- p_{02}p_{28}p_{86}p_{65} + p_{02}p_{76}p_{65} - B_{97}p_{75}p_{85}p_{28}p_{02} - p_{65}B_{96}p_{89}p_{02}p_{28}]$$
$$a_7 = -p_{02}p_{27}b_0 - p_{02}p_{28}p_{89}B_{97}b_0 ; a_8 = p_{02}p_{28}b_0 ; a_9 = p_{89}p_{02}p_{28}b_0$$
$$B_{90} = p_{90\vartheta11} + p_{90\vartheta10} ; B_{96} = p_{96} + p_{96\vartheta11} + p_{96\vartheta11} ; b_0 = 1 - p_{28}p_{89}p_{92}$$

5 Availability Analysis

Let $M_i(t)$ be the probability that the system is up initially in regenerative state S_i, is up at time t then by probabilistic arguments, we have:

$$M_0(t) = e^{-(\lambda+\beta)t}\,\bar{A}(t)\,\bar{B}(t) \; ; \; M_1(t) = \bar{G}_5(t)\,e^{-(\lambda+\beta)t} \; ; M_3(t) = \bar{G}_2(t)\,e^{-(\lambda+\beta)t}$$
$$M_2(t) = \bar{G}_1(t)\,e^{-(\lambda+\beta)t} ; \; M_4(t) = \bar{G}_4(t)\,e^{-(\lambda+\beta)t} \; ; M_5(t) = \bar{H}(t)\,e^{-(\lambda+\beta)t}$$
$$M_7(t) = \bar{G}_6(t)\,e^{-(\lambda+\beta)t} ; M_8(t) = \bar{G}_5(t)\,e^{-\lambda t} ;$$

$$M_9(t) = \bar{H}(t)\bar{G}_1(t)\,e^{-\lambda t} + \int_{u=0}^{t} h(u)\,e^{-\lambda u}\bar{G}_1(t)\,e^{-\lambda(t-u)}e^{-\beta(t-u)}\,du$$
$$+ \int_{u=0}^{t} g_1(u)\,e^{-\lambda u}\,\bar{H}(t)\,e^{-(\lambda+\beta)\,(t-u)}\,du$$

(5.1-5.9)

Recursive relations giving the point wise availability $A_i(t)$ are

$$A_0(t) = M_0(t) + \sum_{j=1,2,3,4} q_{0j}(t) \boxed{c} A_j(t) \quad ; \quad A_1(t) = M_1(t) + \sum_{j=5,6} q_{1j}(t) \boxed{c} A_j(t)$$

$$A_2(t) = M_2(t) + \sum_{j=0,6,7,8} q_{2j}(t) \boxed{c} A_j(t) \quad ; \quad A_3(t) = M_3(t) + q_{35}(t) \boxed{c} A_5(t)$$

$$A_4(t) = M_4(t) + q_{45}(t) \boxed{c} A_5(t) \quad ; A_5(t) = M_5(t) + \sum_{j=0,6} q_{5j}(t) \boxed{c} A_j(t);$$

$$A_6(t) = q_{65}(t) \boxed{c} A_5(t) \quad ; \quad A_7(t) = M_7(t) + \sum_{j=5,6} q_{7j}(t) \boxed{c} A_j(t)$$

$$A_8(t) = M_8(t) + \sum_{j=6,9} q_{8j}(t) \boxed{c} A_j(t)$$

$$A_9(t) = M_9(t) + q_{90}(t) \boxed{c} A_0(t) + q_{96}(t) \boxed{c} A_6(t) + q_{97}(t) \boxed{c} A_7(t)$$

(5.10-5.19)

Taking Laplace transform of the above equations and solving it for $A_0^*(s)$ the steady state availability of the system when the system starts from $S_i \in E$ is obtained as follows:

$$A_0(\infty) = \lim_{s \to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)} \qquad (5.20)$$

where

$$N_2(0) = \mu_0 a_0 + \mu_1 a_1 + \mu_2 a_2 + \mu_3 a_3 + \mu_4 a_4 + \mu_5 a_5 + \mu_7 a_7 + \mu_8 a_8 + M_9^*(0) a_9 \qquad (5.21)$$

$$\begin{aligned} D_2'(0) = {} & \mu_0 c_0 + \mu_1 p_{01} c_0 + \mu_2 p_{02} c_0 + \mu_3 p_{03} c_0 + \mu_4 p_{04} c_0 - \mu_5 [-p_{01} p_{15} - p_{04} p_{45} - p_{03} p_{35} \\ & + p_{02} p_{27} p_{75} - p_{01} p_{16} p_{65} - p_{02} p_{26} p_{65} - p_{03} p_{36} p_{65} - p_{04} p_{46} p_{65} - p_{02} p_{28} p_{86} p_{65} \\ & + p_{76} p_{65} - B_{97} p_{75} p_{89} p_{28} p_{02} - p_{65} B_{96} p_{89} p_{02} p_{28}] - \mu_6 [p_{04} p_{46} + p_{01} p_{16} + p_{02} p_{26} \\ & + p_{02} p_{28} p_{86} + p_{02} p_{28} p_{89} B_{96} + p_{02} p_{27} p_{76} + p_{02} p_{28} p_{76} B_{97} p_{89} + p_{03} p_{36} + p_{04} p_{56} p_{45} \\ & + p_{01} p_{56} p_{15} + p_{03} p_{56} p_{35} + p_{02} p_{27} p_{56} p_{75} + p_{02} p_{28} p_{56} p_{75} p_{89} B_{97}] + \mu_7 [-p_{02} p_{27} c_0 \\ & - p_{02} p_{28} p_{89} B_{97} c_0 + \mu_8 p_{02} p_{28} c_0 + n_9 p_{89} p_{02} p_{28} c_0 \end{aligned}$$

(5.22)

where: $c_0 = (1 - p_{56}\, p_{65})$

6. Busy Period Analysis

(a) Expected Busy Period Analysis of the Repairman in Simple Repair in (0,t]

After developing the recurrence relation for $B_i^1(t)$, and solving it, in the long run, the fraction of time for which the system is under simple repair is given by :

$$B_0^1(\infty) = \lim_{t\to 0} B_0^1(t) = \lim_{s\to 0} B_0^{1*}(s) = \frac{N_3(0)}{D_2'(0)} \quad (6.1)$$

$$N_3(0) = \mu_1[p_{01} + p_{03}b_{31}(1 - p_{24}p_{42}) + p_{02}p_{24}p_{41}] + \mu_2[p_{01}p_{12} + p_{02} + p_{03}b_{31}p_{12}] + \mu_4[p_{01}p_{12}p_{24} + p_{02}p_{24} + p_{03}b_{31}p_{24}p_{12}] \quad (6.2)$$

(b) Expected Busy Period Analysis of the Repairman in Cold Repair in (0,t]

Similarly, after developing the recurrence relation for $B_i^1(t)$, and solving it in the long run, the fraction of time for which the system is under cold repair is given by:

$$B_0^2(\infty) = \lim_{t\to 0} B_0^2(t) = \lim_{s\to 0} B_0^{2*}(s) = \frac{N_4(0)}{D_2'(0)} \quad (6.3)$$

$$N_4(0) = \mu_3 p_{03} c_0 \quad (6.4)$$

(c) Expected Busy Period Analysis of the Repairman in Capital Repair in (0,t]

Similarly, in the long run, the fraction of time for which the system is under capital repair is given by :

$$B_0^3(\infty) = \lim_{t\to 0} B_0^3(t) = \lim_{s\to 0} B_0^{3*}(s) = \frac{N_5(0)}{D_2'(0)} \quad (6.5)$$

$$N_5(0) = \mu_5[p_{04}\ p_{46} + p_{03}b_{31}(1 - p_{24}p_{42}) + p_{02}p_{24}p_{41}] + \mu_2[p_{01}p_{12} + p_{02} + p_{03}b_{31}p_{12}] + \mu_4[p_{01}p_{12}p_{24} + p_{02}p_{24} + p_{03}b_{31}p_{24}p_{12}] \quad (6.6)$$

(d) Expected Busy Period Analysis of the Repairman in Shut Down in (0,t]

Similarly, in the long run, the fraction of time for which the system is under shut down repair of the repairman is given by :

$$B_0^4(\infty) = \lim_{t\to 0} B_0^4(t) = \lim_{s\to 0} B_0^{4*}(s) = \frac{N_6(0)}{D_2'(0)} \quad (6.7)$$

$$N_6(0) = \mu_6[p_{04}p_{46} + p_{01}p_{16} + p_{02}p_{26} + p_{02}p_{28}p_{86} + p_{02}p_{28}p_{89}p_{96} + p_{02}p_{27}p_{76} + p_{02}p_{28}p_{89}B_{97}p_{76} + p_{03}p_{36} + p_{04}p_{45}p_{56} + p_{01}p_{15}p_{56} + p_{03}p_{35}p_{56} + p_{02}p_{27}p_{75}p_{56} + p_{02}p_{28}p_{56}p_{75}p_{89}B_{97}] \quad (6.8)$$

(e) Expected Busy Period Analysis of the Repairman in Preparation in (0,t]

Similarly, in the long run, the fraction of time for which the system is under preparation of the repairman is given by:

$$B_0^5(\infty) = \lim_{t\to 0} B_0^5(t) = \lim_{s\to 0} B_0^{5*}(s) = \frac{N_7(0)}{D_2'(0)} \quad (6.9)$$

$$N_7(0) = \mu_5 a_5 + \mu_7 a_7 \quad (6.10)$$

7 Particular Cases

(1) When all the repair time distributions including shut-down and capital repair time distributions along with preparation time distribution are n-phase Erlang distributed i.e.

$g_i(t) = nr_i(nr_it)^{n-1}e^{-nr_it} / (n-1)!\ ,i = 1,2\ ;\ a_i(t) = na_i(na_it)^{n-1}e^{-na_it} / (n-1)!\ ,i = 1,2$

$b_i(t) = nb_i(nb_it)^{n-1}e^{-nb_it} / (n-1)!\ ,i = 1,2$

and other time distributions to be negative exponential.

Then steady state equations become:

$MTSF = K_0 / K_1$; $AV = K_{01} / K_2$; $BP^1 = K_{02}^1 / K_2$; $BP^2 = K_{02}^2 / K_2$; $BP^3 = K_{02}^3 / K_2$

where :

$K_0 = e_0 + e_1 + e_2 + e_4 + e_6$; $K_1 = e_{01}$

$K_{01} = e_0 + e_1 + e_2 + e_3 + e_4 + e_6$;

$K_{02}^1 = e_1 + e_2 + e_4 + e_6$

$K_{02}^2 = e_3 + e_6$

$K_{02}^3 = e_5$; $K_2 = e_0 + e_1 + e_2 + e_3 + e_4 + e_5 + e_6$

$$e_0 = a_0(1-\eta_1 a_8 a_{13}\}\{(1-\eta_1 a_6 a_{10})-\eta_3 a_4 \eta_1 a_6 a_9\}$$

$$e_{01} = [1-(\eta_1 a_0(a_3+\eta_3 a_4 a_5)-\eta_2 a_0 a_5-\eta_1 a_6(a_{10}+\eta_3 a_4 a_9)+\eta_1 a_6 \eta_1 a_0 a_3 a_{10}$$
$$\eta_1 a_6 \eta_2 a_0 a_3 a_9 - a_1 a_7(1-\eta_1 a_6 a_{10})+a_1 a_7 \eta_3 a_4 \eta_1 a_4 a_9 - \eta_1 a_8 a_{13}+\eta_1 a_0 \eta_1 a_8 a_{13}$$
$$(a_3+\eta_3 a_4 a_5)+\eta_2 a_0 \eta_1 a_8 a_{13} a_5 - a_3 a_1 \eta_1 a_8 a_{12} - \eta_3 a_4 a_1 \eta_1 a_8 a_5 a_{12}+\eta_1 a_6 \eta_1 a_8$$
$$a_{13}\ (a_{10}+\eta_3 a_4 a_9)-\eta_1 a_6 \eta_1 a_8 a_{13} \eta_1 a_0 a_3 a_{10}+\eta_2 a_0 \eta_1 a_6 \eta_1 a_8 a_{13} a_3 a_9 +$$
$$a_1 \eta_1 a_6 \eta_3 a_4 a_9 a_9 +\ a_1 \eta_1 a_6 \eta_1 a_8 a_3 a_{10} a_{12}]$$

$$e_1 = a_4[(1-\eta_1 a_8 a_{13})\{\eta_1 a_0(1-\eta_1 a_6 a_{10})+\eta_2 a_0 \eta_1 a_6 a_9\}+a_1 \eta_1 a_8 a_{12}(1-\eta_1 a_6 a_{10})]$$

$$e_2 = a_6[(1-\eta_1 a_8 a_{13})(\eta_2 a_0+\eta_1 a_0 \eta_3 a_4)+\eta_2 a_0 a_1 \eta_1 a_8 a_{12}]$$

$$e_3 = a_8[a_1(1-\eta_1 a_6 a_{10}-\eta_3 a_4 \eta_1 a_6 a_9)]$$

$$e_4 = a_{19}[(1-\eta_1 a_8 a_{13})(\eta_2 a_0+\eta_1 a_0 \eta_3 a_4)+a_1 \eta_1 a_8 a_{12} \eta_3 a_4]$$

$$e_5 = a_{20}[\eta_1 a_8\{a_{13}(\eta_2 a_0 \eta_1 a_6(a_{11}-1)+\eta_1 a_0 \eta_3 a_4(\eta_1 a_6 a_{11}-\eta_3 a_6))+a_1 \eta_3 a_{14}$$
$$\{[1-\eta_1 a_6 a_{10}+\eta_1 a_6 a_9 \eta_3 a_4)-\eta_3 a_4(\eta_1 a_6 a_{11} a_{12}+\eta_3 a_6 a_{12})\}-\eta_2 a_0 \eta_1 a_6(a_{11}+1)$$
$$+\eta_1 a_0 \eta_3 a_4(\eta_3 a_6+a_{11})+a_1 \eta_2 a_8[1-\eta_3 a_6(a_{10}+\eta_3 a_4 a_9)]$$

$$e_6 = a_1 \eta_1 a_8[1-\eta_1 a_6(a_{10}+\eta_3 a_4 a_9)]$$

where :

$$p_{01} = \eta_1 a_0\ ;\ p_{02} = \eta_2 a_0\ ;\ p_{03} = a_1\ ;\ a_0 = \sum_{j=0}^{n-1}(na)^j/(\eta_1+\eta_2+na)^{j+1}$$

$$a_1 = (na/(\eta_1+\eta_2+na)^n\,;p_{10} = (nr_1/(nr_1+\eta_3)^n = a_3\,;p_{12} = \eta_3 a_4$$

$$a_4 = \sum_{j=0}^{n-1}(nr_1)^j/(\eta_3+nr_1)^{j+1}\ ;\ p_{20} = (nr_2/nr_2+\eta_1+\eta_4)^n = a_5\ ;\ p_{24} = \eta_1 a_6$$

$$p_{25} = \eta_3 a_6\quad ;\ a_6 = \sum_{j=0}^{n-1}(nr_2)^j/(\eta_1+\eta_3+nr_2)^{j+1};\ p_{30} = (nb/nb+\eta_1+\eta_2)^n = a_7$$

$$p_{35} = \eta_2 a_8\quad ;\qquad p_{36} = \eta_1 a_8\quad ;\ a_8 = \sum_{j=0}^{n-1}(nb)^j/(\eta_1+\eta_2+nb)^{j+1}$$

$$p_{41} = [\sum_{j=0}^{n-1}(nr_2)^n(nr_1)^i/(n-1)!\ i!\ (\eta_3+nr_1+nr_2)^{n+i}](n+i-1)! = a_9$$

$$p_{42} = [\sum_{j=0}^{n-1}(nr_1)^n(nr_2)^i/(n-1)!\ i!\ (\eta_3+nr_1+nr_2)^{n+i}](n+i-1)! = a_{10}$$

$$p_{45} = \eta_3 \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (nr_1)^n (nr_2)^i \tau(i+j+1)/(i)!(j)!\ (\eta_3 + nr_1 + nr_2)^{i+j+1} = a_{11}$$

$$p_{61} = \sum_{i=0}^{n-1} (nb)^n (nr_1)^i (n+i-1)!/(n-1)!(i)!\ (\eta_3 + nb + nr_1)^{n+i} = a_{12}$$

$$p_{63} = \sum_{j=0}^{n-1} (nr_1)^n (nb)^j (n+j-1)!/(n-1)!(j)!\ (\eta_3 + nr_1 + nb)^{n+j} = a_{13}$$

$$p_{65} = \eta_3 \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (nr_1)^i (nb)^j \tau(i+j+1)/(i)!(j)!\ (\eta_3 + nr_1 + nb)^{i+j+1} = \eta_3 a_{14}$$

$$p_{50} = 1$$

$$\mu_4 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (nr_1)^i (nr_2)^j \tau(i+j+1)/(i)!(j)!\ (\eta_3 + nr_1 + nr_2)^{i+j+1} = a_{19}$$

$$\mu_5 = \sum_{i=0}^{n-1} (nr_3)^i / (nr_3)^{i+1} = a_{20} \quad ; \quad \mu_6 = \sum_{j=0}^{n-1} (nb)^j \tau(j+1)/(j)!\ (\eta_1 + \eta_2 + nb)^{i+j+1} = a_{11}$$

(2) When $n = 1$,all the repair times follow negative exponential distribution i.e.

$$e_3 = (1/X_4)[(a/X_1)\{1 - (\eta_1/X_3)(r_1/X_5) - (\eta_3/X_2)(\eta_1/X_3)(r_2/X_5)\}]$$
$$e_4 = (1/X_5)[(\eta_1/X_3)\{1 - (\eta_1/X_4)(r_1/X_6)\}\{(\eta_2/X_1) + (\eta_3/X_2)(\eta_1/X_1)\} + (\eta_1/X_4)(a/X_1)(\eta_3/X_2)(b/X_6)]$$

where :

$$X_1 = \eta_1 + \eta_2 + a \ ;\ X_2 = r_1 + \eta_3 \ ;\ X_3 = r_2 + \eta_1 + \eta_3 \ ;\ X_4 = \eta_1 + \eta_2 + b$$
$$X_5 = r_1 + r_2 + \eta_3 \ ;\ X_6 = r_1 + \eta_3 + b$$

8 Cost-Benefit Analysis

The cost benefit analysis of the system can be carried out by considering the expected busy period of the repairman in repair in (0,t] . Therefore,

G(t) = expected revenue earned by the system in (0,t] - expected repair cost of the repair facility in (0,t]

$$= C_1\, \mu_{UP}(t) - C_2 \mu_b^1(t) - C_3 \mu_b^2(t) - C_4 \mu_b^3(t) - C_5 \mu_b^4(t) - C_6 \mu_b^5(t)$$

$\mu_{up}(t)=\int_0^t A_0(t)\,dt$; $\mu_b^1(t)=\int_0^t B_0^1(t)\,dt$; $\mu_b^2(t)=\int_0^t B_0^2(t)\,dt$; $\mu_b^3(t)=\int_0^t B_0^3(t)\,dt$;

$\mu_b^4(t)=\int_0^t B_0^4(t)\,dt$; $\mu_b^5(t)=\int_0^t B_0^5(t)\,dt$

The expected profit per unit of time in steady state is

$$G=\lim_{t\to\infty}\frac{G(t)}{t}=\lim_{s\to 0} s^2 G^*(s) = = C_1\mu_{UP}(t)-C_2\mu_b^1(t)-C_3\mu_b^2(t)-C_4\mu_b^3(t)-C_5\mu_b^4(t)-C_6\mu_b^5(t)$$

where C_1 is the revenue per unit up time and C_2, C_3, C_4 *and* C_5 are the simple repair cost, Cold repair cost, capital repair cost, shut-down repair cost and preparation cost, respectively
.

9 Graphical Representation

Figure 1.2 shows the behavior of the mean- time-to-system-failure of the **Mixer-crane system** with respect to β for varying values of λ. From the graph it can be observed that MTSF of the machine increases as failure rate λ decreases. Same thing happens in case of availability and expected cost or profit as shown in Figures 1.3 and 1.4.

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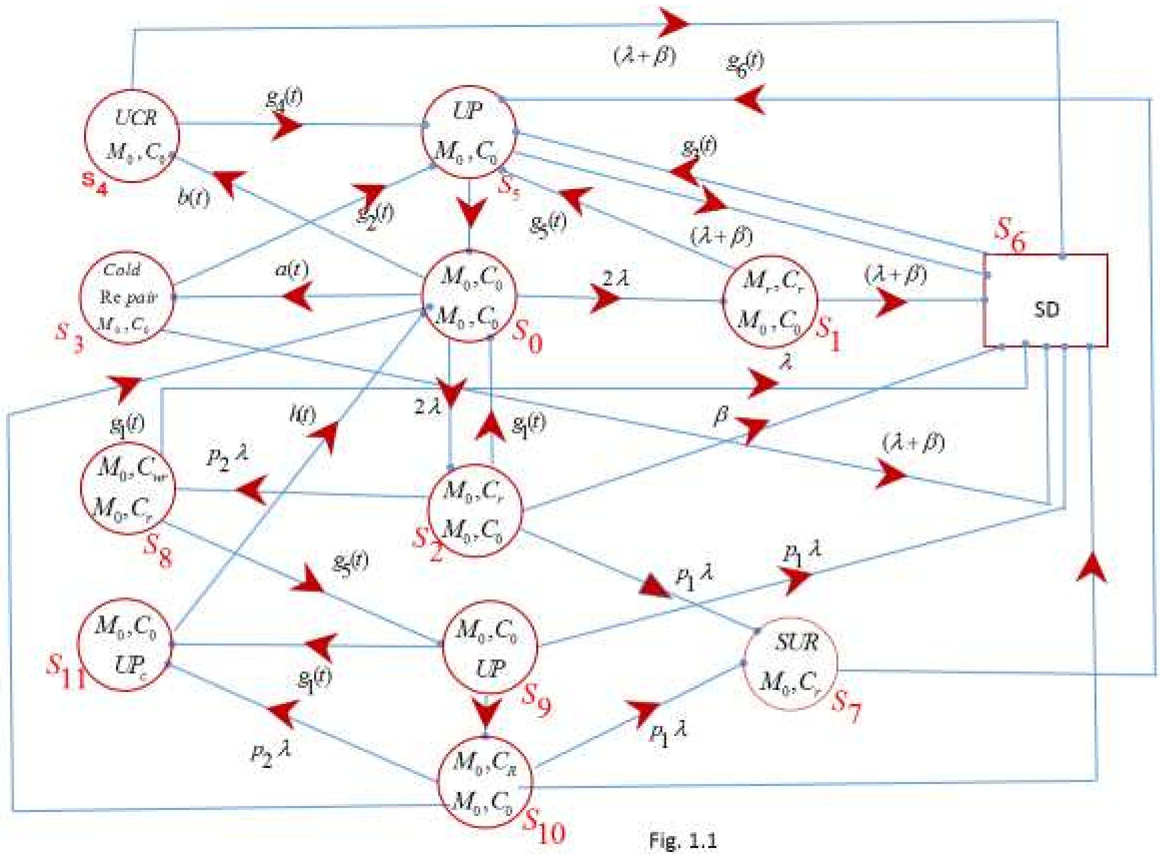

Fig. 1.1

STATE TRANSITION DIAGRAM OF MIXER-CRANE SYSTEM

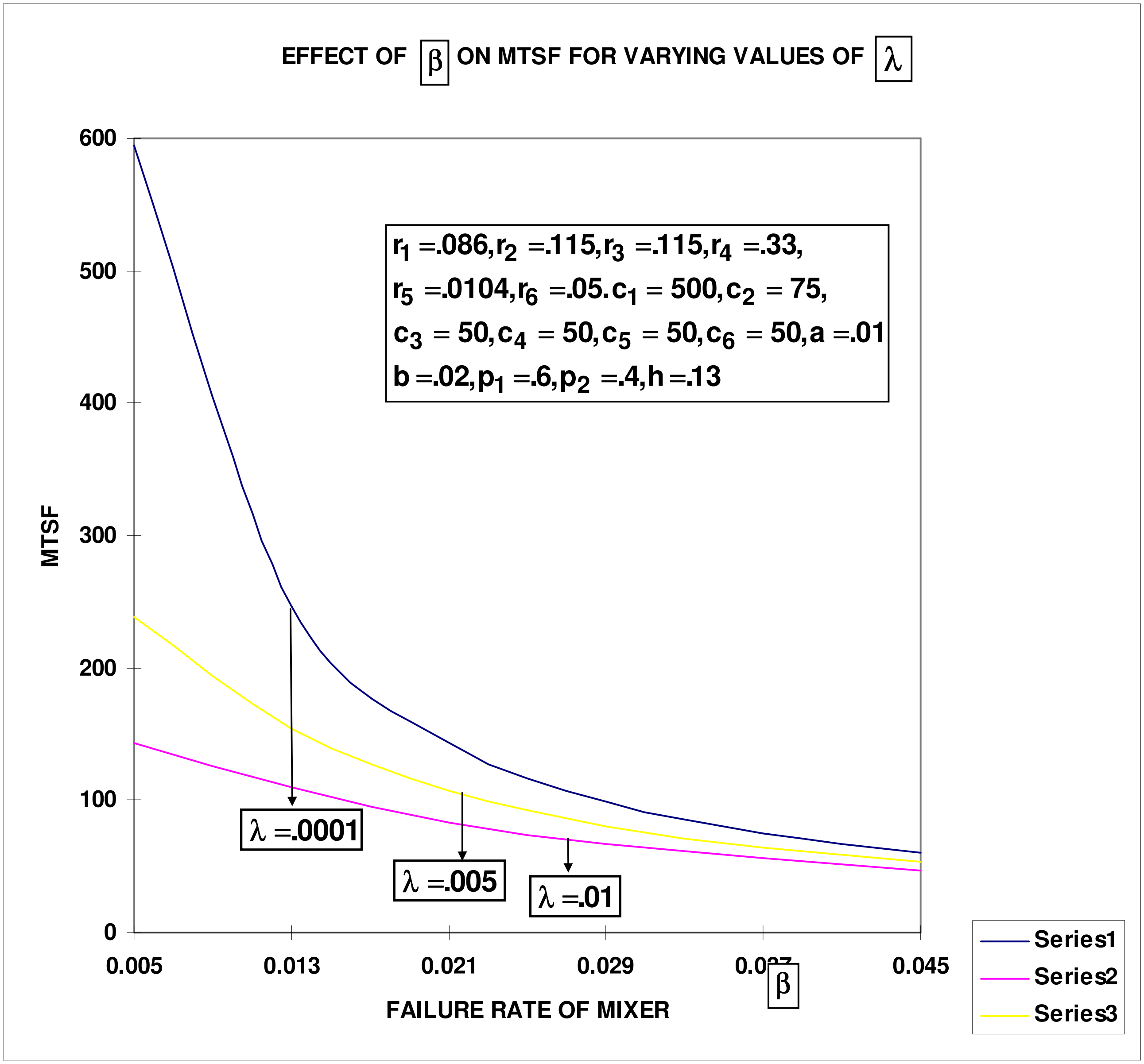
EFFECT OF β ON MTSF FOR VARYING VALUES OF λ
r1 =.086,r2 =.115,r3 =.115,r4 =.33,
r5 =.0104,r6 =.05.c1 = 500,c2 = 75,
c3 = 50,c4 = 50,c5 = 50,c6 = 50,a =.01
b =.02,p1 =.6,p2 =.4,h =.13
λ =.0001
λ =.005
λ =.01
MTSF
600
500
400
300
200
100
0
0.005
0.013
0.021
0.029
0.045
β
FAILURE RATE OF MIXER
Series1
Series2
Series3

FIGURE 1.2

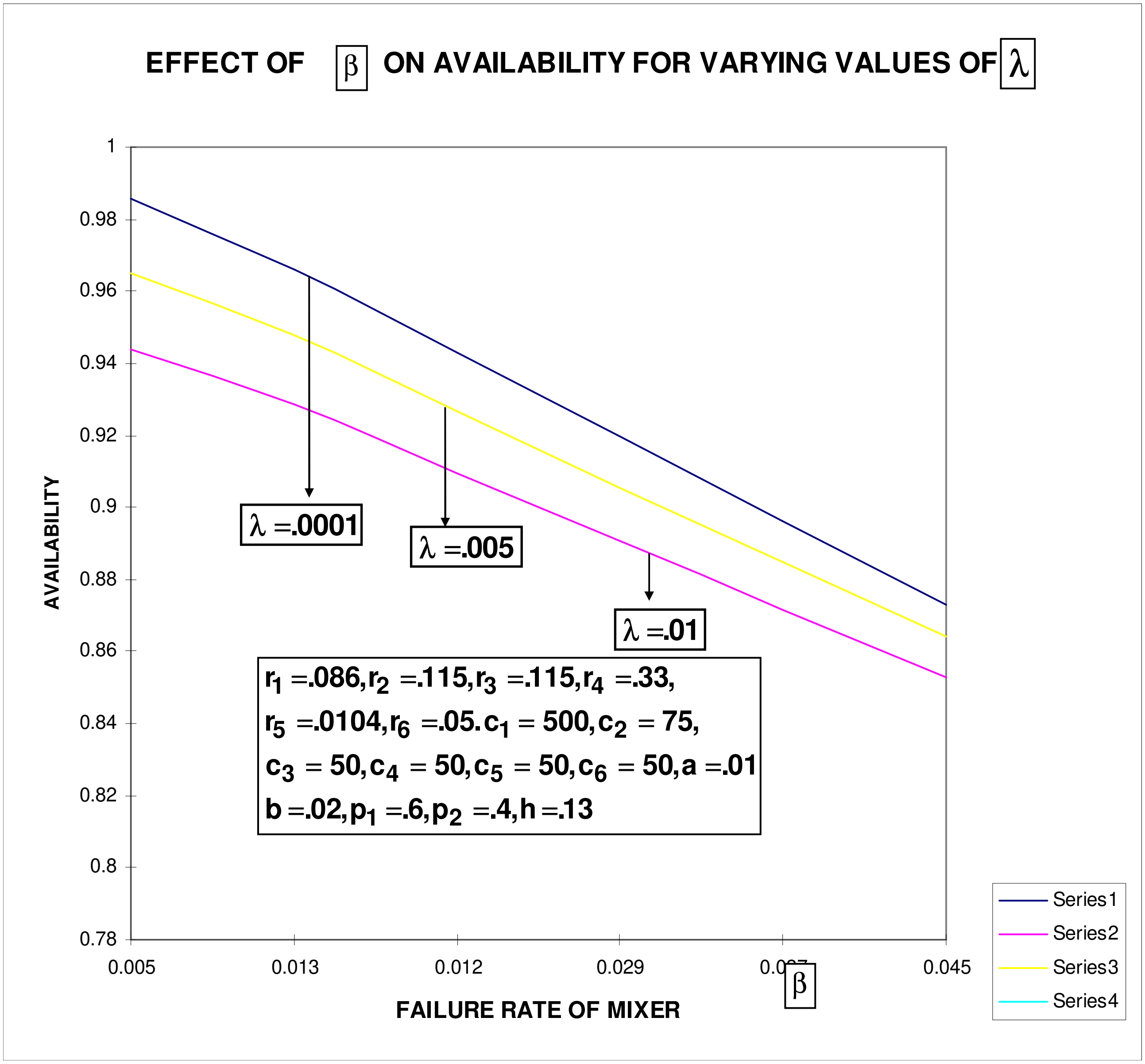
EFFECT OF β ON AVAILABILITY FOR VARYING VALUES OF λ
AVAILABILITY
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
0.78
0.005
0.013
0.012
0.029
0.045
β
FAILURE RATE OF MIXER
λ =.0001
λ =.005
λ =.01
r1 =.086, r2 =.115, r3 =.115, r4 =.33,
r5 =.0104, r6 =.05. c1 = 500, c2 = 75,
c3 = 50, c4 = 50, c5 = 50, c6 = 50, a =.01
b =.02, p1 =.6, p2 =.4, h =.13
Series1
Series2
Series3
Series4

FIGURE 1.3

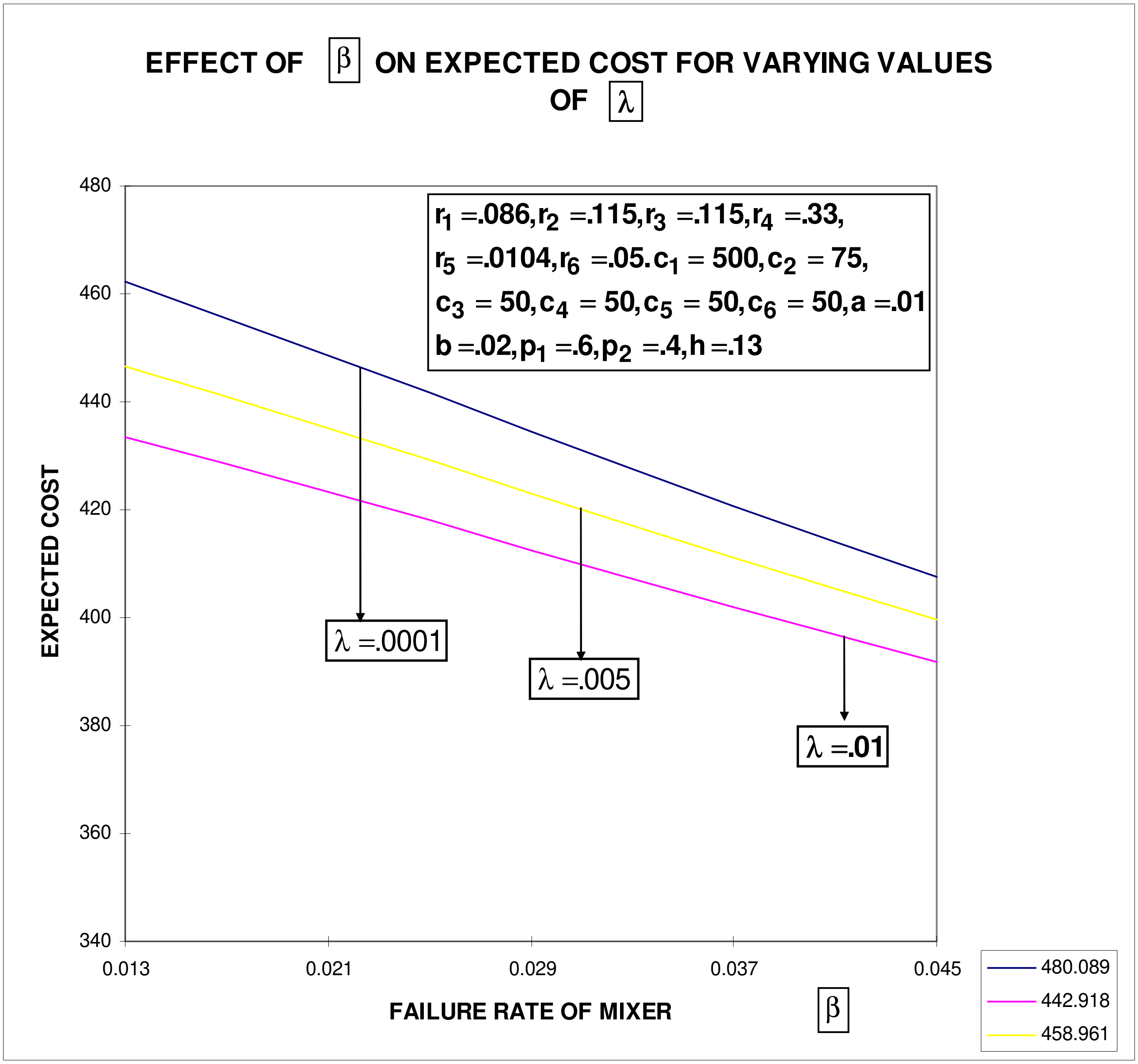
EFFECT OF β ON EXPECTED COST FOR VARYING VALUES OF λ
$r_1 = .086, r_2 = .115, r_3 = .115, r_4 = .33,$
$r_5 = .0104, r_6 = .05. c_1 = 500, c_2 = 75,$
$c_3 = 50, c_4 = 50, c_5 = 50, c_6 = 50, a = .01$
$b = .02, p_1 = .6, p_2 = .4, h = .13$
λ =.0001
λ =.005
λ =.01
EXPECTED COST
480
460
440
420
400
380
360
340
0.013
0.021
0.029
0.037
0.045
FAILURE RATE OF MIXER β
480.089
442.918
458.961

FIGURE 1.4